

# **TRANSIENTS IN ELECTRIC POWER SUPPLY SYSTEMS**



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**G.G. Pivnyak, I.V. Zhezhelenko, Y.A. Papaika**



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Trans Tech Publications Inc.  
PO Box 699, May Street  
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USA

Phone: +1 (603) 632-7377  
Fax: +1 (603) 632-5611  
e-mail: [sales-usa@scientific.net](mailto:sales-usa@scientific.net)

# PREFACE

The need to improve electric supply systems is stipulated by development of industry, high technologies, transport, civil engineering and the agroindustrial complex. These enterprises are the main consumers of electric power in complex infrastructure of industrial centers. They consume electric power with different frequencies, voltages and currents. They are also characterized by high density of electric loads. Their electric supply systems have multilevel stages of power distribution, large-scale load centers with different transformations of electromagnetic energy parameters, power consumers of various types, power supply from power system and local thermal power stations, synchronous compensators, sources of reactive power. In emergencies power is supplied by motors that pass into generator mode. The distribution networks have significant degree of branching. Power supply systems are characterized by availability of reverse effect of electric technological processes and powerful consumers on the system performance in emergencies.

Construction and maintenance of the electric power supply systems involves considerable material resources. That is why it is very important to improve their efficiency and reliability in different operating conditions, including malfunctions and post-emergency modes. On these reasons to the power supply systems, their operation conditions power quality, which may be assessed on the basis of transients investigation, increased requirements are imposed.

In Western developed economies sufficient attention is given to development of methods of investigation and evaluations of transients which are intended to creation of power supply systems of new technological level, providing stability of their operational modes at the needed level of cost effectiveness, power quality, reliability and operational safety. Use of simulation methods and computing facilities greatly contributes to solving tasks arising at analysis and evaluations of transients. That allows to choose the most appropriate circuit designs and electric characteristics of the system components, and also to achieve the highest indicators of their performance economic efficiency and reliability not only in normal, but also in transient modes.

Researches in this area were actively carried out in Universities and in National Academy of Sciences of Ukraine. Widely known technical decisions and means providing reliability and cost effectiveness of modern power supply systems of powerful enterprises were elaborated.

Transients in electric systems are widely elucidated in the technical and educational literature, but a generalizing textbook on transients in electric power supply systems for students of higher educational institutions was absent for a long time. To fill a gap the team of authors from National Technical University of Ukraine “Kyiv Polytechnic Institute” and the State Higher Educational Institution “National Mining University” prepared the textbook for higher educational institutions “Transients in Electric Power supply Systems” printed in limited edition («Вища школа», 1989). Over time, there were several editions of this textbook [44-46] in Ukrainian, Russian and English. Idea of the textbook belongs to Professors V.M. Vinoslavskiy (NTUU “Kyiv Polytechnic Institute”) and G.G. Pivnyak (SHEI “National Mining University”). Considerable contribution at preparation and editing the first edition of the textbook [44] was made by Professor V.M. Vinoslavskiy. Professor of the National Mining University A. Y. Rybalka elaborated sections of the book devoted to calculations of transients [45-46]. Authors of the textbook were awarded with the State Prize of Ukraine in the Field of Science and Engineering for 2005.

Educational achievements of the two leading Universities have been kept and developed in further research of teaching staff and the followers. Experience of the world science, developments in modern electric systems and networks, growth of enterprises and high-tech technologies power supply system capacity, changes in approach to analysis and calculation of transients and operation modes of the power supply systems, wide application of valve inverters, increasing need solving problems of electromagnetic compatibility hut a new task of systematization and generalization of this material taking into account the results of fulfilled research projects. Fruitful cooperation with Prof. I.V. Zhezhelenko, who is internationally renowned scientist in the field of electromagnetic

interferences generation and electromagnetic compatibility in industrial enterprises power supply systems, contributed to this task solving. So the proposal for reissue the textbook in Ukrainian and in English for students trained in specialty “Electric Power, Electrical Engineering and Electromechanics” arose. It is important to use the textbook fulfillment Masters’ graduation projects.

In preparation of the textbook new issue more attention to special features of power supply systems in comparison with power systems is paid. Emphasis is made on calculation of electromagnetic transients in distribution networks caused by high technologies. Considerable material to calculations of power supply systems load nodes stability at different operation conditions and different types of disturbances. Possible accuracy of fault currents calculations is determined. Calculations of electromagnetic transients with account of electromagnetic compatibility requirements have been carried out.

Feature of the textbook is availability in it both electromagnetic and electromechanical transients description. At this, authors tried to show more clearly the unity of electromagnetic and electromechanical processes at their analysis and in calculations of electric power supply systems stability. That contributes to self-study the book material by students, development of innovative thinking and skills of scientific research and calculations, and to making responsible decisions.

In the textbook the fundamental works of the scientists P.S. Zhdanov, S.O. Ulyanov, V.A. Vyenikov, I.A. Syromiatnikov, results of research and works by Academicians A.K. Shidlovskiy and G.G. Pivnyak and Professors D.O. Arzamastsev, V.M. Vinoslavskiy, I.V. Zhezhelenko, G.Y. Vagin, A. Y. Rybalka, et al. The recommendations and suggestions to authors by Academician A.K. Shidlovskiy, Professors D.O. Arzamastsev, O.Y. Lozinskiy, V.O. Grabko, Assistant Professor V.A. Ladenzon were taken into account.

The book consists of introduction and two parts. In the first part electromagnetic transients caused by shorts, longitudinal and lateral asymmetry, technological process and conditions of electromagnetic compatibility are considered. The part two is devoted to electromechanical transients and to combined influence of the electromagnetic and electromechanical transients on power supply system operation stability.

There are control questions and topics for essays at the end of the chapters for better comprehension of the given material. They will help students' self-dependent activities in learning the discipline.

Authors will gratefully accept remarks and readers’ opinion, which will assist to further improvement of this text-book.

*Authors*

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## INTRODUCTION

### 1. Short historical information

At the beginning of electric power use the operating conditions of electrical generators, motors and other components of power equipment were analyzed only by their ordinary operation. First power installations being of low-capacity had sufficient stability margin to mechanical, thermal and other types of influence in standard operation modes and under emergency.

When electric installations capacity increased their faults and significant deviations of mode parameters had serious sequences. It became necessary to develop specific measures and means to provide the operation of power equipment under emergency. As successful solving this problem depended on depth of understanding phenomena taking place in emergencies it was needed to elaborate appropriate calculation methods and introduce for transients and introduce protective means for power equipment against faults with account of transients, and solve the problem of operation conditions stability.

During the first period of power industry development, research of transients was at initial stage. Applied methods of transient calculation did not have appropriate theoretical background and were approximate on the results. It was especially felt while implementing GOELRO plan. There was the necessity for substantiation of large electric systems creation and solving the problems of their stable operation provision, refinement transients calculation methods in complex branched electric networks, development of methods for limiting short circuit currents and means for protection of electric installations and solving other problems of power industry development.

During short period of time a number of significant research works concerning transients had been carried out. From 1926 till 1930 L.I. Sirotinskiy, V.P. Khashchinskiy, M.M. Shchedrin and A.A. Smurov investigated transients in synchronous electric machines. For the first time in the world (1933-1934) books on electric systems stability had been published. In them the results of research were described and developments of world scientists were generalized.. A.A. Gorev and R. Park independently of each other composed differential equations of electromagnetic transient in a synchronous electric machine (1930-1935).

The pre-war years were the period of intensive development of scientific fundamentals, practical methods of transients research, and control of electric systems operation. More perfect methods for short circuit currents calculation in electric systems were specified and developed basing on research by M.M. Shchedrin, S.O. Ulyanov, A.B. Chernin, B.I. Rosenberg and other scientists. I.M. Markovich and S.A. Sovalov grounded the before proposed and introduced new practical criteria for evaluation of static stability of electric systems (1937-1938). At the same time P.S. Zhdanov and K.A. Smirnov discovered the nature of “voltage avalanche” and proposed methods of electric load stability analysis. Later S.A. Lebedev, I.A. Syromyatnikov and other scientists brought off theoretical and experimental investigations of synchronous machines automatic excitation control, which later on were widely used in electric systems (1938-1940).

To investigations of transients in electric installations, research and educational institutions (VEI, MEI, LPI, etc.) and also large production associations (Mosenergo, Lenenergo) were involved.

In wartime of 1941 -1945 when power systems of the western part of former USSR were destroyed, the power industry of the Urals and eastern regions started working intensively. That motivated intense continuation of research to increase reliability of these regions power systems having significant electric load and intensively working.

Basing on research results (1944) the “Guidelines for short circuit currents calculations and apparatus and conductors in high voltage devices selection on the basis of continuous short circuit mode “ were published which being supplemented, changed and revised are valid in the nowadays as well. Implementing on a mass scale means of line and anti-fault automation equipment - automatic regulators of generators excitation and frequency unload of electrical systems were developed.

After the war (during 1950-1955) automatic means of frequency unload, voltage control, automatic reclosing and reserve connection were being constantly improved by I.A. Syromyatnikov, L.G. Tsukernik, S.S. Rokotyan, D.I. Azar'ev, S.V. Usov and others and became obligatory for use in all electric systems.

Establishment of territorial joint power systems and forming of integrated power system of the former Soviet Union helped to carry out specific theoretical and experimental research concerning further implementation of anti-fault automation and providing operation modes stability of interconnected power systems being created. The necessity to standardize requirements for interconnected power systems operation modes stability arose. In 1964 the "The regulations and temporary guidelines to determine power systems stability" were published.

As power systems and their interconnections were developed, the solved problems became more complicated. In the 1960<sup>th</sup> design models for direct current and static models for alternative current were widely used. The use of continuous operating analog computers and electrodynamic (physical) models promoted further development of research in transients. Analog computers were used to study the effects of self-excitation, generators excitation automatic control, influence of synchronous machines parameters on transients etc. Physical models were used to research transients in complex power systems, principles of operation and optimization of relay protection tuning and anti-fault automation, power lines operation modes peculiarities etc.

In recent years digital computers are used in calculations and analysis of transients in electric installations as the primary means. Great attention is also paid to development and application of hybrid complexes which contain a physical model, analog components simulating adjusting devices and control computer. That makes possible automating control processes and application modern methods of processing and monitoring the investigation results.

The software elaborated in IED of NANU, IPE of NANU, institutes "Energomerezhproject", SibNDIE and others is spread for computer calculation of fault currents. In calculations and investigations of electromechanical transients, determination of fault currents and solving a number of other tasks, great attention is paid to replacement the electric power supply system with the equivalent system that simplifies its mathematical model in which the most important system properties needed for achievement the goal are saved.

Investigations related to the use of equivalent models at determination parameters and characteristics of separate system elements are carried out at SibNDIE, MEI, VNDIE, IED of NANU and other institutions. At this, great attention is paid to modeling of complex loads to take into account the system behavior under variation of operation modes parameters, electric consumer composition, power supply circuit etc.

## 2. Operation modes of electric power supply systems

Changes in operation conditions of electric power supply systems are accompanied with transients that cause changes in operation modes. Combination of processes which characterize operation conditions of electric power supply system and its state at any moment of time is called **the operation mode** of the system. Quantitative performances of operation mode (**operation mode parameters**) are values of the power, voltage, current and other quantities related to each other by dependences via appropriate parameters of the system.

**System parameters** include resistances and conductivity of elements, transformer ratios, time constants, gain factors and other parameters stipulated by physical properties, elements connection and design data.

Both steady states and transient (unsteady) conditions can take place in the system of electric power supply. The first ones are characterized by constant (slow) or insignificant variations of the system parameters, and the latter - with their fast variations in time.

Electric power systems operation modes are divided into four types depending on their parameters variation behavior:

- **standard steady operation** at which operation mode parameters vary within limits meeting normal operation of the customers and are stipulated by their main technical and economical performances data;
- **standard transient modes** which take place upon ordinary operation changes (turn on, turn off, switch, load change etc). Such operation is characterized with fast and sharp change of operation parameters of some electric power supply system elements under insignificant variations of mode parameters in its nodes;
- **emergency steady and transient operation** which arise under the influence of such changes of the circuits elements electric connections when values of operation mode parameters differ significantly from nominal ones;
- **after-emergency steady-state operation** which arise when the fault elements were turned off to eliminate emergency. Values of operation parameters in this case can be close to ordinary operation mode parameters or differ significantly. Accordingly either favorable or unfavorable recovery from emergency takes place.

The main task at keeping up the necessary operation of electric power supply system is to maintain the parameter values under which the stability of definite operation mode is provided. **Operation stability** is ability of a power supply system to keep up allowable values of operation mode parameters in the nodes under disturbance.

**The static stability of a power supply system operation mode** is the system ability to return into initial stable operation mode after parameters small deviations being in allowable limits.

**The dynamic stability of a power supply system operation** is the system ability to return into initial stable operation mode or close to it after sudden temporary sharp disturbance when values of parameters in the load nodes are in allowable limits. A kind of dynamic stability – **the resulting stability** – is ability of a supply system to restore synchronous operation after short-term allowable by operating conditions asynchronous operation of the sources generators providing assumable electric power quality. If operation mode parameters in the system nodes under after-emergency conditions do not differ significantly of the standard steady operation parameters, it is taken that dynamic stability of the system was not violated.

Violation of a power supply system dynamic stability may be caused by such a sharp disturbance as switching on or off one of its important elements (any of long-distance transmission lines, powerful motors etc.), that essentially changes operation conditions of other supply system elements.

In the case of static or dynamic stability of a power supply system violation, the voltage frequency can be reduced to not allowable for most of consumers value that leads to economic losses.

### 3. Causes of transients

Transients in electric power supply systems is the result of changes caused by operation conditions or insulation fault of electric installations current-carrying parts.

The transients can arise under following conditions:

- turn on, turn off, or switch of sources of electric power, transformers, transmission lines, using equipment and other elements;
- appearance of phase currents and voltages unbalance caused by swithinf off separate phases, occurrence of non-symmetrical load variation, a phase failure etc.;
- asymmetrical phase currents and voltages as a result of some load phases turn off, asymmetrical changes of phase loads, phase conductors breaks etc.;
- short circuits in elements of electric power supply system;
- synchronous machines excitation forcing or their field killing;
- sudden load surge and shedding;
- asynchronous start of motors and synchronous compensators;
- induction motors reversal;
- asynchronous running of synchronous machines after their drop-out;

- atmospheric and climatic influences on electric power supply system elements;
- short circuits reclosing on and turn off.

Transients caused by the system elements switching, tests and modes control belong to standard operation conditions. Short circuits, phase breaks, reclosing on and turn off the short circuits, generators fall out of step and other faults of ordinary operation modes belong to emergency situations.

Generally limiting values of power installations operation parameters under transients are taken into consideration while producing power equipment, under designing and constructing electric power supply systems and at operation modes substantiation.

In the textbook only the methods of tasks solution on determination of allowable operation modes under emergency and post-emergency situations are considered. Among them are study of electromagnetic transients (calculations of fault currents, open-phase operating conditions etc.), determination of static stability and stability margin, analysis of dynamic stability, investigation of long-continued processes caused by emergency violation the power balance, calculations of asynchronous operating conditions and others.

The emergencies arise both under stable faults of current-carrying parts insulation, and unstable ones.

Examples of unstable insulation faults in parts of power supply system:

- *in overhead transmission lines* - flashover of suspension insulators strings, wires closing under critical climatic conditions, closeness of tree branches to wires, and also foreign objects throwing on wires;

- *in cables* - insulation break-downs which are kept themselves aloof thanks to specific peculiarities of paper-oil insulation (the conditions which favor arc quenching in discharge gaps);

- *in switch gears* – objects throwing on current-carrying buses or surface discharge flashover under increased humidity or insulation gaps dirtying.

According to statistical data the number of unstable faults exceeds greatly those of stable ones. Thus, for overhead transmission lines which voltage is 110 to 500 kV stable faults are 16 per cent in average, and for cable lines which voltage is 6 to 10 kV - 11 to 20 per cent.

In cable transmission lines faults are accumulated gradually. With it 82 per cent result in insulation break-down, an 18 per cent in other faults.

To maintain serviceability under unstable faults within the majority of overhead transmission lines as well as some cable lines the availability of devices for their re-closing is provided which successful operation is from 45 to 90 per cent of turns off.

#### 4. The purpose of transients calculations

Transients in power supply systems are studied on the basis of knowledge of general fundamentals and special subjects in which elements of electric power supply system are considered. Transients in elements are studied with account of connections between them and current course of system operation.

The aim of transients study and evaluations is to have learnt the course of transients to foresee and to control them on the basis of operation peculiarities and qualitatively new characteristics at quantitative variations in the power supply system. For that it is necessary to be able to calculate transients, to make forecast of the quantitative changes in the system operation, using data of the parameters changes, and to affect the transient with regulating devices.

The goal of study and calculations transients is to find out features of operation conditions and new qualitative properties under quantitative variations in electric power supply systems, to learn to predict the course and probability of transients and to control them. To do that it is necessary to be able to calculate transients, forecast the system operation modes quantitative changes and exert influence the transient by means of regulating devices.

Transients study and calculations are necessary condition for solving many problems arising when electric power supply systems are designed and in the process of their operation. In particular, it is necessary for selection of operating principles and settings of automatic anti-emergency means,

analysis of electromechanical transients for determination conditions of electric loading stability and development measures providing continuous functioning of industrial enterprises under various operation modes of electric power supply systems.

On the basis of transients study and calculations a number of the most important problems concerning design, construction and maintenance power supply systems are solved:

- economy expedient ways for electric power transmission, distribution and consumption substantiation;
- ensuring appropriate operation mode after transients in the system;
- meeting the requirements concerning qualitative performances of transients;
- stability of transition while changing operation mode;
- operation mode stability evaluation after transients have finished;
- determination of transient duration and its influence the system operation mode parameters variation;
- electric apparatuses and electric power supply systems testing in a transient mode.

On the basis of the results of the studies and calculations it is advisable to design the electric power supply systems in which transients would finished by allowable steady-state operation mode. With it transients have to be considered from the point of view of:

- ✓ electric power supply system reliability;
- ✓ both system and its elements behavior after working conditions have been changed.

With account of transient, changes of operation mode parameters under which qualitative indicators of the power supply system consumers wouldn't be reduced essentially have to be provided. That's why transient duration reduction, new transients arise exception, transient finish with rather reliable operation mode play important role. While analyzing transients analytically the method of coordinate transformation, complex quantities for description instantaneous values of variables, method of symmetrical components, equivalent circuits of electric power supply systems for different operation modes and other techniques are used. Besides, grapho-analytical methods of transient representation are applied.

Methods of modeling and experimental investigations of actual electric power supply systems give large-scale opportunities to study and calculate transients.

### **Test questions**

1. Importance of the discipline and its role in formation the theoretical and practical competences in the field of transients.
2. What are the main stages of transient study and their calculations improvement?
3. What kinds of operation modes and processes do take place in the systems of electric power supply?
4. What is the difference between "the operation mode parameters" and "the parameters of a system"?

### **Topics for essay**

1. Emergency situations in the systems of electric power supply, their consequences and ways to avoid them.
2. Types of transients in the systems of electric power supply and features of possible operation modes.





# **Part 1: Electromagnetic Transients**



## **CHAPTER 1: GENERAL INFORMATION ABOUT TRANSIENTS**

- 1.1. Kinds, reasons and consequences of short circuits
- 1.2. Purpose of electromagnetic transients calculation, design conditions
- 1.3. Basic regularities of short circuit currents calculation
- 1.4. Design circuit of electric power supply system description
- 1.5. Equivalent circuit and methods of its parameters calculation
- 1.6. Transformations of equivalent circuits
- 1.7. Application of approximate reduction in per unit

Test questions

Topics for essay

### 1.1. Kinds, reasons and consequences of short circuits

A *short circuit* is a random or intentional (unprovided for normal operating mode) electric closing of conducting circuits (phases, terminals) of electrical installation with each other or to ground. The short circuit causes sharp increase of the current in the circuit parts adjoining to the point of its occurrence, so that their values exceed the maximum allowable values for the circuit continuous operation. When one of the phases (or terminals) of the electric installation, which operates with insulated or resonance grounded neutral, is connected to the ground, it is said that a *closure* takes place.

Short circuit occurring under maintenance is the main reason for electromagnetic transients, which significantly derange normal operating modes. That is why short circuits are considered as a typical influencing disturbance when power supply systems are designed.

Depending on type and peculiarities of electric networks (voltage level, kind of current, number of a source phases and terminals, type of neutral, etc.) the following kinds of short circuits (SC) are possible (Tab. 1.1).

*Three-phase SC* is a fault that occurs in three phase AC network as a result of connection across all three phases.

*Three-phase SC to the ground* is the circuit closing to ground or neutral main in AC electric network with dead-grounded or effectively grounded neutrals of power equipment, when three phases are closed with each other and ground.

*Three-phase SC with the ground* occurs in three-phase AC network with insulated or resonant grounded neutrals of power equipments when the fault point contacts with ground.

*Two-phase SC* is the closing of two phases in a three-phase AC electric network.

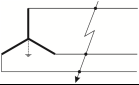
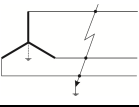
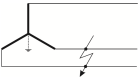
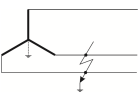
*Two-phase ground SC* occurs under two-phase closing in three-phase AC network with dead-grounded or effectively grounded neutrals of components when phases contact ground, or in two-phase AC traction network when one phase is connected to the trolley line and other one, grounded, to the track circuit.

*Two-phase SC with ground* is a closing of two-phases in a three-phase AC network with insulated or resonant grounded neutrals of power equipment when fault point has contact with ground.

*Single-phase ground SC* is a closing of one phase to ground or grounded neutral main in three-phase AC network with dead-grounded or effectively grounded neutrals of power equipment.

Table 1.1

Kinds of short circuits in electrical installations

Key scheme	Type of neutral (midpoint, terminal)			
	dead (effectively) grounded		insulated (resonant grounded)	
	Kind of SC	designation	Kind of SC	designation
	Three-phase SC	$SC^{(3)}$	Three-phase SC	$SC^{(3)}$
	Three-phase ground SC	$SC^{(1,1,1)}$	Three-phase SC with ground (contact with ground is present)	$SC^{(3g)}$
	Two-phase SC	$SC^{(2)}$	Two-phase SC	$SC^{(2)}$
	Two-phase ground SC	$SC^{(1,1)}$	Two-phase SC with ground (contact with ground is present)	$SC^{(2g)}$

Key scheme	Type of neutral (midpoint, terminal)			
	Two-phase ground SC	$SC^{(1,1)}$	–	–
	Single-phase ground SC	$SC^{(1)}$	Single-phase ground SC	$G^{(1)}$
	Single-phase ground SC	$SC^{(1)}$	Single-phase SC	$SC^{(1)}$
	Double-terminal ground SC	$SC^{(1,1)}$	Double-terminal ground SC	$SC^{(2)}$
	Single-terminal ground SC	$SC^{(1)}$	Single-terminal ground SC	$G^{(1)}$
	Double SC to ground	$SC^{(1+1)}$	Double SC to ground	$G^{(1+1)}$
	–	–	Double SC to ground	$G^{(1+1)}$

*Single-phase SC* is a closing of the phase with neutral main in a single-phase AC electric network with insulated or resonant grounded neutral (terminal)

*Single-phase SC with ground* is a closing of phase to ground in three-phase or single-phase (with neutral main) AC electric network with insulated resonant grounded neutrals (terminals) of power equipment.

*Double-terminal ground SC* occurs in direct (rectified) current network with grounded power source center when fault point contacts ground.

*Double-terminal SC* is a closing of terminals in DC (rectified) network with insulated power source center.

*Single-terminal ground SC* is a closing of insulated terminal to ground in DC (rectified) network grounded power source midpoint.

*Single-terminal SC with ground* occurs in DC network with insulated midpoint when one of the terminals contacts with ground.

*Double ground SC* is a simultaneous occurrence of two single-phase short circuits in different but electrically connected parts of electric installation.

*Double SC with ground* is a simultaneous occurrence of two single-phase (single-terminal) short circuits with ground in different but electrically connected parts of electric installation.

Symmetrical and asymmetrical, stable and unstable, modifiable short circuits are defined according to current's path. Under *symmetrical short current* all phases of electric installations are under the same conditions. Short circuit is defined as *asymmetrical* if conditions at least in one phase differs from those in others. *Stable* short currents occur when reasons for SC remain while dead time of switching unit. *Unstable* short current occurs when reasons for SC are removed while dead time of switching unit installed in the short circuit current path. *Modifiable* is short circuit changing its kind during the process.

There are several possible points of short current:

inter-path SC – between winding's paths in one phase;

inter-coil or inter-section SC – between coils or sections of winding in one phase;

inter-turn SC – between different turns of one coil of transformer or electric motor winding.

The weights of main short circuit kinds in electric networks are:  $SC^{(3)} \approx 5\%$ ;  $SC^{(2)} \approx 10\%$ ;  $SC^{(1)} \approx 65\%$ ; ( $SC^{(1,1)}$  и  $SC^{(1+1)}$ )  $\approx 20\%$ . Statistically proved that occurrence of different kinds of short circuits depends on mains voltage (Table 1.2).

Depending on reason for short circuits they are defined as random or intentional. *Intentional short circuits* are essential aspect of technological equipment functioning (electric welders, arc furnaces, etc.) and electric installations (air-break dischargers or lightning-arresters, short-circuiters, switching units with forced arc extinction)

Table 1.2.

The weights of different kinds of short circuit depending on mains voltage

Kind of SC	Weight (%) in networks with mains voltage, kV					
	6-20 (distribution network)	6-20 (unit network)	35	110	220	330
$SC^{(1)}$	61	60	67	83	88	91
$SC^{(2)}$	17	20	18	5	3	4
$SC^{(1,1)}$	11	15	7	8	7	4
$SC^{(3)}$	11	5	8	4	2	1

The reasons for *random short circuits* are: failures of current path's insulation because of its aging under operation; overvoltages, direct thunderbolts; mechanical failures; disposal of the outside subjects at the current paths; insufficient maintenance of the electrical equipment, inaccurate service.

Some kinds of short circuits can cause different dangerous consequences.

- Unallowable overheating of electric equipment and its thermal failure because of significant rise of currents (10-15 rated values).
- Strong mechanical forces between phases of current-carrying paths, causing electric installations to failure or serious damage.
- Voltage reduction and phase voltage symmetry breakdown, which harms on demand-side. So, voltage decrease for 30-40% for 1 second makes electric motors stop thus breaking manufacturing process, leading to waists of production and quality decrease.
- Generating of electromotive forces in nearby transmission and signaling lines under asymmetrical short currents, which can be dangerous for maintenance staff and automatic equipment;
- Instability of electric supply resulting in emergencies with overload switching off many power consumers;
- Fire in electric installations.

The most dangerous consequences usually occur in power equipment nearby to short circuit point. If short circuit occurred at the point remote from power source, generators react on it as to certain load buildup. Voltage drops significantly only close to point of three-phase short circuit.

To provide uninterruptible supply of all consumers it is necessary to design and construct electric networks considering possible short circuits, to comply electric equipment operating rules, continually improve technical level and quality of electric equipment.

To avoid dangerous consequences and provide stable operation of equipment in electric supply network the following measures are applied: relay protection, special automatic load transfer devices, non-simultaneous motor groups' automatic start, synchronous machines' excitation control devices.

## 1.2. Purpose of electromagnetic transients calculation, design conditions

Estimation of electromagnetic transients under short circuit (which are typical disturbance) is very important for supply lines design and maintenance. Estimation provides calculation of currents and voltages at points of short circuits or any other electric supply network's points or shorted network branches under given conditions.

Estimation emergencies caused by short circuits are meant for the following *tasks*:

- Identification of operating conditions on demand-side under possible short circuits and certain mode permissibility.
- Selection of electric equipment and its verification for short circuit conditions.
- Design and tuning of relay protection and system control relays, automatic switching units settings estimation.
- Comparison, estimation and choosing of connection patterns of electric supply components.
- Fitting and optimization of currents and power of short circuits.
- Estimation of electric supply system's and its load centers' stability.
- Ground grids' design.
- Estimation of short currents' effect on communication lines.
- Design of dischargers for electric installations' overvoltage protection.
- Analysis of emergencies in electric installations.
- Accomplishing various tests in electric supply networks.

According to purpose of calculation it is necessary to estimate design conditions for each network's component. They are the basis for calculation of short circuit mode parameters or their transients.

*Short circuit design conditions* are heaviest, but rather possible conditions of electric installation's component under short circuit. The initial design conditions are: short circuit design scheme, short circuit design point (location), design kind of short circuit, design duration of short circuit.

A *short circuit design scheme* is an electric circuit realizing boundary conditions under short circuit in given component or other task examined. Choosing the design circuit for short circuit the long term operation of the electric installation must be taken into account, its short-term modification unprovided by normal maintenance must be ignored (e.g. switches). Short-term modifications are not those concern remedial maintenance and postemergency states. Short circuit design circuit must consider further development of external networks and power sources electrically coupled with given electric supply network. One should consider at least 5 years of further development since putting network in operation.

*Calculated short circuit current* must be evaluated assuming that possible location of short-circuit results in heaviest conditions of equipment and current-conductors or protection means are least sensitive under these conditions. Simultaneous ground short circuits of two different phases at different points may be ignored, if this is not a special task.

*Design point (location) of short circuit* is a point (or circuit branch) of electric installation, which shorting meets the design short circuit conditions for given component.

*Design kind of short circuit* is a kind of short circuit which realizes design short circuit conditions under given component shorting. For design kind of short circuit should be taken:

three-phase SC – for estimation of thermal stability of devices and conductors on all steps of voltage, except generator voltage;

three-phase or two-phase SC (that resulting in most overheating) – for estimation of devices' and conductors' thermal stability under generator voltage step;

three-phase SC – for estimation of electrodynamic stability of devices and rigid buses and their supporting structures;

three-phase and single-phase ground short circuit – for choosing of apparatus by their switching capability.

*Design duration of short circuit* is stated as designed one according to allowable affect of short current on given component. For example, design duration for thermal stability estimation is a total time, obtained by summing the time from operation of main protection (including automatic reclosure switch respond time) installed at the nearest to short circuit switcher, and full time of this switch turning off (including arc dying out time). If main protection system has dead zone (in current, voltage, impedance etc) the thermal stability must be additionally verified according to operating time of backup protection, which is sensitive to faults in this zone, and to switcher's break time. For design short circuit current the current corresponding short circuit point must be taken.

*The required accuracy* of short current mode's parameters depends on evaluations' purpose. For instance, for choosing and verification of electric apparatus the required accuracy may be lower, then for selection of protection and automatic devices. In the latter the upper and lower values of voltages and currents and possible displacement between them at different phases or their symmetrical components must be defined. Arc impedance and other aspects must be considered as well.

### 1.3. Basic regularities of short circuit currents calculation

In general case, taking into account all processes characterizing transient is laborious for calculation of short circuit mode's parameters in large industrial network. Determination of exact location of network's short circuit is an awkward problem that can be solved only using computer-aided applications.

Most practical tasks of electric supply network design and maintenance permit a certain error of short circuit current evaluations, which must be in acceptable range for specific purpose of calculations. That is why a set of restrictions and assumptions concerning transients' abstraction calculations simplifying are applied, they allow using exact or approximate evaluation methods.

*The exact evaluation methods* (for example, for relay protection and system relay control design) must account the following:

a) for initial instant of short circuit ( $t = 0$ ) magnitudes and angles of generators' and capacitors' electromotive forces are considered to be equal to electromotive force apart from subtransient impedance in previous mode (subtransient electromotive force); for arbitrary instant for transients caused by short circuit ( $t > 0$ ) the changes of magnitude and phase of electromotive force should be estimated by method of rectified characteristics, including affect of generators' automatic excitations control;

b) complex impedance of power components of short circuit design diagram, ohmic resistances of current-limiting chokes attached in power transformers' neutrals should be considered;

c) complex character of load is to be accounted;

d) mutual inductance between parallel electric mains in zero sequence design circuits is to be taken into consideration;

e) it is necessary to account shunt capacitive susceptance of electric mains in supply network of 330-750 kV with at least 150 km extent and of 110-220 kV with at least 200-250 km.

Above mentioned factors may not be fully taken into account in *simplified evaluation methods*. These methods are used to choose and verify electric equipment on short circuit conditions, if their error do not exceed 5..10%.

- Evaluation of short circuit current, aimed to the apparatus and conductors selection, and estimation of the currents' effect on bearing structures in electric installations over 1 kV bases on the following.

- All power sources feeding given shorted branch operate simultaneously under rated load.
- All synchronous machines have automatic voltage regulation and excitation forcing devices.



- Short circuit occurs at the instant that provides its maximum current.
- Electromotive forces of all sources are assumed to coincide on phase. Variation of synchronous motor's speed is neglected if short circuit duration doesn't exceed 0.5 sec.
- Design voltage for each step is taken on 5% above rated voltage.
- Affect of synchronous capacitors, connected to the network, synchronous and asynchronous (induction) motors went into generating mode on short circuit current should be considered. The affect of induction motors is not considered in the following cases: if rated power of each of them is under 100 kW in a unit; if they are separated from the point of short circuit by two or more transformation stages, when their current can flow to point of short circuit through elements carrying the main current of the short circuit and which have considerable impedance (supply lines, transformers and the like).
- Saturation of magnetic cores of all shorted circuit components (generators, transformers and electric motors) is neglected.
- Active resistances of circuit's components are neglected if ratio of resultant impedances from the power source to short circuit point is  $r_{res} / x_{res} \leq 1/3$ . Active resistances are taken into account only for determination of currents' aperiodic components' decaying.
- Line-to-ground capacitive susceptance of overhead (aerial) transmission lines with voltage up to 220 kV is neglected. Capacitive susceptance of cable lines over 110 kV must be taken into account.
- Phases of all networks' electric installations are considered to be symmetrical, phase symmetry breakdown occurs only in the short circuit point;
- Short circuit current's aperiodic component decaying is considered approximately in circuits with several independent paths;
- Complex load affect is neglected if it causes current less than 5% of total short circuit current, evaluated under neglecting this load.
- Difference between subtransient direct- and quadrature-axis inductive reactances of synchronous motors is neglected.
- Magnetizing currents of transformers and autotransformers are neglected.
- Under evaluations of short circuit currents of the stage above 1 kV short circuit point infed by electric motors of the stages up to 1 kV is neglected.

In electric installations of voltage above 1 kV, only inductive reactances of electric motors, power transformers and autotransformers, chokes, overhead and cable lines, and current conductors should be taken into account. Active resistances should be considered only for overhead (aerial) transmission lines with small cross-section area or with steel wires, and for lengthy cable lines with small cross-section, having significant resistances.

• Electric installations of voltage up to 1 kV are mainly distribution divided networks, they contain significant quantity of power components, control and regulation apparatus. As a rule, they are supplied by one powerful source. Thus evaluation of short circuit in electric installations up to 1 kV supplied by step-down transformers must be accomplished under condition that the voltage, stable and is equal to rated one.

Evaluations of short circuit current in such network is done with assumptions the same for networks above 1 kV, but considering active components of power equipment's paths. The following components must be accounted in the design circuit:

- active resistances and inductive reactances of all components in shorted circuit, including wires and current conductors of switchovers, current coils of automatic switchers;
- active resistances of contacts along the short circuit current path;
- transient impedance of electric arc at short circuit point;
- network's and mode's parameters of synchronous and induction motors that went to generating modes;

It is recommended to take into account the following:

affect of complex load (electric motors, converters, thermal processing installations, incandescent lamps) on short circuit current if the electric motors' rated current exceeds 10% of initial value of short circuit current's periodic components, evaluated neglecting load;

variation of pure active resistances of shorted subcircuit components and wires because of their heating caused by short circuit current;

capacitor batteries infeed at the short current evaluation for fuses choosing.

Active or inductive component of impedance can be neglected if it doesn't stipulate more than 10% of total impedance reduction.

#### 1.4. Design circuit of electric power supply system description

The design circuit for analysis of emergencies caused by short circuit can be obtained on the basic one-line electric supply network diagram. The basic circuit must correspond to electric installations scheme under normal maintenance with maximal switched on power sources. To the design short circuit diagram only components that generate power under emergencies or introduce impedance in short circuit path, and short circuit point are included. I.e. it contains only the following: power sources (generators, synchronous capacitors, static sources of reactive power); local sources under generating mode (generalized loads, motors); power transformers and autotransformers, chokes, overhead and cable transmission lines – connecting power sources with points of short circuits. Description and nominal parameters of components necessary for evaluations are also shown at design circuit.

Depending on task statement several points and kinds of short circuits can be marked at the design circuit. The final goal of evaluations can be determination of maximum (for verification of electric equipment's stability under short circuit) or minimum (for relay protection verification) emergency currents and residual voltages. Thus design conditions to be considered at the design circuit development must include: necessary components, points and kinds of short circuits, their durations for evaluating fault parameters. The design mode is given a meaning that proceeds from the final goal of the short circuit currents calculation. Analysis of design modes with short circuits should foresee further network development.

Each component of design circuit is characterized by the following set of rated data necessary for short circuit mode parameters evaluation.

*Synchronous motor* (limited power electric system, generator, capacitor, electric motor) is defined by: rated total  $S_{\text{rated}}$ , MV·A, or active  $P_{\text{rated}}$ , MW power, power factor  $\cos\varphi_{\text{rated}}$  and rated voltage  $U_{\text{rated}}$ , kV; direct-axis and quadrature-axis impedances: subtransient -  $x''_d$  и  $x''_q$ ; transient -  $x'_d$  and  $x'_q$ ; synchronous -  $x_d$  и  $x_q$ ; stator winding leakage inductive reactance  $x_{*\sigma}$ ; negative sequence inductive reactance  $x_{*2}$ ; resistance  $R_f$ , and inductive  $x_f$  components of the excitation winding's impedance, Ohm; resistance  $R_{1d}$ , Ohm, and inductive  $x_{*1d}$  components of impedances of direct-axis damper winding; active  $R_{1q}$ , Ohm, and inductive  $x_{*1q}$  components of impedances of quadrature-axis damper winding; maximum field (excitation) current  $I_{f,\text{lim}}$ , A, and its value under no-load run at the rated voltage  $I_{f0}$ , A; time constants of stator's current aperiodic component decay under three-phase  $T_a^{(3)}$  and single-phase  $T_a^{(1)}$  short circuit at the machine's terminals, s; voltage  $U_{ph(0)}$ , kV, stator's current  $I^{(0)}$  and power factor  $\cos\varphi^{(0)}$  at instant of time previous to short circuit; efficiency (for electric motors)  $\eta$ , (%).

If an electric network is supplied from a powerful system, influence of the latter can be considered with current and power of short circuit on input buses. If these data are unavailable,

approximate evaluation of this connection is accomplished by rated current of electric motors breakers installed at connection point. Current or power under three-phase short circuit right behind breakers' is supposed to be equal to rated breaking current  $I_{br, rated}$  or rated breaking power  $S_{br, rated}$  correspondingly under given voltage. These parameters are the basis for estimation of system's equivalent impedance.

*Induction motor* is characterized by rated power  $P_{rated}$ , kW, rated voltage  $U_{rated}$ , kV, rated power factor  $\cos \varphi_{rated}$ , starting current– to- rated current ratio  $I_{*start}$ ; rated slip  $s_{rated}$ , %; maximal torque–to rated torque ratio  $M_{*max}$ ; direct-current resistance of stator  $R$ , Ohm; voltage  $U_{ph(0)}$ , kV, current  $I^{(0)}$ , kA; and power factor  $\cos \varphi^{(0)}$  at the instant previous to the short circuit.

*Power transformers and autotransformers* are defined by a set of performances: rated power  $S_{rated}$ , MVA; rated windings voltage  $U_{H, rated}$ ,  $U_{M, rated}$ , and  $U_{L, rated}$ , kV, and actual transformation ratios  $n$ ; voltage between pairs of windings under short circuit  $u_{s, H-L}$ ,  $u_{s, H-M}$ ,  $u_{s, M-L}$ , %, and their dependences on transformation ratio; voltage regulation range that determine short circuit voltage, %; short circuit losses in windings  $\Delta P_{s, H-L}$ ,  $\Delta P_{s, H-M}$ ,  $\Delta P_{s, M-L}$  or transformer  $\Delta P_S$ , kW; windings' connection pattern.

*Choke* is characterized by rated voltage  $U_{rated}$ , kV; rated inductive reactance  $x_{rated}$ , Ohm or %, coupling factor  $K_{cpl}$  (for double choke); rated current  $x_{rated}$ , A; rated power loss  $\Delta P_{rated}$ , kW, or by ratio  $x_{rated}/r$ .

*Overhead transmission lines* are characterized by: number of parallel lines, their length  $l$ , km, positive sequence inductive impedance  $x_1$ , zero sequence inductive impedance  $x_0$ , Ohm/km; positive sequence active resistance  $r_1$ , Ohm/km, or ratio  $x_1/r_1$ ; zero sequence active resistance  $r_0$ , Ohm/km. Impedances  $x_1$  and  $r_1$  are given in reference books depending on type and average distance between wires. Average design values of  $x_1$  are 0,4 Ohm/km for aerial lines which voltage is 6-220 kV, 0,33 Ohm/km for aerial lines with voltage 330 kV (two wires per phase) and 0,3 Ohm/km for transmission lines of 500 kV (three wires per phase). Impedance  $x_0$  depends on wires' cross-section area, distance between phases, presence or absence of grounded nearby ropes.

*Cable lines* are defined by their length  $l$ , km; by quantity of cables in line, positive sequence inductive impedance  $x_1$  and zero sequence inductive impedance  $x_0$ , Ohm/km, positive sequence active resistance  $r_1$ , Ohm/km. Impedances of cable lines depend on their type and vary within wide range. Average design values of  $x_1$  are: 0,12 Ohm/km for triple-core cable lines of voltage 35 kV; 0,08 Ohm/km for cable lines with voltage 6 and 10 kV; 0,07 Ohm/km for cable lines with voltage 3 kV. Values of  $x_0$  and  $r_0$  depend on cable laying and its type. For three-core cables approximately may be taken  $x_0 = (3,5 \dots 4,6) x_1$ .

*Complex load* can be defined by performances of specific electric power consumers: total power  $S_{rated}$  MV·A; power factor  $\cos \varphi_{rated}$  and supply voltage  $U_{rated}$ , kV. Complex load reduction is permitted for approximate evaluations. It is presented as generalized load with equivalent electromotive force and equivalent impedance. Recommended values of positive and negative sequences complex loads are given in Table 1.3.

Table 1.3  
Performances of complex load components

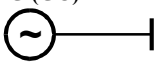
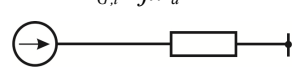
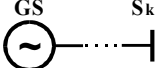
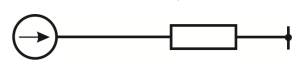
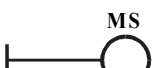
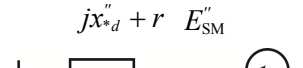
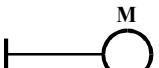
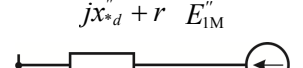
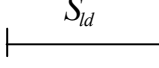
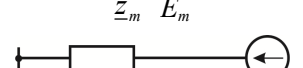
Component of complex load	COS $\varphi_{ld}$	Impedances, per unit	
		positive sequence $Z_{1ld}$	negative sequence $Z_{2ld}$
High-voltage synchronous electric motors	0.9	0.04+ j 0.22	0.04+ j 0.22
High-voltage induction electric motors	0.9	0.06+ j 0.18	0.06+ j 0.18
Low-voltage induction electric motors	0.8	0.09+ j 0.154	0.09+ j 0.154
Filament lamps	1.0	1.0	1.13
Gas-discharge light sources	0.85	0.85 + j 0.53	0.382+ j 0.24
Converters	0.9	0.9 + j 0.44	1.66 + j 0.814
Thermal-electric installations	0.9	1.0 + j 0.49	0.4 + j 0.196

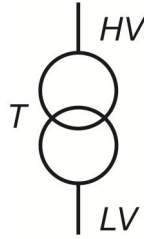

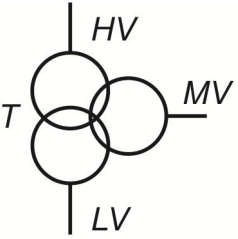
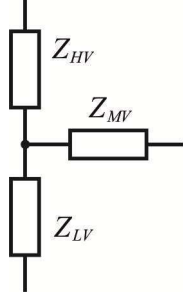
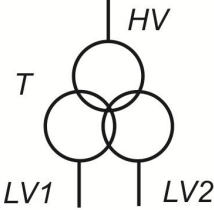
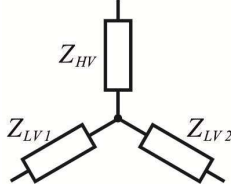
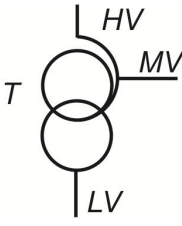
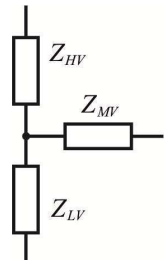
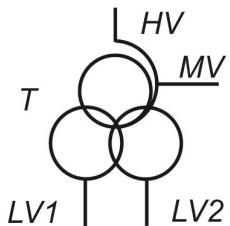
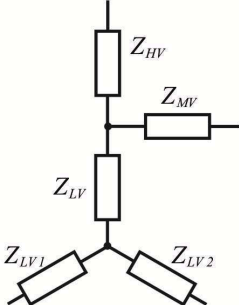


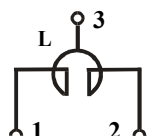
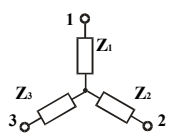
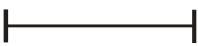



**1.5. Equivalent circuit and methods of its parameters calculation**

A short circuit design circuit can not be used directly for calculation of short circuit current analytically. It contains real electromagnetically coupled components with their impedances equivalent to losses and leakage inductive reactances. Components' parameters are given for different steps of voltages. To make theoretical electric circuit calculation methods suitable for determination of short circuit current the supply network diagram must be presented as electric circuit (path). Short circuit is assumed to be symmetrical (asymmetrical short circuits will be dealt in Chapters 4-6), thus transient is analyzed only in one phase. To do that, design circuit of short current is transformed to equivalent, where components are substituted by their equivalents, connected by the same order (Table 1.4).

Table 1.4

Components of electric supply networks and their electric equivalents in equivalent circuit

Component	Components designation	
	in design circuit	in equivalent circuit
Generator (synchronous capacitor)	$G (GC)$ 	$E_{G,t} \quad jX''_d + r$ 
Equivalent source "electric system"	$GS \quad S_k$ 	$E'' = U_{av, rated}$  $X''_C = X''_{start}$
Synchronous motor	$MS$ 	$jX''_d + r \quad E''_{SM}$ 
Induction motor	$M$ 	$jX''_d + r \quad E''_{IM}$ 
Generalized load	$S_{ld}$ 	$Z_m \quad E''_m$ 

<p>Double-wound transformer</p>		
<p>Triple-wound transformer</p>		
<p>Three-phase transformer with split secondary</p>		
<p>Three-phase autotransformer</p>		
<p>A group of single-phase transformers with split secondary</p>		
<p>Single choke</p>		
<p>Double choke</p>		
<p>Overhead (aerial) transmission line</p>		
<p>Cable transmission line</p>		

The design circuit is composed for each point of short circuit. For each component the data are indicated as the ratio, in the nominator: the element ordinal number in Arabic numerals (this number should be repeated on the design circuit of the component); in the denominator – its impedance or other performance.

The following should be considered under evaluation of equivalent circuit component's parameters:

o In simplified evaluation methods active components (generating sources) of design circuits are substituted by equivalent source of electromotive force and its impedance, determined for instant  $t = 0$  (Table 1.4). Components impedances and distribution of currents in circuit for  $t > 0$  are assumed to be constant during all transient and equal to those evaluated for  $t = 0$ .

o In exact evaluation methods values of  $E_t$  and  $x_t$  for generating sources are evaluated at short circuit transient time  $t > 0$ , for passive components the variation of their impedance because of short circuit current heating must be taken into account.

Performances of components' equivalents in equivalent circuits can be determined by the following methods:

1) in concrete units with performances of design circuits' components reduced to base supply voltage step using actual value of power transformers' and autotransformers' transformation ratios;

2) in per unit with performances of design circuits' components reduced to given base conditions using actual value of power transformers' and autotransformers' transformation ratios;

3) in concrete units with performances of design circuits' components reduced to base supply voltage level using approximate values of power transformers' and autotransformers' transformation ratios;

4) in per unit with performances of design circuits' components reduced to base conditions using approximate values of power transformers' and autotransformers' transformation ratios;

5) in concrete units keeping transformer couplings (i.e. without reducing of performances of design circuits components to specific supply voltage step) using actual value of power transformers' and autotransformers' transformation ratios.

Let us consider each method in detail. Methods (1)-(2) are known as exact reduction in concrete and per unit correspondingly, methods (3) and (4) - approximate reduction in concrete and per unit correspondingly

1. The point of **exact reduction in concrete units** is in calculation of components' performances for voltage step named as *base step*. Any step of supply network's voltage can be considered as base. A dummy voltage level can be used too.

Reduction of mode parameters  $\dot{E}_i, \dot{U}_i, \dot{I}_i$  and impedance  $\underline{Z}_i$  of component in concrete units from  $i$  – step of voltage, separated from base step by several series-plugged transformers with actual transformation ratios  $n_1, n_2, \dots, n_m$ , is accomplished according to equations:

$$\begin{aligned}\widehat{E} &= \underline{E}_i \cdot n_1 n_2 \dots n_m; \\ \widehat{U} &= \underline{U}_i \cdot n_1 n_2 \dots n_m; \\ \widehat{I} &= \underline{I}_i / (n_1 n_2 \dots n_m); \\ \widehat{Z} &= \underline{Z}_i \cdot (n_1 n_2 \dots n_m)^2.\end{aligned}\tag{1.1}$$

Here  $\widehat{E}, \widehat{U}, \widehat{I}, \widehat{Z}$  are reduced values and transformation ratio of each transformer is determined as ratio of no-load run voltage of winding series with base voltage step to no-load run voltage of winding series connected with voltage step where components which parameters are being reduced are situated.

If initial mode's parameters  $\underline{E}_{*(rated)}$ ,  $\underline{U}_{*(rated)}$ ,  $\underline{I}_{*(rated)}$  and impedance of component are defined in per unit in relation to rated conditions (rated voltage  $U_{rated}$  and rated power  $S_{rated}$  at  $i$ -step of voltage) then voltages reduced to base step in per unit are determined by equations:

$$\begin{aligned}\widehat{\underline{E}} &= \underline{E}_{*(rated)i} U_{rated} \cdot n_1 n_2 \dots n_m, \\ \widehat{\underline{U}} &= \underline{U}_{*(rated)i} U_{rated} \cdot n_1 n_2 \dots n_m \\ \widehat{\underline{I}} &= \underline{I}_{*(rated)i} S_{rated} / (\sqrt{3} U_{rated} n_1 n_2 \dots n_m), \\ \widehat{\underline{Z}} &= \underline{Z}_{*(rated)i} U_{rated}^2 (n_1 n_2 \dots n_m)^2 / S_{rated}\end{aligned}\quad (1.2)$$

Evaluated parameters for equivalent circuit with components reduced using (1.1) and (1.2) will equal to actual only for base voltage step. Actual currents and voltages for other steps are defined by transformation ratios between base step and estimated one.

2. The point of **exact reduction in per unit** is in reduction of design circuit's components' quantities and parameters to base conditions, which are determined by:

arbitrary taken base power  $S_b$  (to simplify calculations it is recommended to use value of the same order as total power sources and multiplied by  $10^k$ , i.e. 100, 1000);

base voltage of voltage step taken for main  $U_{b,main}$ ;

base current on the base voltage step,

$$I_{b,main} = S_b / (\sqrt{3} \cdot U_{b,main}) \quad (1.3)$$

Base voltages for other  $i$ -steps of electric supply system voltage are evaluated considering actual transformation ratios, using formula

$$U_{b,i} = U_{b,main} / (n_1 n_2 \dots n_{i-1}), \quad (1.4)$$

and base current as

$$I_{b,i} = I_{b,main} (n_1 n_2 \dots n_{i-1}), \quad (1.5)$$

i.e. for each voltage step the following ratio should be valid:

$$S_b = \sqrt{3} U_{b,i} I_{b,i} \quad (1.6)$$

If datum quantities of design the circuit  $\underline{E}_i$ ,  $\underline{U}_i$ ,  $\underline{I}_i$  and  $\underline{Z}_i$  are given in concrete units at  $i$ -th voltage step of the network, than their reduction is accomplished by expressions:

$$\begin{aligned}\underline{E}_{*(b)} &= \underline{E}_i / U_{b,i}, \\ \underline{U}_{*(b)} &= \underline{U}_i / U_{b,i}, \\ \underline{I}_{*(b)} &= \underline{I}_i / U_{b,i}, \\ \underline{Z}_{*(b)} &= \underline{Z}_i S_b / U_{b,i}^2,\end{aligned}\quad (1.7)$$

where  $U_{b,i}, I_{b,i}$  - correspondingly base current and base voltage of network voltage step at which the design circuit component is located.

If initial performances  $E_{*(rated)i}, U_{*(rated)i}, I_{*(rated)i}$  and  $Z_{*(rated)i}$  of the element that belongs to  $i$ -th step of short circuit design diagram are given in per unit in relation to rated performances of this element (rated power  $S_{rated}$ , voltage  $U_{rated}$ , current  $I_{rated}$ ), then their reduction is accomplished by formulae:

$$\begin{aligned} \underline{E}_{*(b)} &= \underline{E}_{*(rated)i} U_{rated} / U_{b,i}; \\ \underline{U}_{*(b)} &= \underline{U}_{*(rated)i} U_{rated} / U_{b,i}; \\ \underline{I}_{*(b)} &= \underline{I}_{*(rated)i} (S_{rated} / (\sqrt{3}U_{rated})) (\sqrt{3}U_{b,i} / S_b); \\ \underline{Z}_{*(b)} &= \underline{Z}_{*(rated)i} (U_{rated}^2 / S_{rated}) (S_b / U_{b,i}^2); \\ \underline{Z}_{*(b)} &= \underline{Z}_{*(rated)i} (U_{rated} / I_{rated}) (I_{b,i} / U_{b,i}). \end{aligned} \quad (1.8)$$

Actual mode parameters (short circuit current, for example) in equivalent circuit with components' parameters expressed in per unit reduced to base conditions are determined by formulae:

at base voltage step

$$\underline{I}_{s,\text{main}} = \underline{I}_{*(b),s} I_{b,\text{main}}. \quad (1.9)$$

at other  $i$ -steps of voltage

$$\underline{I}_{s,i} = \underline{I}_{*(b),s} I_{b,i}. \quad (1.10)$$

**3.** The point of *approximate reduction in concrete units* is in reduction of performances of design circuit's components to one voltage step, taken for base. The reduction is based on use of average transformation ratios of transformers (autotransformers). For each transformation step and its components the average rated voltage is defined according to the scale, kV: 1150; 750; 515; 400; 340; 230; 158; 115; 37; 24; 20; 18; 15.75; 13.8; 10.5; 6.3; 3.15; 1.21; 0.69; 0.4; 0.23; 0.133. Average transformation ratios are assumed to be equal to ratio of average rated voltages of correspondent windings of transformer or autotransformer. Than products of transformation ratios in reduction formulas (1.1) for  $m$  transformation steps connected in series between  $i$ -th and the base one transformers steps:

$$n_{av,i} n_{av,i+1} \dots n_{av,m} = \frac{U_{av,i+1}}{U_{av,i}} \cdot \frac{U_{av,i+2}}{U_{av,i+1}} \dots \frac{U_{av,\text{main}}}{U_{av,m}} = \frac{U_{av,\text{main}}}{U_{av,i}}, \quad (1.11)$$

where  $U_{av,i}$  - average rated voltage of  $i$ -th transformation step, from which the reducing is being accomplished;  $U_{av,\text{main}}$  - the same for base voltage step.

Reduction of mode parameters and impedances in concrete units from the  $i$ -step of voltage to base one is accomplished by expressions:

$$\underline{\hat{E}} = \underline{E}_i (U_{av,\text{main}} / U_{av,i});$$



$$\begin{aligned}
\widehat{U} &= \underline{U}_i (U_{av,main} / U_{av,i}), \\
\widehat{I} &= \underline{I}_i / (U_{av,main} / U_{av,i}), \\
\widehat{Z} &= \underline{Z}_i (U_{av,main} / U_{av,i})^2.
\end{aligned} \tag{1.12}$$

If reduction to base voltage step is done for performances, given for the  $i$ -step in per unit (referred to rating conditions), then the following expressions are used:

$$\begin{aligned}
\widehat{E} &= \underline{E}_{*(rated)i} U_{av,i} (U_{av,main} / U_{av,i}) = \underline{E}_{*(rated)i} U_{av,main}, \\
\widehat{U} &= \underline{U}_{*(rated)i} U_{av,i} (U_{av,main} / U_{av,i}) = \underline{U}_{*(rated)i} U_{av,main}, \\
\widehat{I} &= \underline{I}_{*(rated)i} [S_{rated} / (\sqrt{3} \cdot U_{av,i})] / (U_{av,main} / U_{av,i}) = \\
&= \underline{I}_{*(rated)i} S_{rated} / (\sqrt{3} \cdot U_{av,main}); \\
\widehat{Z} &= \underline{Z}_{*(rated)i} (U_{av,i}^2 / S_{rated}) (U_{av,main}^2 / U_{av,i}^2) = \underline{Z}_{*(rated)i} U_{av,main}^2 / S_{rated}.
\end{aligned} \tag{1.13}$$

Mode parameters, determined by (1.12) and (1.13) will be actual for base voltage step. Parameters of other  $i$ -steps are defined using average transformation ratios, for example, short circuit current,

$$\dot{I}_{s,i} = \underline{I}_{s,main} / (U_{av,i} / U_{av,main}) \tag{1.14}$$

4. The point of **approximate reduction in per unit** is in reduction of electric installations' components' performances to base conditions using average transformation ratios (1.11). The base conditions are defined by: base power  $S_b$ ; base voltage, which is assumed equal to average rated voltage of base step  $U_{b,main} = U_{av,main}$ ; base current  $I_{b,main}$ , evaluated by (1.3).

For others  $i$ -th voltage steps base conditions are defined by:

base voltage of  $i$ -th step, which according to expression (1.4), considering (1.11), will be equal to average rated voltage  $U_{b,i} = U_{av,i}$   
base current of  $i$ -th step

$$I_{b,i} = S_b / (\sqrt{3} \cdot U_{av,i}) \tag{1.15}$$

If initial parameter values of short circuit design diagram components are given at the  $i$ -th step of voltage in concrete units, then their reduction is accomplished by expressions:

$$\begin{aligned}
\underline{U}_{*(b)} &= \underline{U}_i (U_{av,main} / U_{av,i}) / U_{av,main} = \underline{U}_i / U_{av,i}, \\
\underline{I}_{*(b)} &= [\underline{I}_i / (U_{av,main} / U_{av,i})] / [S_b / (\sqrt{3} \cdot U_{av,main})] = \\
&= \underline{I}_i / [S_b / \sqrt{3} \cdot U_{av,i}] = \underline{I}_i / \underline{I}_{b,i}, \\
\underline{Z}_{*(b)} &= \underline{Z}_i (U_{av,main} / U_{av,i})^2 (S_b / U_{av,main}^2) = \underline{Z}_i S_b / U_{av,i}^2.
\end{aligned} \tag{1.16}$$

Recalculation of initial performances of components of short circuit design diagram given in per unit (referred to rating conditions) and the  $i$ -th step of voltage is accomplished by expressions:

$$\begin{aligned} \underline{E}_{*(b)} &= \underline{E}_{*(rated)}_i U_{av,i} (U_{av,main} / U_{av,i}) / U_{av,main} = \underline{E}_{*(rated)}_i, \\ \underline{U}_{*(b)} &= \underline{U}_{*(rated)}_i U_{av,i} (U_{av,main} / U_{av,i}) / U_{av,main} = \underline{U}_{*(rated)}_i, \\ \underline{I}_{*(b)} &= \{ \underline{I}_{*(rated)}_i [S_{rated} / (\sqrt{3} \cdot U_{av,i})] / (U_{av,main} / U_{av,i}) \} / \\ & / [S_b / (\sqrt{3} \cdot U_{av,main})] = \underline{I}_{*(rated)}_i S_{rated} / S_b, \\ \underline{Z} &= (\underline{Z}_{*(rated)}_i U_{av,i}^2 / S_{rated}) (U_{av,main} / U_{av,i})^2 / (U_{av,main}^2 / S_b) = \\ & = \underline{Z}_{*(rated)}_i S_b / S_{rated}. \end{aligned} \quad (1.17)$$

In final form equations (1.16) and (1.17) do not include the voltage of base transformation step that simplifies evaluations. Intermediate transformations reveals that it still present in hidden form.

**5. In equivalent circuits with retaining the transformer couplings** evaluation of their components' performances is accomplished in concrete units. Retaining transformer couplings are used when:

transformer or autotransformer taps are changed over for voltage regulation;

the design circuit contains subcircuits with transformer couplings that has different transformation ratios;

if mode parameters evaluation is accomplished using AC models, where transformer couplings are introduced directly.

For this principle of evaluation:

components' performances had been defined for voltage step, where they are located; thus, under voltage regulation, impedances must be recalculated only for this transformation step;

evaluated mode parameters are actual for each step of voltage.

Equivalent circuit of transformer with equivalent retained coupling is formed: without consideration of magnetizing path. It consists of ideal transformer with transformation ratio  $n_T = U_2 / U_1$  and transformer leakage impedance  $\underline{Z}_T$  connected in series (Fig 1.1,a). If parallel

paths contain only transformers with different transformation ratios, accordingly  $n_{T1}$  and  $n_{T2}$ , the

equivalent circuit of such parallel paths with transformer coupling (Fig 1.1,b) contains in

one circuit, second, for example, additional ideal transformer with transformation ratio

$n_{T2} / n_{T1}$  and in both circuits ideal

transformers with transformation ratio  $n_{T1}$

with leakage impedances  $\underline{Z}_{T1}$  and  $\underline{Z}_{T2}$ , connected as in the design circuit. When there

is no need to consider individual voltage regulation or transformation ratios of parallel transformers are almost equal, in approximate

evaluations the average transformation ratio

$n_{T,av} \approx \sqrt{n_{T1} n_{T2}}$  may be applied.

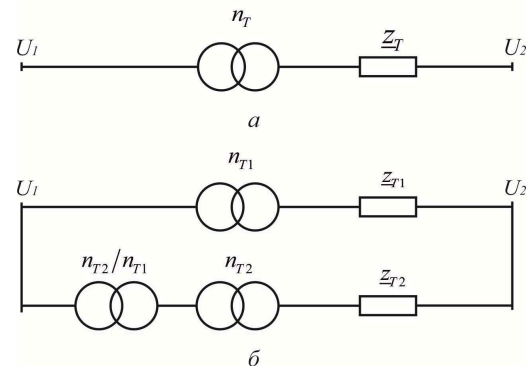


Fig. 1.1. Equivalent circuits with equivalent transformer couplings: a – for single transformer; b – for parallel transformers with different transformation ratios

1.6. Transformations of equivalent circuits

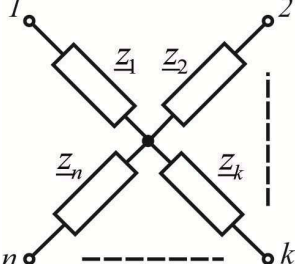
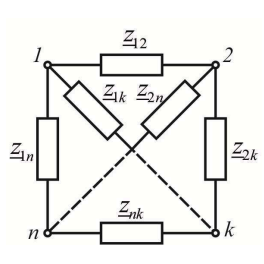
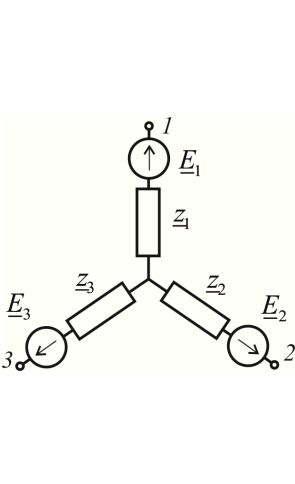
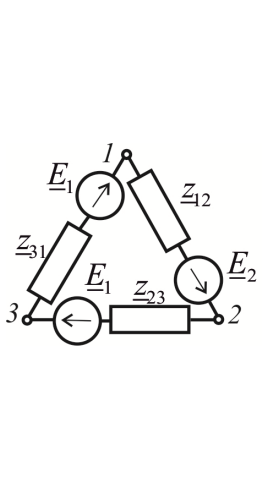
The most common target of short circuit transients calculations is determination of current in faulted subcircuit or short circuit point. Thus equivalent circuit must be transformed in order to keep faulted subcircuit to the finish of transformation.

Using equivalent transformations equivalent circuit is reduced to a simplest type for determination or resultant impedance of shorted subcircuit. The following transformation methods for linear circuits are used for this purpose: series and parallel connection of impedances, replacement of several sources with different electromotive forces and impedances, connected with one network's point by single equivalent source; transformations of delta-connection into equivalent star connection, star – to equivalent delta, and multi-rayed star to full polygon with diagonals (Table 1.5).

A complex equivalent circuit with several sources (Fig. 1.2, a) can be transformed, using coefficients of current distribution, into multi-rayed equivalent circuit with generating rays and short circuit point in rays junction (Fig. 1.2,c). The reduced electromotive forces of sources are assumed to be equal. Coefficients of current distribution under no-load mode characterize the role of each source in short circuit point infeed.

Table 1.5  
Equivalent transformations of equivalent circuits

Kind of transformation	Circuits		Relationships
	initial	equivalent	
Series connection			$Z_e = \sum_{k=1}^n Z_k$
Parallel connection			$Z_e = 1 / \sum_{k=1}^n (1/Z_k)$
Replacement of sources' group with equivalent one			$E_e = Z_e \sum_{k=1}^n (E_k / Z_k);$ $Z_e = 1 / \sum_{k=1}^n (1/Z_k)$
Replacement of delta with star			$Z_{123} = Z_{12} + Z_{31} + Z_{23};$ $Z_1 = Z_{12} Z_{31} / Z_{123};$ $Z_2 = Z_{12} Z_{23} / Z_{123};$ $Z_3 = Z_{23} Z_{31} / Z_{123}$
Replacement of star with delta			$Z_{12} = Z_1 + Z_2 + Z_1 Z_2 / Z_3;$ $Z_{23} = Z_2 + Z_3 + Z_2 Z_3 / Z_1;$ $Z_{31} = Z_3 + Z_1 + Z_3 Z_1 / Z_2$

<p>Replacement of multi-rayed star with polygon with diagonals</p>			$Z_{12} = Z_1 Z_2 Y_{1-n};$ $Z_{23} = Z_2 Z_3 Y_{1-n};$ <p>.....</p> $Z_{k(k-1)} = Z_k Z_{k-1} Y_{1-n},$ <p>where <math>Y_{1-n} = \sum_{k=1}^n (1/Z_k)</math></p>
<p>Replacement of star with EMFs in rays with delta with EMFs in sides</p>			$E_{12} = (E_2 (z_2 + z_3) - (E_2 + E_3) z_1) / z_{123};$ $E_{23} = (E_2 (z_1 + z_2) - (E_1 + E_3) z_2) / z_{123};$ $E_{31} = (E_3 (z_1 + z_2) - (E_1 + E_2) z_3) / z_{123},$ <p>where <math>z_{123} = z_1 + z_2 + z_3</math>.</p> <p>Impedances <math>z_{12}</math>, <math>z_{23}</math>, <math>z_{31}</math> are evaluated the same way as for replacement of star to delta</p>

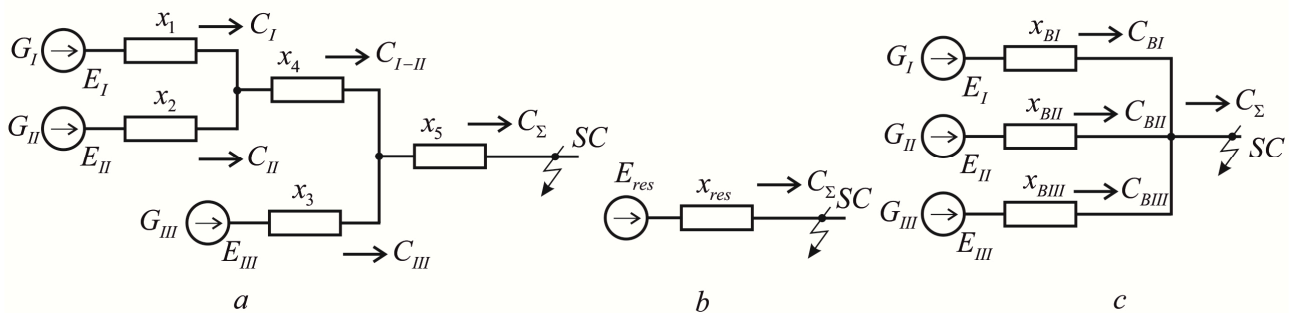


Fig. 1.2. Transformation of equivalent circuit using coefficient of current distribution: a – initial complex circuit; b – equivalent; c – multi-rayed circuit

The algorithm and rules of transformation are:

current in short circuit point is conditionally assumed being equal to one  $C_{\Sigma} = 1$ ;

coefficients of current distribution (they show the relative weight of short circuit current in subcircuit) in generating arms of initial equivalent circuit (Fig. 1.2,a) are equated with those in corresponded rays of new equivalent circuit (Fig. 1.2, c), i.e.  $C_I = C_{BI}$ ;  $C_{II} = C_{BII}$ ;  $C_{III} = C_{BIII}$ . The following identity is valid for generating arms of both equivalent circuits

$$C_I + C_{II} + C_{III} = C_{\Sigma}; \quad C_{BI} + C_{BII} + C_{BIII} = 1;$$

the resultant impedance of initial equivalent circuit is determined (Fig. 1.2,b) by joining of zero points of power sources  $G_I, G_{II}, G_{III}$ , i.e.

$$x_{res} = x_5 + x_3 \left[ x_4 + x_1 x_2 / (x_1 + x_2) \right] / \left[ x_3 + x_4 + x_1 x_2 / (x_1 + x_2) \right];$$

coefficients of current distribution in parallel paths of initial equivalent circuit are inversely proportional to their impedances

$$C_{III} / C_{\Sigma} = [x_4 + x_1 x_2 / (x_1 + x_2)] / [x_3 + x_4 + x_1 x_2 / (x_1 + x_2)];$$

$$C_{I-II} / C_{\Sigma} = x_3 / [x_3 + x_4 + x_1 x_2 / (x_1 + x_2)];$$

$$C_I / C_{I-II} = [x_1 x_2 / (x_1 + x_2)] / x_1;$$

$$C_{II} / C_{I-II} = [x_1 x_2 / (x_1 + x_2)] / x_2;$$

coefficients are estimated, starting from SC point and to power sources' arms of the initial equivalent circuit,

$$C_{III} / C_{\Sigma} = [x_4 + x_1 x_2 / (x_1 + x_2)] / [x_3 + x_4 + x_1 x_2 / (x_1 + x_2)];$$

$$C_{I-II} / C_{\Sigma} = x_3 / [x_3 + x_4 + x_1 x_2 / (x_1 + x_2)];$$

$$C_I = C_{I-II} x_2 / (x_1 + x_2);$$

$$C_{II} = C_{I-II} - C_I;$$

the impedances of rays in new equivalent circuit are determined by equalities

$$x_{BI} = C_{\Sigma} x_{res} / C_{BI};$$

$$x_{BII} = C_{\Sigma} x_{res} / C_{BII};$$

$$x_{BIII} = C_{\Sigma} x_{res} / C_{BIII}.$$

If three-phase short circuit occurs in a node with several branches meeting at it then this node can be divided into several ones with remaining of short circuit at the end of each node. Then, obtained equivalent circuit can be transformed relative to any shorted branch, considering other arms with or without load's electromotive forces. This principle is especially suitable when it is necessary to estimate current in one of subcircuits, connected to shorted node.

If equivalent circuit is symmetrical relative to short circuit point or examined circuit section is symmetrical relative to some intermediate point, then equipotential points can be connected while the circuit transformations transformations. Impedances that are not in short current's path can be excluded from circuit.

In certain cases transformation of circuits can be simplified by replacing three-rayed star into delta and cutting its corner with electromotive force applied.

Replacement of two or more power sources of one type by equivalent one is possible if they are practically under the same conditions relative to short circuit point. That is verified by condition

$$S_{Irated} x_{resI} / (S_{IIrated} x_{resII}) = 0,4 \dots 2,5.$$

Here  $S_{Irated}, S_{IIrated}$  are rated powers of sources;  $x_{Ires}, x_{IIres}$  - resultant inductive reactances between correspondent power sources and short circuit point.

The source of less power can be neglected under transformations if

$$x_{Ires} / x_{IIres} \geq 20 \quad \text{and} \quad S_{Irated} / S_{IIrated} \leq 0,05.$$

It is not necessary to transform complex equivalent circuits analytically. They can be made equivalent using design DC or AC models. To reduce equivalent circuit to its simplest type the resultant impedance between each source and short circuit point is determined by direct measuring.

Equivalent electromotive forces of sources obtained by considered transformations and resultant impedances of shorted subcircuits is a basis for calculation of currents and voltages of the transient by short circuit.

### 1.7. Application of approximate reduction in per unit

Performances of shorted circuit components of a design circuit are reduced to base conditions by (1.16) and (1.17). In practical calculations impedances of certain components of shorted circuit in per unit, reduced to base conditions, are determined by formulae given below (design circuits and equivalent circuits of components are given in Table 1.4).

On switching on the synchronous generators, capacitors and electric motors at  $i$ -step of voltage with base voltage  $U_{b,i} = U_{av,i}$

$$x_{*(b)}'' = x_{*d}'' S_b / S_{\text{rated}}; \quad (1.18)$$

At voltage step with  $U_{av} \neq 1,05U_{\text{rated}}$

$$x_{*(b)}'' = (x_{*d}'' U_{\text{rated}}^2 / S_{\text{rated}}) / (U_{b,i}^2 / S_b); \quad (1.19)$$

$$r_{*(b)} = x_{*(b)}'' / (\omega T_a).$$

For synchronous motors  $x_{*(\text{rated})}'' = x_d'' \approx 1 / I_{*start}$ , where  $I_{*start} = I_{\text{start}} / I_{\text{rated}}$  is starting current-to-rated current ratio for direct on starting under rated voltage. For induction motors  $x_{*(\text{rated})}'' = x_*'' = 1 / I_{*start}$  and  $x_*''$  should be substituted instead of  $x_{*d}''$  in (1.18), (1.19).

Reduced impedance of electric power supply system equals to

$$x_{*(b)}'' = x_{*GS}'' S_{\sigma} / S_{\text{rated}} \quad (1.20)$$

or

$$x_{*(b)}'' = S_b / S_{\kappa(t=0)} \approx I_{b,i} / I_{br,\text{rated}}. \quad (1.21)$$

Under plugging of three-phase double-wound transformers at  $i$ -step of voltage  $U_{b,i} = U_{av,i}$

$$Z_{*(b)} = (u_s / 100) S_b / S_{\text{rated}}; \quad (1.22)$$

$$r_{*(b)} = \Delta P_s S_b / S_{\text{rated}}^2; \quad (1.23)$$

$$x_{*(b)}'' = \sqrt{Z_{*(b)}^2 - r_{*(b)}^2}. \quad (1.24)$$

For three-phase transformers (autotransformers)

$$\left. \begin{aligned} Z_{*(b)H} &= 0,5(u_{s,H-L} + u_{s,H-M} - u_{s,M-L}) S_b / (100 S_{\text{rated}}); \\ Z_{*(b)M} &= 0,5(u_{s,H-M} + u_{s,M-L} - u_{s,H-L}) S_b / (100 S_{\text{rated}}); \\ Z_{*(b)L} &= 0,5(u_{s,H-L} + u_{s,M-L} - u_{s,H-M}) S_b / (100 S_{\text{rated}}); \end{aligned} \right\} \quad (1.25)$$

$$r_{*(b)H} = r_{*(b)M} = r_{*(b)L} = 0,5 \Delta P_s S_b / S_{\text{rated}}^2. \quad (1.26)$$

For three-phase double-wound transformers with split secondary and separated operation of low-voltage winding sections LV1, LV2

$$\left. \begin{aligned} Z_{*(b)H} &= u_{s,H-LV} (1 - k_{split} / 4) S_b / (100 S_{rated}); \\ Z_{*(b)LV1} &= Z_{*(b)LV2} = u_{s,H-M} k_{split} S_b / (200 S_{rated}); \end{aligned} \right\} \quad (1.27)$$

$$\left. \begin{aligned} r_{*(b)H} &= \Delta P_{s,H-LV} S_b / S_{rated}^2; \\ r_{*(b)LV1} &= r_{*(b)LV2} = 2r_{*(b)H}, \end{aligned} \right\} \quad (1.28)$$

where  $k_{split}$  - splitting factor ( $k_{split} = Z_{split} / Z_{s,H-LV}$ );  $Z_{LV1} = Z_{LV2} = Z_{split} / 2$ . According to test data  $k_{split} = 3, 5$ . Besides

$$\left. \begin{aligned} Z_{*(b)H} &= 0,125 u_{s,H-LV} S_b / (100 S_{rated}); \\ Z_{*(b)LV1} &= Z_{*(b)LV2} = 1,75 u_{s,H-LV} S_b / (100 S_{rated}). \end{aligned} \right\} \quad (1.29)$$

In case of parallel operating of the sections LV1 and LV2 transformer has through inductive reactance

$$Z_{*(b)thr} = (u_{s,H-LV} / 100) S_b / S_{rated}. \quad (1.30)$$

Impedance of transformers with voltage regulation under load device is evaluated depending on regulated voltage  $U_{H,n}$  for this tap  $n$  and short circuit voltage  $u_{s,n}$  by formula

$$Z_{*(b)n} = (u_{s,n} / 100) (U_{H,n}^2 / S_{rated}) / (U_{b,i}^2 / S_b). \quad (1.31)$$

Short circuit voltage and correspondent tap voltage in (1.31) are defined for three positions of voltage regulator: central and outer ones. Design values of these voltages referred to transformer rated power and correspondent tapping voltage are estimated. For transformer with split secondary, value of  $u_{s,n}$  is referred to power  $S_{LV1(LV2)} = 0,5 S_{rated}$ . For triple-wound transformer impedances of three-rayed equivalent circuit are evaluated by (1.31) after estimation of short circuit voltages by given voltages

$$u_{s,H-M,n}, u_{s,H-L,n}, u_{s,M-L,n}, u_{s,H,n}, u_{s,M,n}, u_{s,L,n}$$

for central and outer positions of the voltage regulation under load device.

Under switching of single-phase double-wound transformer with split low-voltage winding

$$\left. \begin{aligned} Z_{*(b)H} &= 0; \\ Z_{*(b)LV1} &= Z_{*(b)LV2} = 2(u_{s,H-LV} / 100) S_b / S_{rated}, \end{aligned} \right\} \quad (1.32)$$

and for switching of single-phase autotransformers with the same winding

$$\left. \begin{aligned} Z_{*(b)H} &= 0,5(u_{s,H-L} + u_{s,H-M} - u_{s,M-L}) S_b / (100 S_{rated}); \\ Z_{*(b)M} &= 0,5(u_{s,H-M} + u_{s,M-L} - u_{s,H-M}) S_b / (100 S_{rated}); \\ Z_{*(b)LV1} &= Z_{*(b)LV2} = 2Z_{*thr} S_b / S_{rated}; \\ Z_{*(b)LV} &= (Z_{*LV} - Z_{*thr}) S_b / S_{rated}, \end{aligned} \right\} \quad (1.33)$$

where

$$Z_{*thr} = (u_{s,H-L} / 100) \parallel (u_{s,M-L} / 100);$$

$$Z_{*LV} = 0,5(u_{s,H-L} + u_{s,M-L} - u_{s,H-M}) / 100.$$

The reduced impedance of a single choke

$$x_{*(b)} = x_{rated} I_b U_{ch,rated} / (100 I_{ch,rated} U_{b,i}); \quad (1.34)$$

$$r_{*(b)} = \Delta P_{rated} S_b / (I_{ch,rated}^2 U_{b,i}^2). \quad (1.35)$$

For a double choke

$$\left. \begin{aligned} x_{*(b)1} = x_{*(b)2} &= (1 + k_{cpl}) x_{rated} I_b U_{ch,rated} / (100 I_{ch,rated} U_{b,i}); \\ x_{*(b)3} &= -k_{cpl} x_{rated} I_b U_{ch,rated} / (100 I_{ch,rated} U_{b,i}); \end{aligned} \right\} \quad (1.36)$$

$$\left. \begin{aligned} r_{*(b)1} = r_{*(b)2} &= \Delta P_{rated} S_b / (I_{ch,rated}^2 U_{b,i}^2); \\ r_{*(b)3} &= 0. \end{aligned} \right\} \quad (1.37)$$

Reduced supply line impedances

$$\left. \begin{aligned} r_{*(b)} &= r_{sp} l S_b / U_{av,i}^2; \\ x_{*(b)} &= x_{sp} l S_b / U_{av,i}^2. \end{aligned} \right\} \quad (1.38)$$

### Test questions

1. Reasons for electromagnetic transients in electric power supply system and their possible consequences.
2. Main kinds of short circuits, possibilities of their arise in components of different voltage supply networks.
3. What is “short circuit”, “closing”? What are designations of their kinds depending on neutral conditions?
4. What are conditions and base assumptions for evaluation of short-circuit current?
5. How to select and reduce base conditions for different steps of supply network voltage?
6. Does the result of short circuit current calculations depend of base conditions selection?
7. What is the basis of exact and approximate reduction of shorted circuit component parameters reduction (for generators, transformers, lines and chokes)?
8. What are goals of evaluation of short circuit mode's parameters? What is the transformation algorithm of equivalent circuits for short circuit current calculations?
9. What is electrical remoteness of a short circuit point from the power source?

### Topics for essay

1. Kinds, reasons and consequences of transients in electric supply system.
2. Estimation errors of short circuit current determination by exact and approximate reduction of equivalent shorted circuit parameters.



## **CHAPTER 2: TRANSIENTS AT THREE-PHASE SHORT CIRCUITS**

- 2.1. Short circuit in radial network without transformer coupling
- 2.2. Short circuit on generator's terminals
- 2.3. Short circuit at remote points of electric supply network
- 2.4. Initial value of periodic component of short circuit current
- 2.5. Periodic component of short circuit current
- 2.6. Short circuit current of emergency steady state

Test questions

Topics for essay

**2.1. Short circuit in radial network without transformer coupling**

To understand the character of current variation in time under three-phase short circuit let's consider the simplest radial network without transformer coupling powered from a source with constant voltage. Such source is usually called a *source of infinite power* and its limit power value is theoretically independent of external conditions (load variations, number of consuming units connected etc.). Practically it is possible when the power supply system is energized from powerful electric supply system and short circuit occurs in low-powered electric installations or long-distance grids.

Fig. 2.1 shows both radial network where sudden three-phase short circuit occurred and its three-phase equivalent circuit with lumped impedances of the network and load. The network is powered from infinite power source with phase voltage  $\underline{U}_{max,A}, \underline{U}_{max,B}, \underline{U}_{max,C}$ . Before short circuit occurrence currents  $I_A, I_B, I_C$  flow through the network, their instant values are determined by applied voltage of feeding network and can be found by projections of rotating current vectors  $\underline{I}_{ld,max,A}, \underline{I}_{ld,max,B}, \underline{I}_{ld,max,C}$  onto the time axis t-t (Fig. 2.2. presents projections for phase A only).

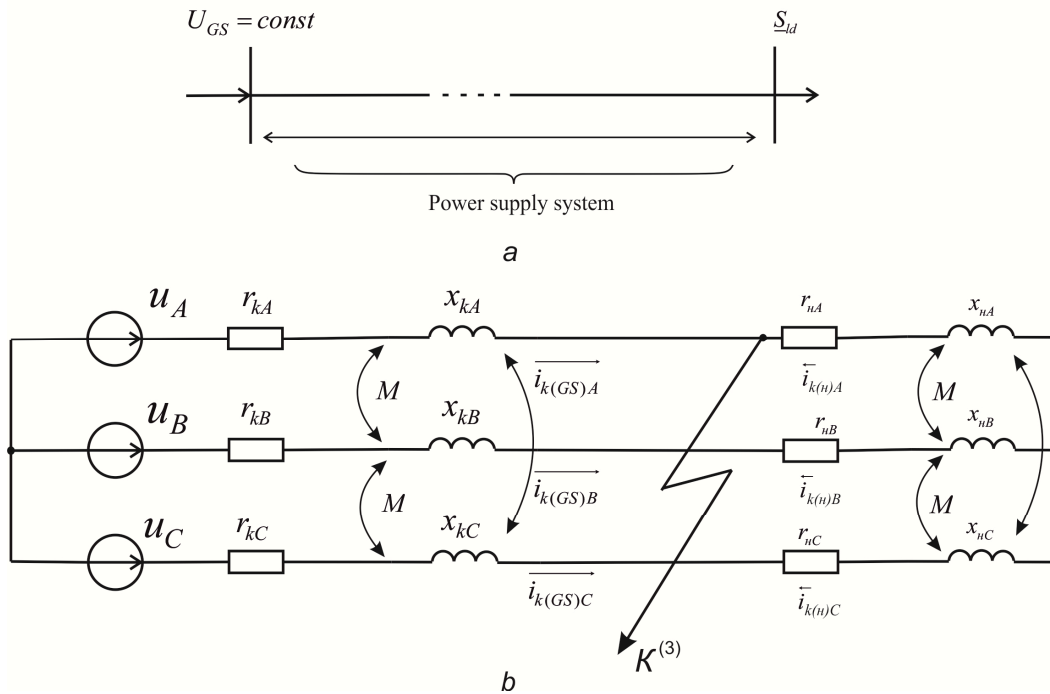


Fig. 2.1. Equivalent representation of the most simple electrical network: a) design circuit; b) three-phase equivalent circuit at three-phase short circuit.

When short circuit occurs the analyzed network splits into 2 parts. The first one is shunted from the source by the point of short circuit and has no internal supply but it has power supply in the components of electric installations.

The left part is still powered from the source of infinite power with constant voltage. The current in the shunted part flows until the power of electro-magnetic field transforms into the heat in circuit resistances.

The equation can be written for any phase, e.g. phase A

$$u_A = i_A r_{ld,A} + L_{ld,A} di_A / dt + M di_B / dt + M di_C / dt,$$

that, taking into account conditions

$$u_A = 0,$$

$$i_A = -(i_B + i_C);$$

$$L_{ld} = L_{ld,A} - M; r_{ld,A} = r_{ld,B} = r_{ld,C} = r_{ld},$$

can be transformed into the equation of a general type for each phase

$$i_{s(ld)} r_{ld} + L_{ld} di_{s(ld)} / dt = 0. \quad (2.1)$$

The equation solution for current is

$$i_{s(ld)} = i_{a(t=0)} \exp(-t / T_{a(ld)}), \quad (2.2)$$

That is free current component dying out according to the exponential law with the time constant

$$T_{a(ld)} = x_{ld} / (\omega r_{ld}). \quad (2.3)$$

Initial values of the current in phases  $A, B, C$  of a shunted circuit part are equal to their previous instant values as sudden current variations cannot occur when there is inductivity in the circuit. Despite the fact that free currents in phases damp out with the same time constant, their initial values are different and defined by displacement angles between phase currents. If, for example, at the time of short circuit current in one phase passes through zero there is no free currents in this phase and free currents in two other phases are equal in magnitudes but directed opposite to each other.

On the circuit part with a power source (Fig. 2.1.), where short circuit occurred, besides the free current a new forced current occurs caused by applied voltage. As resultant impedance of short-circuited network is reduced in comparison with the impedance of the previous operation, the forced currents  $I_{F,max,A}, I_{F,max,B}, I_{F,max,C}$  are higher than currents of the previous operation and displaced relatively to them (Fig. 2.1.). Voltage equations for any phase of short-circuited network can be given as

$$u = i_{s(GS)} r_s + L_s di_{s(GS)} / dt, \quad (2.4)$$

where  $L_s = L_{s,A} - M$  - are resultant inductance of a phase (considering influence of two other phases). The solution of equation (2.4) is

$$i_{s(GS)t} = (U_{max} / Z_s) \sin(\omega t + \alpha - \varphi_s) + i_{a(t=0)} \exp(-t / T_{a(GS)}), \quad (2.5)$$

where  $Z_\kappa$  is total impedance of the circuit;  $\varphi_\kappa$  is angle of current displacement relative to voltage;  $T_{a(GS)} = x_s / (\omega r_s)$  is circuit time constant;  $\alpha$  - the voltage phase angle at the initial instant of short circuit.

As it follows from the equation (2.5), that the first term of right side is a *forced (periodic) current component*  $i_{F(GS)t} \equiv i_{Ft}$  of constant maximum value  $I_{Fmax} = U_{max} / Z_s$ , the second term is a *free or aperiodic current component*  $i_{a(GS)t} = i_{at}$ , damping out exponentially with time constant  $T_{a(GS)} \equiv T_{a,s}$ .

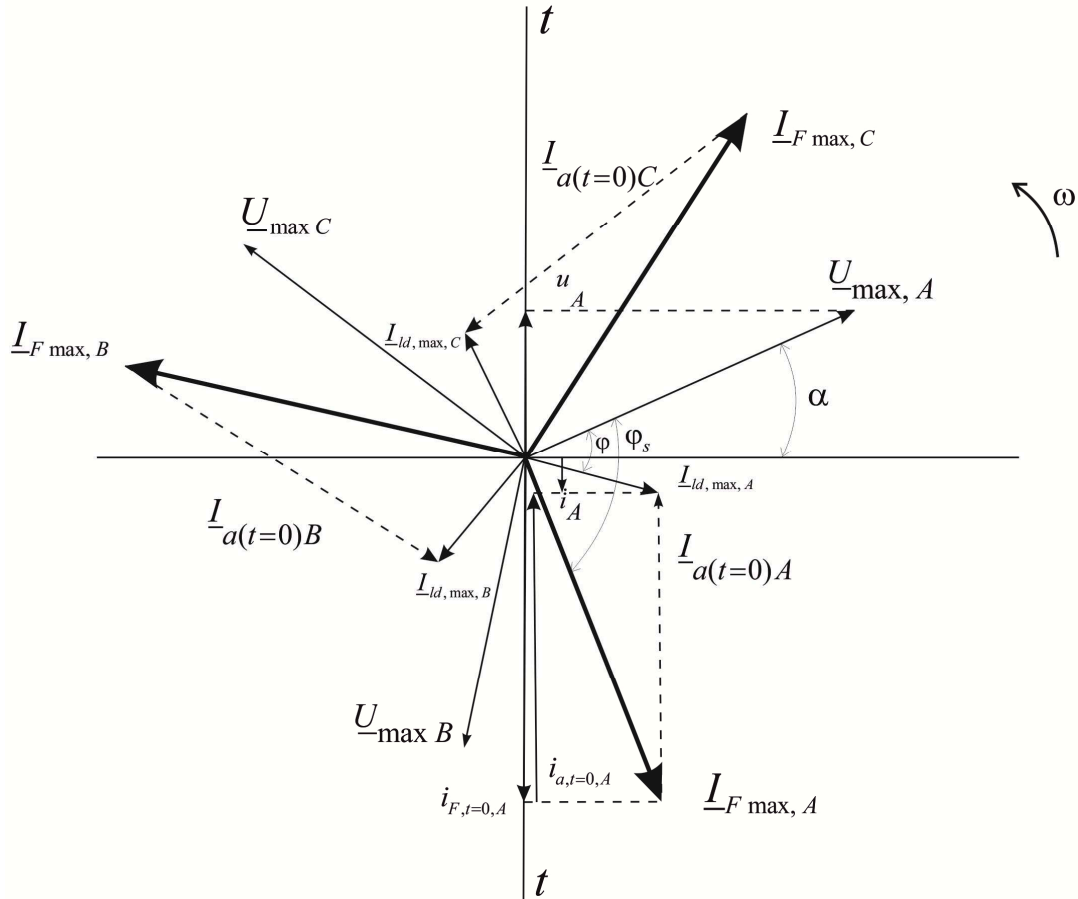


Fig. 2.2. Vector diagram of phase currents at the initial instant of three-phase short circuit occurrence for the circuit part with a power source

The initial value of total short circuit current is equal to the value of current of the previous operation at the time of short circuit occurrence. It consists of periodic and aperiodic components (Fig. 2.2.)

$$i_s(t=0) = i_{ld}(t=0) = i_F(t=0) + i_a(t=0), \quad (2.6)$$

and consequently

$$i_a(t=0) = i_s(t=0) - i_F(t=0) = I_{ld,max} \sin(\alpha - \varphi) - I_{Fmax} \sin(\alpha - \varphi_s) \quad (2.7)$$

As instantaneous values of currents  $i_F(t=0)$  and  $i_s(t=0)$  are projections of vectors  $I_{Fmax}$  and  $I_{ld,max}$  onto the time axis,  $i_a(t=0)$  is a projection of aperiodic current component  $(I_{ld,max} - I_{Fmax})$  onto the same axis (Fig. 2.2). Depending of the phase angle of switching  $\alpha$  (the angle between voltage  $U_{maxA}$  and real axis), initial aperiodic current  $i_a(t=0)$  can vary from the maximum value, when the vector  $(I_{ld,max} - I_{Fmax})$  is parallel to time axis  $t-t$ , to zero when this vector is perpendicular to the time axis.

Fig. 2.3. presents curves of current and its components time variation in phases of short-circuited network at three phase short circuit. The higher the value of short circuit aperiodic current component is, the more current curve is displaced relatively to the time axis.

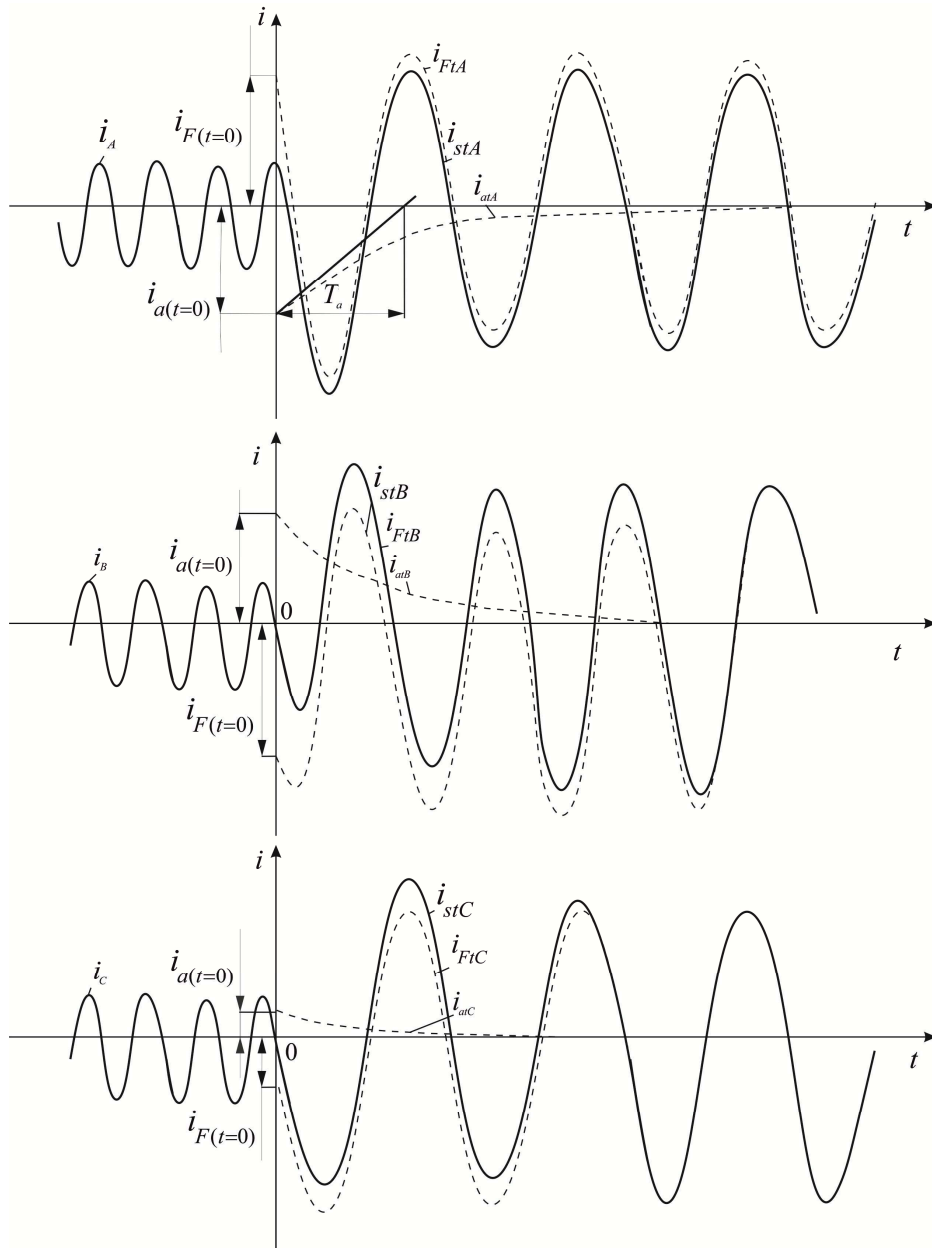


Fig. 2.3. Dependencies of total current and its components of time in phases A, B, C at three-phase short circuit

So in both parts of power supply system separated by the point of short circuit, the summands of the total current at the point of short circuit have been found:

$$i_{st} = i_{s(GS)} + i_{s(ld)} \quad (2.8)$$

or

$$i_{st} = I_{Fmax} \sin(\omega t + \alpha + \varphi_s) + i_{a(GS)(t=0)} \exp(-t/T_{a(GS)}) + i_{a(ld)(t=0)} \exp(-t/T_{a(ld)}) \quad (2.9)$$

In calculations connected with selection and check of electrical installation elements on operation conditions in transient, the maximum current passing through electrical equipment is determined. In this case the largest of equation (2.8) components is taken as unknown quantity; mostly it is the current passing from the source (hereinafter indices of belonging to the circuit part "GS", "ld" are omitted).

The given above mathematical description of current transient in both parts of power supply system relative to the point of short circuit is identical for all three phases of three-phase network. It proves sufficiency of use one phase equivalent circuit for analysis of processes at the three phase short circuit.

At calculations of the transient that occurs as a result of short circuit, usually the limiting values of the quantities are found.

The curve of aperiodic current component can be considered as curved axis of symmetry of short circuit total current curve. Due to influence of aperiodic component, short circuit total current varies according to the law different from the sinusoidal one. Maximum value of short circuit aperiodic component depends not only on switching angle  $\alpha$ , and a phase angle of network load current in the operation previous to short circuit. Under load current at short circuit equal to  $i_{ld(t=0)} = 0$ , the aperiodic component at zero time of transient  $i_{a(t=0)}$  is equal to maximum value of periodic component  $I_{Fmax}$ , if it goes trough its highest positive or negative value at that instant (Fig. 2.4).

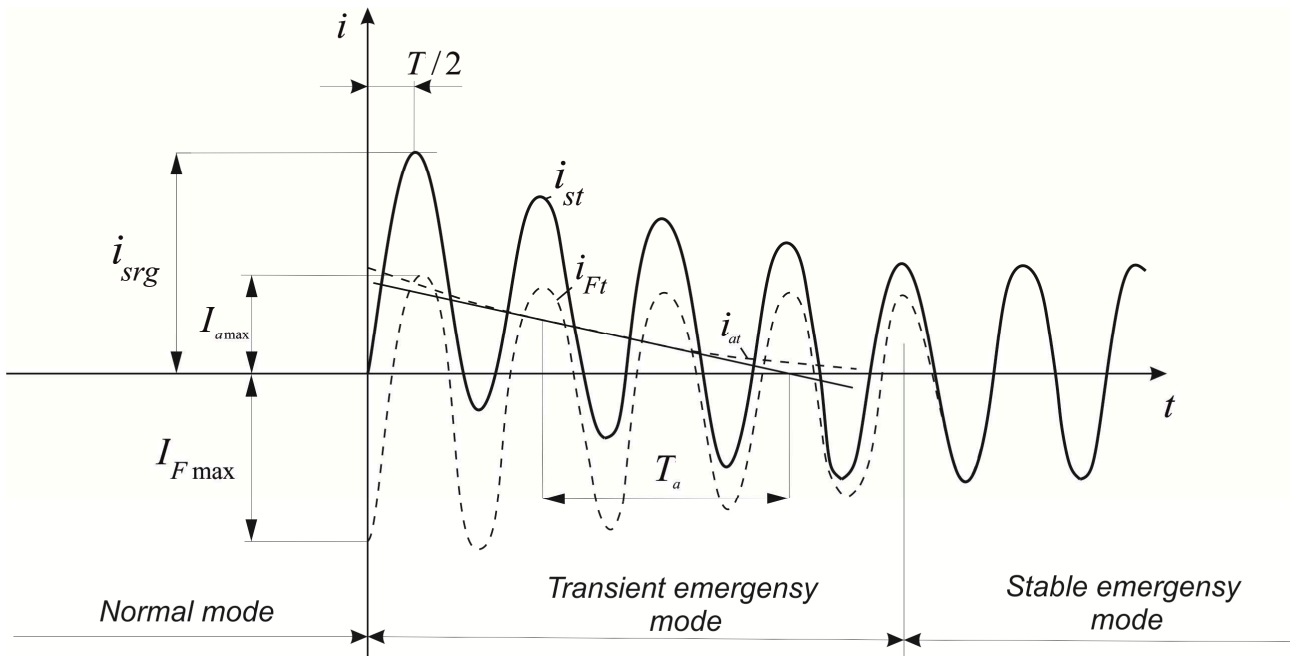


Fig. 2.4. Short circuit current and its components variation in time at maximum initial value of aperiodic component

If the value of  $I_{Fmax}$  of the expression (2.7) is substituted into the expression (2.5) on condition that the value of load current  $I_{ld,max} \sin(\alpha - \varphi) = 0$ , then total current of short circuit is a function of independent variables - time  $t$  and switching angle  $\alpha$ .

Simultaneous solution of the partial derivatives equations obtained from the previous equation

$$\begin{cases} \frac{\partial i_{st}}{\partial t} = \omega \cos(\omega t + \alpha - \varphi_s) + (1/T_a) \sin(\alpha - \varphi_s) \exp(-t/T_a) = 0; \\ \frac{\partial i_{st}}{\partial \alpha} = \omega \cos(\omega t + \alpha - \varphi_s) - \cos(\alpha - \varphi_s) \exp(-t/T_a) = 0, \end{cases}$$

determines the conditions when total short circuit instant current gets maximum value

$$tg(\alpha - \varphi_s) = -\omega T_a = -x_s / r_s = tg(-\varphi_s),$$

which is true under  $\alpha = 0$  (at the point of short circuit the source voltage variation curve passes zero value).

In networks with predominant inductive reactance  $\varphi_s \rightarrow 90^\circ$ . So, the conditions of appearance of maximum values of aperiodic component and instantaneous value of total current almost coincide. Calculating short circuit current, the maximum value of the instantaneous total current is determined at the largest value of the aperiodic component. It is assumed that it takes place roughly after a half-period (i.e. at  $t = 0,01s$  if  $f = 50Hz$ ) from the instant of the short circuit occurrence.

The first highest instant value of short circuit total current in the phases is named a "short circuit surge current",

$$i_{srg} = I_{Fmax} + i_{a(t=0,01c)}$$

As  $i_{srg}$  has the highest value when  $I_{Fmax} = I_{amax}$ , and  $i_{a(t=0,01c)} = I_{amax} \exp(-0,01/T_a)$ , expression (2.9) changes to

$$\begin{aligned} i_{srg} &= I_{Fmax} + I_{Fmax} \exp(-0,01/T_a) = \\ &= I_{Fmax} [1 + \exp(-0,01/T_a)] = k_{srg} I_{Fmax}, \end{aligned} \quad (2.10)$$

where

$$k_{srg} = 1 + \exp(-0,01/T_a) \quad (2.11)$$

Parameter  $k_{srg}$  is named *surge coefficient* that characterizes excess of the surge current over short circuit periodic current component maximum value. It lies in the limits  $1 < k_{srg} < 2$  that corresponds to limit values of time constant  $T_a$   $[0; \infty]$ .

Dependence of the surge coefficient on time constant  $T_a$  or ratio  $x_s/r_s$  is shown at Fig. 2.5. For the time equal to  $3T_a$ , short circuit aperiodic current component practically damps out (it accounts for less than 5% of its initial value).

In multibranch network accurate determination of time constant needs a lot of calculation. When there are several circuits in the network, the resultant short circuit aperiodic current component consists of their aperiodic current components sum. Free current component in any branch of such circuit is defined by decomposition of short circuit current expressed in operator form. If short circuit current expression in operator form is

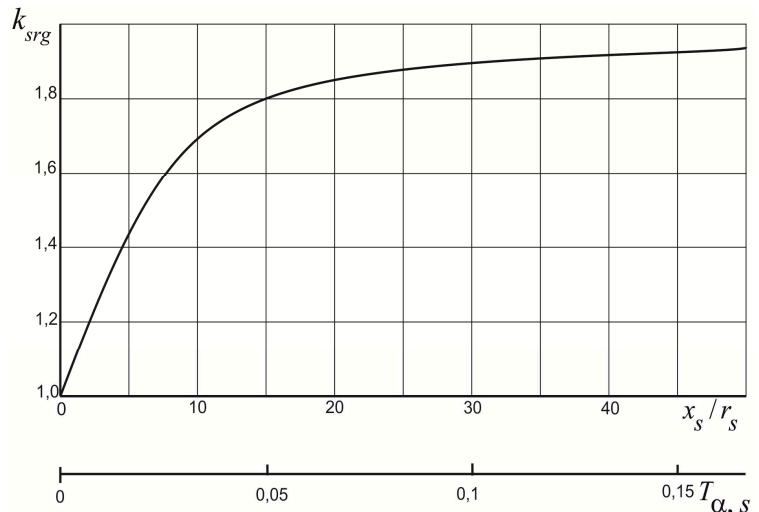


Fig. 2.5. Dependence of surge coefficient on the network's total resultant impedance components ratio or on the network time constant.

$$I_{st}(p) = F_1(p) / [pF_2'(p)], \quad (2.12)$$

then, according to Laplace expansion formula,

$$i_{st} = \hat{\lambda}^{-1} \left\{ F_1(0)/F_2(0) + \sum_{s=1}^n [F_1(p_s)/p_s F_2'(p_s)] \exp(p_s t) \right\} =$$

$$= i_{Ft} + i_{at} = i_{Ft} + \sum_{s=1}^n i_{a(t=0)s} \exp(-t/T_{a,s}), \quad (2.13)$$

From here aperiodic short circuit current component

$$i_{at} = i_{a(t=0)1} \exp(-t/T_{a1}) + i_{a(t=0)2} \exp(-t/T_{a2}) + \dots +$$

$$+ i_{a(t=0)s} \exp(-t/T_{a,s}),$$

where  $T_{a1} = -1/p_1; T_{a2} = -1/p_2; \dots; T_{a,s} = -1/p_s$  - are time components of particular aperiodic current components;  $p_1, p_2, \dots, p_s$  - are roots of characteristic equation.

Initial values of particular aperiodic current components  $i_{a(t=0)1}, i_{a(t=0)2}, \dots, i_{a(t=0)s}$  and their time constants are functions of variable currents of analyzed circuit all elements.

As this method of time constants determination, even for comparatively simple circuit, causes significant difficulties, approximate solution is used for practical purposes. Aperiodic short circuit current component is assumed to die out in accordance with exponential law with equivalent time constant:

$$T_{a,e} \approx x_{s,e} / (\omega r_{s,e}), \quad (2.14)$$

where  $x_{s,e}$  - is resultant inductive reactance of the circuit relative to the point of short circuit determined on condition that all active resistances equal to zero;  $r_{s,e}$  - is resultant ohmic resistance determined on condition that  $x = 0$  for all inductive elements.

When  $T_{a,e}$  is determined by this method the equivalency of electric charge both in actual and assumed conditions is obeyed. So, aperiodic short circuit current component of a complex circuit is presented by one equivalent exponential curve

$$i_{at} = i_{a(t=0)} \exp(-t/T_{a,e}). \quad (2.15)$$

Ratios of inductive reactance and resistance for elements of electric network have the following values:

Component name	Ratio x/r
Turbo-generators	15...150
Water-wheel/hydro generators	40...90
Transformers	7...50
Chokes 6-10kV	15...80
Aerial (overhead) transmission lines	2...8
Cable lines	0.2...0.8
Generalized load	2.5



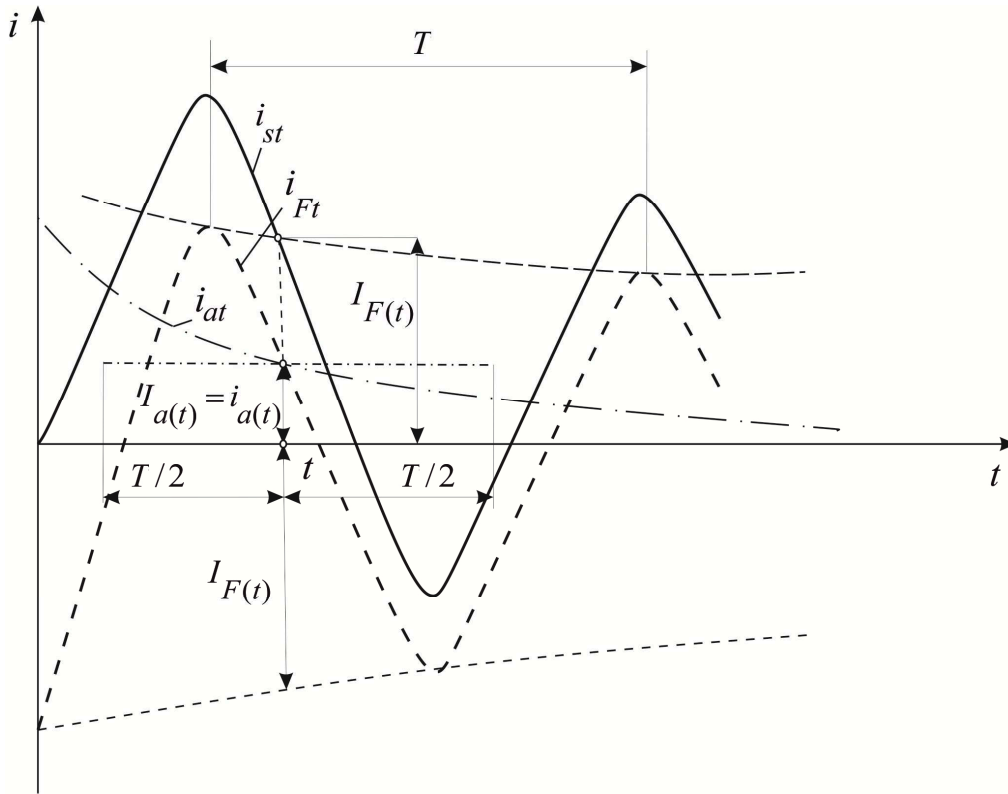


Fig. 2.6. Determining transient current r.m.s. value

The given values of  $x/r$  are used for estimation of equivalent constants of aperiodic component damping time at calculation of short circuit current. Rough values of  $x/r$  ratios, surge coefficient  $k_{srg}$ , and time constant  $T_a$  for typical cases can be taken from Table 2.1. To determine r.m.s. value of short circuit total current and its components it is necessary to know the law of their variation in time. As in general case the source voltage can vary in maximum value (a source of limited power), and total current and its periodic component are complex non-sinusoidal time functions, determining of their r.m.s. value involves significant difficulties.

Effective value of total short circuit current at any instant  $t$  is determined as root mean-square current value for period  $T$ , the given instant of time being in the middle of this period. It is assumed that for period  $T$  maximum value of the periodic and aperiodic components do not change and are equal to their values at analyzed time  $t$  (Fig. 2.6). For the given instant the maximum value of periodic component is determined by the envelope curve in the middle of the period  $T$ , and its r.m.s. value at this point is  $I_{F(t)} = I_{Fmax(t)} / \sqrt{2}$ . R.m.s. value of aperiodic component during this period is equal to its instant value in the middle of the period, i.e.  $I_{a(t)} = i_{a(t)}$ .

R.m.s. value of the total short circuit current for any instant of time  $t$  is

$$I_{st} = \sqrt{I_{F(t)}^2 + I_{a(t)}^2} \tag{2.16}$$

that corresponds to the known expression for non-sinusoidal current effective value determination.

Table 2.1

Average values of parameters ( $x/r$ ,  $k_s$ ,  $T_a$ ) for typical circuit branches, adjacent to the point of short circuit

Branch name or a point of short circuit point	$x/r$	$k_s$	$T_a$ , s
Generator-transformer branch	30...50	1.9...1.95	0.1...0.2
Branch with asynchronous (induction) motor	6.3	1.6	0.02
After the line choke at the power plant	30	1.9	0.1
After the line choke at the substation	18...20	1.85	0.06
After the cable line	3	1.4	0.01
After the transformer	6.3	1.6	0.02
At the connection of substation primary voltage	15	1.8	0.05
At the connection of substation secondary voltage	20	1.85	0.06

The largest r.m.s. value of the surge current  $I_{\text{srg}}$ , used for selection and check of electrical equipment, takes place in the first period of transient. It is determined when aperiodic component during this period is assumed to be equal to its instant value when  $t = 0,01\text{ s}$  and periodic component is assumed to be equal to its maximum value. On this condition

$$I_{\text{srg}} = \sqrt{I_{\text{F}}^2 + i_{a(t=0,01\text{c})}^2} = \sqrt{I_{\text{F}}^2 + [I_{a,\text{max}} \exp(-0,01/T_a)]^2} \quad (2.17)$$

Assuming  $I_{a,\text{max}} = I_{\text{Fmax}} = \sqrt{2}I_{\text{F}}$  and taking into account that  $\exp(-0,01/T_a) = (k_{\text{srg}} - 1)$ , expression (2.17) can be presented as

$$I_{\text{srg}} = I_{\text{F}} \sqrt{1 + 2(k_{\text{srg}} - 1)^2}$$

When surge coefficient lies in the limits of  $1 < k_{\text{srg}} < 2$  currents ratio is in the limits  $1 < I_{\text{srg}} / I_{\text{F}} < \sqrt{3}$ .

## 2.2. Short circuit on generator's terminals

Sudden three-phase short circuit on generator terminals causes the most hazardous emergency operation. Resultant impedance of a formed shorted circuit is equal to internal impedance of a generator and transient is characterized by maximum changes of voltage and current. The process is accompanied with aperiodic current occurrence that is superposed with periodic (forced) current generated by a power source. To calculate these currents, the previous relationships are valid (2.1). The curves of the current and its components variation in time at one of short-circuited network phases at three-phase short circuit on generator terminals, without device of automatic excitation control, are shown in Fig. 2.7.

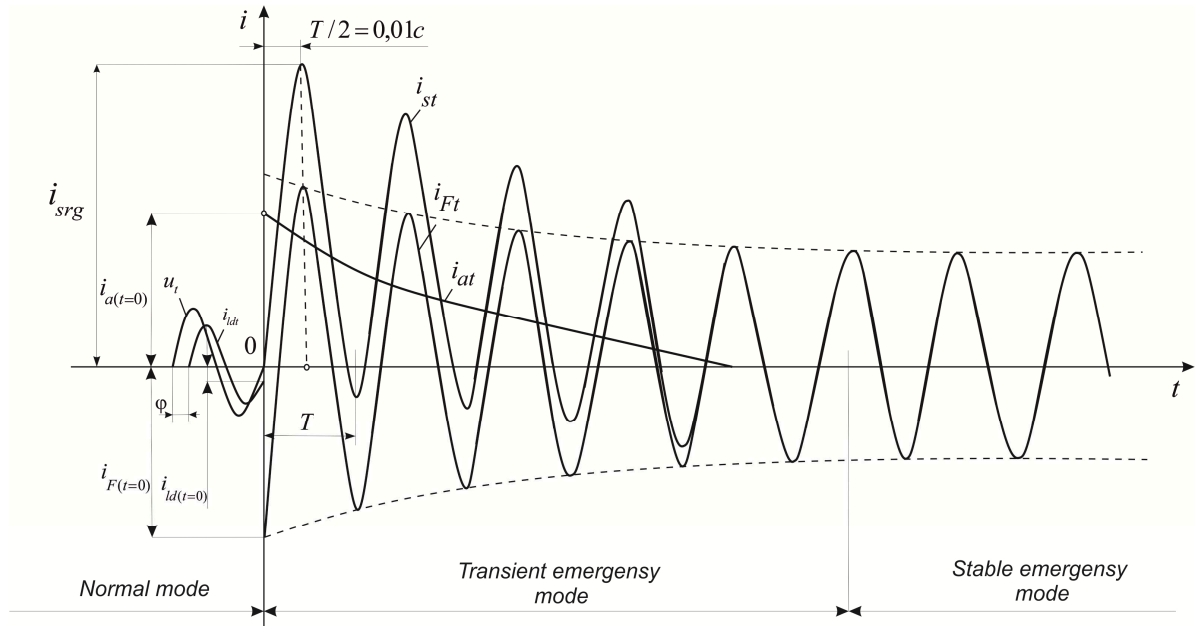


Fig. 2.7. Variations in time of phase current and its components for a generator without device of automatic excitation control at three-phase short circuit on terminals

Before short circuit (point  $t=0$ ) the generator worked in normal operation, when the current  $i_{ld}$  passes through the load circuit. At the time when the load current has value  $I_{ld}(t=0)$  the short circuit occurred that resulted in transient condition manifested by current increase. Load current and short circuit periodic current component of zero time can be found using vector (phasor) diagram (Fig. 2.2) with expressions

$$\left. \begin{aligned} i_{ld}(t=0) &= I_{ld,max} \sin(\alpha - \varphi); \\ i_{F}(t=0) &= I_{F,max} \sin(\alpha - \varphi_s). \end{aligned} \right\} \quad (2.18)$$

At short circuit on generator terminals or nearly located points of the circuit, the resultant impedance is practically inductive, with the angle  $\varphi_s$  that is close to  $90^\circ$  and always bigger than  $\varphi$ .

As generator in this case is a source of limited power and works without automatic excitation control, voltage on its terminals and consequently periodic component of short circuit current are reduced in the course of transient in comparison with their initial values. It is explained by the fact that free currents induced at zero time of short circuit in the field winding, damper windings and rotor core die out, the stator reaction magnetic flux attenuate resultant magnetic flux in generator air gap at the field current being flat.

The maximal value of short circuit periodic current component remains constant if the source voltage does not vary during transient (as in a source of unlimited power). Short circuit periodic current component during transient without automatic excitation control is shown in Fig. 2.7 as a sine curve with decreasing maximum value. Duration of transient with current frequency  $f = 50$  Hz and presence of resistance in shorted circuit is 0.1-0.2 s. Average value of aperiodic component decay time constant  $T_a = 0,05$  s and its decay time is  $t \approx 0.15$  s.

Because of aperiodic component decay, short circuit current for the time  $t \geq 0.15$  s can be assumed equal to its periodic component. When transient have been ceased, steady operation begins (instant and r.m.s. values of steady short circuit current are given as  $i_\infty$  and  $I_\infty$ ). Short circuit current initial value is more than steady-state current ( $I_{F}(t=0) > I_\infty$ ).

Modern generators are equipped with devices of automatic excitation control to keep voltage on generator terminals constant or changing in permissible limits when power supply network operation changes. If short circuit occurs on terminals of generator with device of automatic field

current control, it significantly affects on transient current. Curves of current and current components variation in time at short circuit on terminals of a generator with automatic field control are given in Fig. 2.8.

At the initial instant of short circuit occurrence, due to inertia of magnetic fluxes linked with generator winding, device of automatic excitation control has no significant effect on current transient. But with time passing when device of automatic excitation control starts operation, field current and coupled with it components of stator electromotive force and current, and damper windings increase. This process is comparatively slow, mostly emf of generator and stator periodic current component stipulated by it are changed.

As it is shown in Fig. 2.8 when short circuit occurs on terminals of generator with the device of automatic excitation control, at the beginning of transient, varies in the same manner as in generators without automatic excitation control device. This time interval is characterized by time of voltage decrease to the value when automatic excitation control device starts operation and its own response time. After coming into operation of automatic excitation control device, the voltage on generator terminals and short circuit periodic current component start increasing and gain their steady values corresponding limiting field current.

As action of generator device of automatic excitation control appears in several time periods after short circuit occurrence, initial values of periodic and aperiodic components as well as surge current of short circuit in transient remain the same as in the circuit without automatic excitation control device. So free components of the stator and excitation winding occurring at sudden short circuit damping out is compensated to some extent by short circuit current increase because of operation of generator device of automatic excitation control.

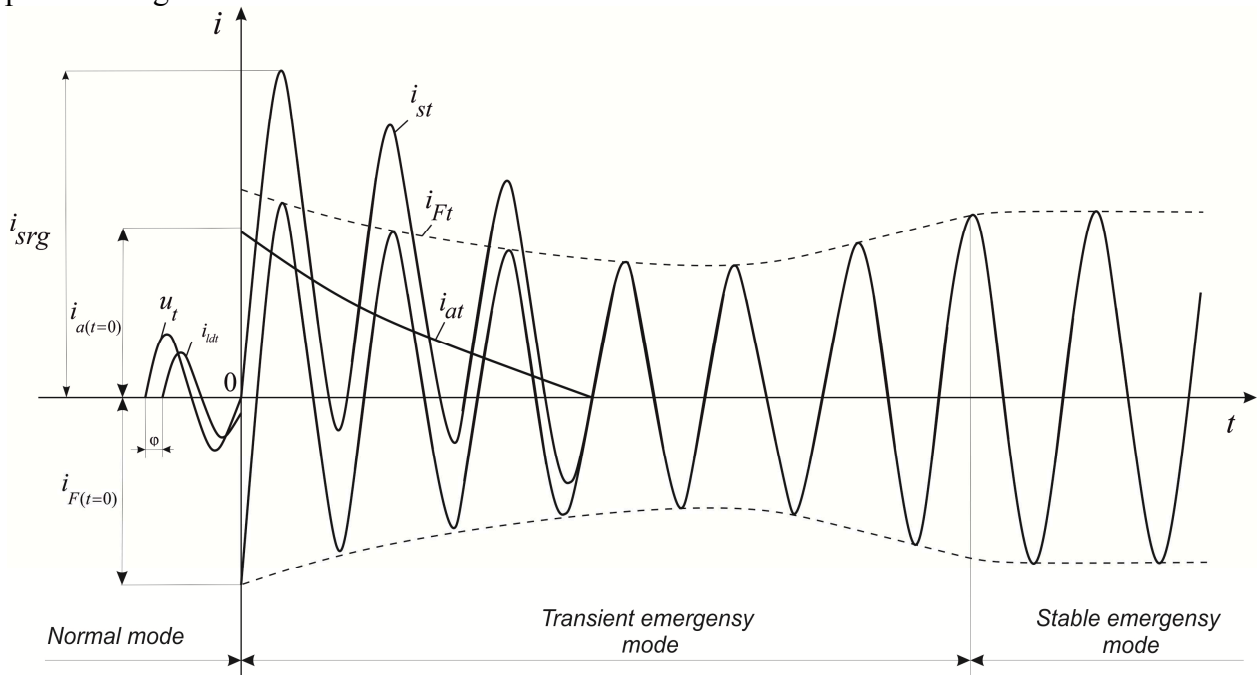


Fig. 2.8. Time variation of phase current and its components for a generator with automatic excitation control device at three-phase short circuit on terminals

The curve of short circuit current has different shape depending on the ratio of the current values and their variation character.

Aperiodic component  $i_{at}$  practically remains the same as in the circuit without automatic excitation control device.

Periodic component  $i_{Ft}$  can die out, or increase, or remain constant as it is shown at fig. 2.9 depending on the ratio of initial and steady-state short-circuit current values at boundary excitation current. Short circuit current remains constant if generator voltage gains rated or permissible limiting value under the effect of the automatic excitation control device.

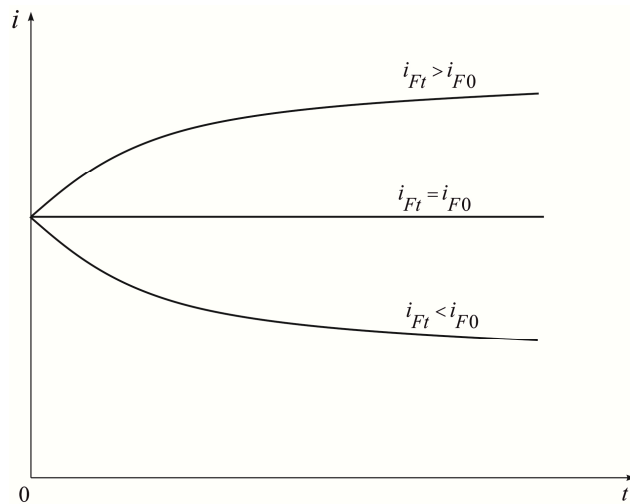


Fig. 2.9. Time variation of short circuit current periodic component for a generator with automatic excitation control at different values of limiting excitation current and excitation winding time constant  $T_e = 0$

Changes of r.m.s. values of stator current periodic component and aperiodic current component in excitation and rotor direct-axis damper windings during transient caused by short circuit on generator terminals are given in Fig. 2.10, a, b, c correspondingly. Dashed curves show currents in the circuit without automatic excitation control device and solid curves show currents with account effect of automate excitation control device.

The figure shows that under the effect of the automatic excitation control device current values in a stator and excitation winding gradually increase and in direct-axis damper winding current decreases as current change in this winding under the effect of automatic excitation control is opposite to free current. At reaching limit of automatic excitation control, analyzed currents in generator windings take their final steady values.

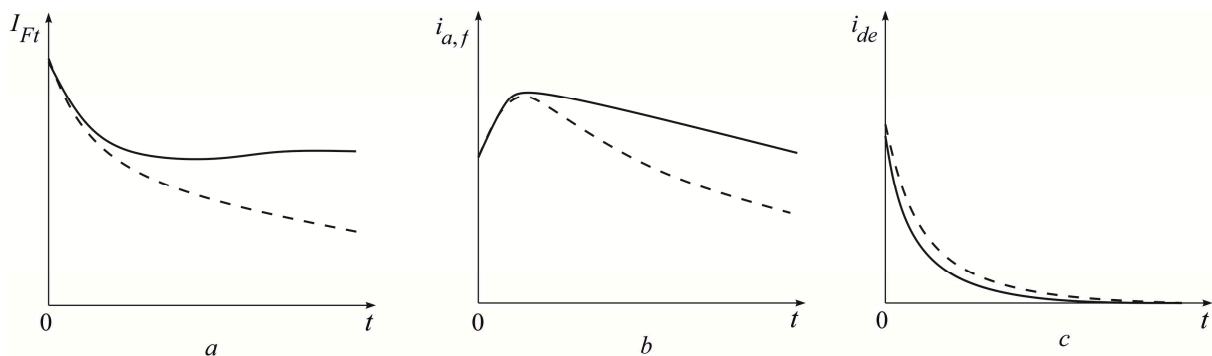


Fig. 2.10. Effect of automatic excitation control device at short circuit on generator terminals on the current variation in winding: a-in stator winding; b-in excitation winding; c -in direct-axis damper winding

### 2.3. Short circuit at remote points of electric supply network

When short circuit occurs in supply system short circuit current is significantly less in comparison with currents that occur at short circuit on generator terminals, and resultant impedance of short circuit current path increases. Influence of the short circuit to the generator operation becomes weaker. Duration of transient that depends on electrical remoteness of the short circuit point shortens. The more distant the point to short circuit is the less is the transient duration.

Periodic current component remains constant in maximum value when short circuit occurs at the electrically remote point of power supply network (Fig. 2.11). In this case the generator emf

variation may be neglected and voltage on its terminals can be assumed to be constant and equal to rated one. The short circuit current significantly exceeds the rated one and is dangerous for power supply network components.

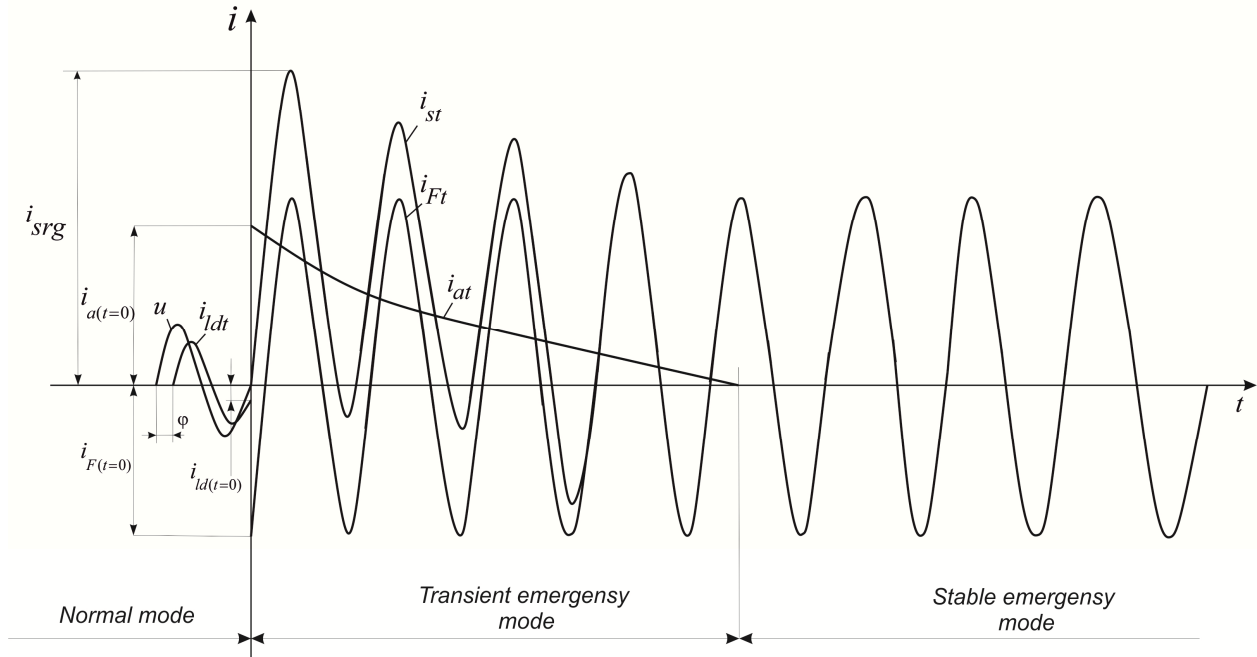


Fig. 2.11. Variation in time of current and its components at short circuit at the remote point of electric supply network

So, when short circuit occurs at electrically remote point of power supply network periodic current component is assumed to be constant, and since the initial instant the current in the point of short circuit is equal to

$$I_{F(t=0)} = I_{Ft} = I_{\infty} \quad (2.19)$$

Aperiodic current component occurs at any remoteness from the source and damp out as soon as higher the active resistance component of shorted circuit is. At short circuit after which initial value of periodic current component and steady-state current  $I_{\infty} I_{F(t=0)}$  are the same, external reactance of a generator circuit  $x_{\text{ext}}$  can be found expressing it by corresponding emf and reactances with the use of the equation

$$E_q'' / (x_d'' + x_{\text{ext}}) = E_q / (x_d + x_{\text{ext}}), \quad (2.20)$$

where  $E_q''$  and  $E_q$  - are electromotive forces of generator at zero time of short circuit occurrence and in steady-state operation.

From (2.20) follows that

$$x_{\text{ext}} = (E_q'' x_d - E_d x_d'') / (E_d - E_q'')$$

If the dependence of ratio  $I_{F(t=0)} / I_{\infty}$  and external reactance  $x_{ext}$  is built (Fig. 2.12), it can be found out that when there is no voltage control the ratio is always more than one and ends to one only in the limit (curve 1). When generator voltage is controlled with automatic excitation control device (curve 2) the ratio  $I_{F(t=0)} / I_{\infty}$  decreases to minimum value (0.6...0.8), then starts increasing and approaches one in the limit too. Limiting value of external reactance of a generator shorted circuit at which ratio  $I_{F(t=0)} / I_{\infty}$  in the system of electric supply with voltage control starts increasing is called critical and denoted by  $x_{cr}$ . The value  $x_{cr}$  depends on synchronous machine parameters and its previous operation mode.

Fig. 2.13 shows variation of stator periodic current component r.m.s. values and generator voltage as the function of time with switched on (solid lines) and switched off (dotted lines) device of automatic excitation control and short circuit remoteness characterized with values of external impedance of a short circuit current path

$$x_{ext} = 0; x_{ext} = x_{cr}; x_{ext} > x_{cr}.$$

Comparison of the obtained curves shows that, at slight electrical remoteness of short circuit, the generator current curve is flat owing to the action of a generator automatic excitation control. It has the shape of a falling curve. With increase of  $x_{ext}$ , the current curve is at first reduced to some minimum value than it starts increasing gaining final steady current value that can exceed initial value of short circuit current.

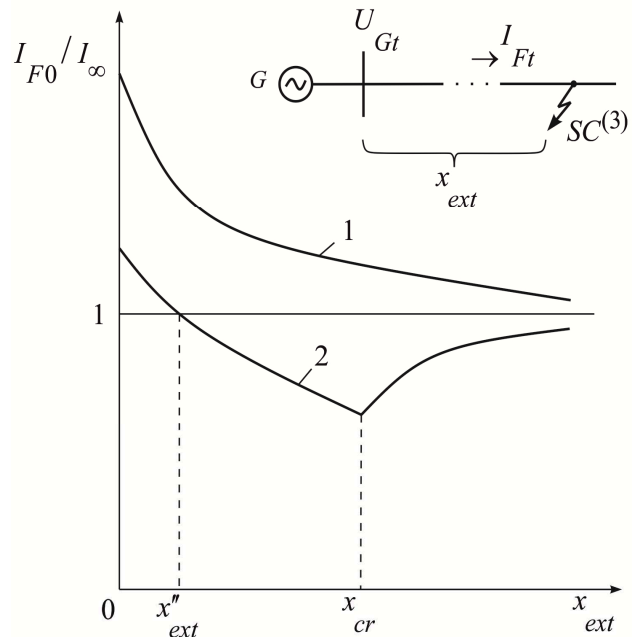


Fig. 2.12. Dependence of ratio  $I_{F(t=0)} / I_{\infty}$  on electrical remoteness of a short circuit point and the presence of automatic excitation control device on generator: 1) generator without device; 2) generator with device

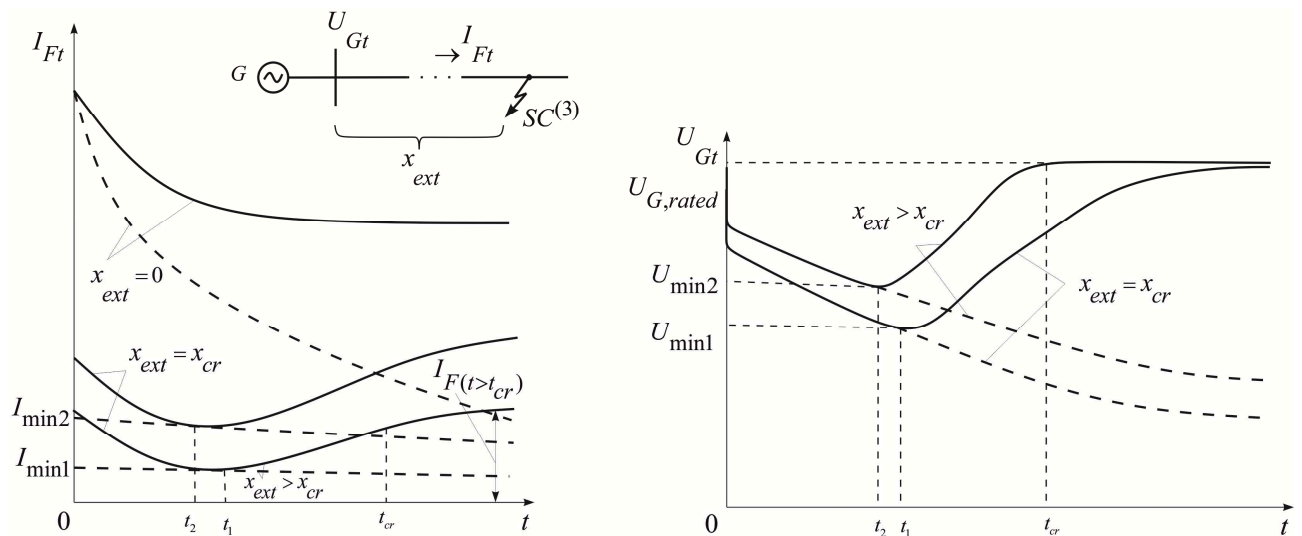


Fig. 2.13. Effect of automatic excitation control on current and voltage variation at different electrical remoteness of a short circuit point ( $x_{ext}$ )

When  $x_{\text{ext}} > x_{\text{cr}}$  generator voltage gains the rated value in time  $t = t_{\text{cr}}$  and farther remains constant. Short circuit current for  $t \geq t_{\text{cr}}$  remains constant too and is defined by expression

$$I_{Ft} = U_{G,\text{rated}} / x_{\text{ext}}. \quad (2.21)$$

Time of generator voltage increase to its rated-value under the effect of automatic excitation control device is called *critical time*  $t_{\text{cr}}$ . This time is reduced when short circuit electrical remoteness increases, getting in limits  $t = 0$ .

#### 2.4. Initial value of periodic component of short circuit current

Periodic current component at zero time of short circuit occurrence is defined using the principle of initial resultant rotor flux linkage invariability. As rotor flux linkage remains constant at the time of sudden short circuit occurrence, emf induced in the stator doesn't vary too. As it is shown in chapter 3, transient emf and inductive reactance of a synchronous machine without damper winding are defined by expressions

$$E'_q = E_{qf} x_{ad} / (x_{\sigma f} + x_{ad}) = E_{qf} x_{ad} / x_f; \quad (2.22)$$

$$x'_d = x_{\sigma} + x_{\sigma f} x_{ad} / (x_{\sigma f} + x_{ad}). \quad (2.23)$$

If parameters of the previous mode (and for the zero time of occurrence) are denoted with indices "(O)" and "O" correspondently, transient emf and initial transient current of short circuit can be presented as

$$E'_{qO} = E'_{q(0)} = U_{q(0)} + I_{d(0)} x'_d; \quad (2.24)$$

$$I'_{dO} = E'_{dO} / (x'_d + x_{\text{ext}}). \quad (2.25)$$

For synchronous generators with damper contours, subtransient emf

$$E''_q = (E_{qf} / x_{\sigma f} + E_{q,de} / x_{\sigma,de}) / (1/x_{\sigma} + 1/x_{\sigma,de} + 1/x_{ad}), \quad (2.26)$$

and subtransient reactance

$$x''_d = x_{\sigma} + x_{ad} \parallel x_{\sigma f} \parallel x_{\sigma,de}. \quad (2.27)$$

Similarly to (2.22) and (2.23) subtransient emf and initial value of subtransient current of short circuit are determined by expressions

$$E''_{qO} = E''_{q(0)} = U_{q(0)} + I_{q(0)} x''_d; \quad (2.28)$$

$$I''_{dO} = E''_{dO} / (x''_d + x_{\text{ext}}). \quad (2.29)$$

Using the expressions (2.22) and (2.24) for determining  $E'_{q(0)}$  and  $E''_{q(0)}$  is inexpedient as currents and voltages of synchronous machine should be decomposed on its axes components.

Nevertheless analysis of vector diagrams of salient-pole and nonsalient pole synchronous machines in the previous mode (Fig. 2.14) shows that values  $E'_{q(0)}$  and  $E''_{q(0)}$  can be determined with accuracy sufficient for practical calculation by expressions:

for machines without damper contours

$$E'_{q(0)} \approx E'_{(0)} \approx U_{(0)} + I_{(0)} x'_d \sin \varphi_{(0)} \quad (2.30)$$



for machines with damper contours

$$E''_{q(0)} \approx E''_{(0)} \approx U_{(0)} + I_{(0)}x''_d \sin \varphi_{(0)} \quad (2.31)$$

For machines of power less than 100 MW at their full load, rated voltage and  $\cos \varphi = 0,8$  before the beginning of the transient (short circuit), average values  $x'_d, x''_d$  and  $E'_{q(0)}, E''_{q(0)}$  in per unit:

- for turbogenerators – 0.2; 0.13 and 1.12; 1.078;
- for hydrogenerators – 0.35; 0.25 and 1.15; 1.21.

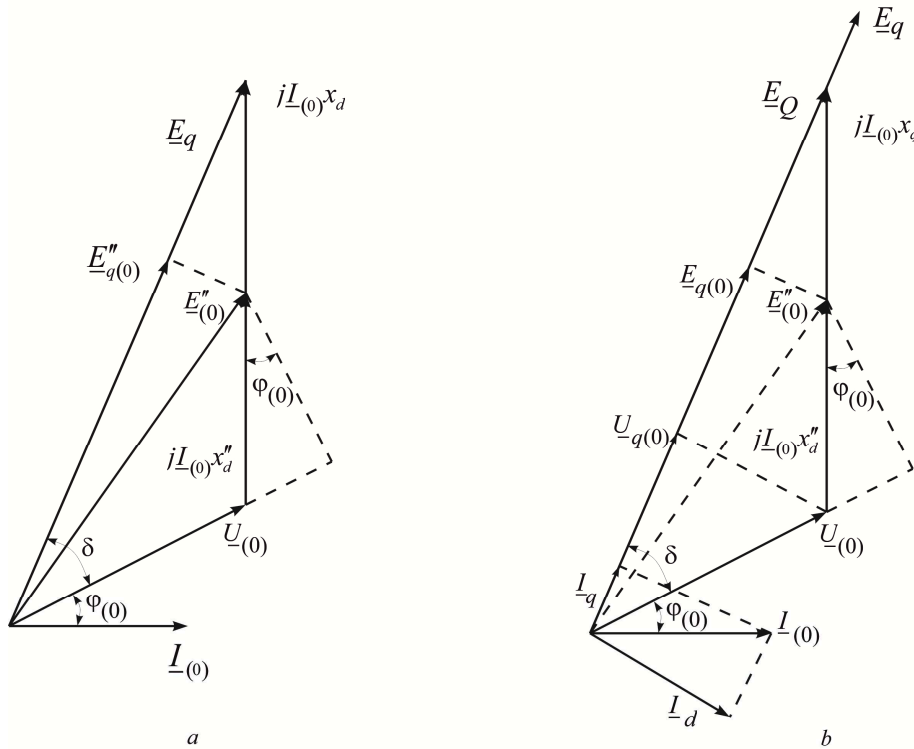


Fig. 2.14. Vector diagrams for synchronous machine with rotors of different types before the beginning of transient  
 a) generator with non-salient rotor  
 b) salient pole generator

In approximate calculations it is often assumed that  $E'_{*q(0)} = 1, E''_{*q(0)} = 1$ , despite on value of a load preceding short circuit. When it is necessary to consider the load of the machine previous operation expressions (2.24) and (2.28).are used to determine emf.

Initial values of periodic current component of three-phase short circuit are determined by resultant reactance of a short-circuited network in concrete units or per unit  $x_{*(b)res}$  and transient emf  $E'_{(0)}$  emf when damper contours are absent, and subtransient emf  $E''_{(0)}$  when damper contours are available.

If short circuit network is powered from several sources with equivalent emf  $E''_{\Sigma}$ , and resultant reactance of the network is expressed in concrete units: initial value of short circuit periodic current component is

$$I_{F(t=0)}^{(3)} = E''_{\Sigma} / (\sqrt{3}x_{(b)res}) \quad (2.32)$$

If emf and circuit reactance are expressed in per unit

$$I_{F(t=0)}^{(3)} = E''_{*\Sigma} I_b / x_{*(b)res}, \quad (2.33)$$

where  $I_b$  - is base current, corresponding the voltage of the transformation step, where short circuit current is determined average relative values of power supply network component parameters given in Table 2.2 can be used for approximate calculations of short circuit current.

Table 2.2

Average relative values  $x''_{*d}$  and  $E''_{*}$  of power supply system components

Circuit component	$x''_{*d}$	$E''_{*}$
Turbogenerator with power up to 100 MW	0.125	1.08
Turbogenerator with power up to 100...500 MW	0.2	1.13
Hydrogenerator with damper windings	0.2	1.13
Hydrogenerator without damper windings	0.27	1.18
Synchronous motor	0.2	1.2
Synchronous capacitor	0.2	1.1
Induction (asynchronous) motor	0.2	0.9
Generalized load	0.35	0.85

To determine initial value of short circuit periodic current component at the fault point at the given preceding operation of power supply system the *principle of superposition* can be applied. The desired current is found by superposition of emergency current (itself) on the current of previous operation. Root-mean square current is got as a result of superposition of a number of conditional currents each corresponding the action of one or several electromotive forces, when all remaining circuit components are switched on.

When the number of emf is significant the initial value of short circuit periodic current component determining becomes more simple if there is used the two-terminal circuit theorem.

According to the theorem current at the point of short circuit can be found as a sum of previous current  $I^{(0)}$  in the circuit branch and component  $I_{F,fault}$  of emergency current from the emf action equal to the value of voltage applied to this point in previous operation  $U_{s(0)}$ .

Emergency current component under the given condition is

$$I_{F,fault} = -U_{s(0)} / x_{input,s}, \quad (2.34)$$

where  $x_{input,s}$  - is input reactance of the circuit from the side of short circuit at condition that all electromotive forces in the circuit are equal to zero. Current and voltage of the analyzed  $j$ -branch of the diagram at short circuit in the point  $K$  are determined by expressions

$$I_j = I_{js} + I_{j(0)}; \quad (2.35)$$

$$U_j = U_{js} + U_{j(0)}, \quad (2.36)$$

where  $I_{js} = I_s c_{js}$ ;  $c_{js}$  - is coefficient of current distribution for  $j$ -branch of the circuit at short circuit in the point  $K$ .

Influence of the network load on initial value of short circuit periodic current component depends on the value of residual voltage at the point of its connection. The farther the power source from the point of short circuit located and the closer to that point the load is, the more the load has influence on the growth of the short circuit current.

When short circuit occurs behind low-power transformers, chokes, long cable lines (short circuit is significantly remotod from the power source), mostly impedance of these components effects resultant circuit impedance. It can be assumed that short circuit point is powered from the source of infinite power is ( $S_{GS} = \infty$ ;  $x_{GS}'' = 0$ ;  $E_{*GS}'' = U_{*GS} = 1 = const$ ). Short circuit periodic current is sustained damped ( $I_{F(t=0)} = I_{Ft}$ ) and at zero time

$$I_{F(t=0)}^{(3)} = I_b / x_{*(b)res} \quad (2.37)$$

Similarly the power of short circuit is determined

$$S_{s(t=0)}^{(3)} = S_b / x_{*(b)res} \quad (2.38)$$

Current and power values obtained by (2.37) and (2.38) are rather overestimated compared to their real values, because actually  $x_{GS}'' > 0$ .

## 2.5. Periodic component of short circuit current

Short circuit current variation during transient is stipulated by demagnetizing effect of synchronous generator stator reaction, free current damping out and affect of generator automatic excitation control device

R.m.s. value of main frequency periodic current component

$$I_{Ft} = \sqrt{I_{d,Ft}^2 + I_{q,Ft}^2} \quad (2.39)$$

If short circuit occurs on generator clamps and produces excitation rise the synchronous machine periodic components on the axis  $d$  and  $q$  at any instant can be found according to formulae

$$\begin{aligned} I_{d,Ft} &= E_{q(0)} / x_d + (E'_{q(0)} / x'_{d,de} - E_d / x_d) \exp(-t / T'_d) + \\ &+ (E''_{q(0)} / x''_d - E'_{q(0)} / x'_{d,de}) \exp(-t / T''_d) + \Delta I_{lim} F_d(t) = \\ &= I_\infty + (I' - I_\infty) \exp(-t / T_d) + (I'' - I') \exp(-t / T''_d) + \\ &+ [(E_{q,np} - E_{q(0)}) / x_d] F_d(t); \end{aligned} \quad (2.40)$$

$$I_{q,Ft} = (U_{d(0)} / x''_q - U_{d(0)} / x_q) \exp(-t / T''_q) \quad (2.41)$$

where  $I_\infty$ ;  $I'$ ;  $I''$  - are periodic components of short circuit steady-state current, transient and subtransient ones respectively.

$$x''_{d,de} = x_d (T'_d / T_{d0}) \approx x_d (T'_f + T'_{de}) / (T_{f0} + T_{de0}) \approx (0,9 - 1) x_d;$$

$$x''_q = x_q - x_{aq}^2 / x_{qe};$$

Time constants of the current dieing:

$$T'_d \approx T'_f + T'_{de}; \quad T''_d \approx \sigma' T'_f + T'_{de} / (T'_f + T'_{de});$$

$$T''_q = T_{qe0} (x''_q / x_q);$$

Time constants of excitation winding in the case of closed stator and open damper winding:

$$T_f = T_{f0}(x'_q/x_q) = T_{f0}(1 - x_{ad}^2)/(x_d x_q). \quad (2.42)$$

Time constant of the damper winding, when the stator winding is closed, and excitation winding is opened

$$T_{de} = T_{de0}(1 - x_{ad}^2)/(x_d x_{de}); \quad (2.43)$$

Function  $F_d(t) = \varphi(T'_d, T''_d, T_e, T_{\sigma, de})$ , where  $T_e$  - is time constant of excitation system and  $T_{\sigma, de} = x_{\sigma, de}/r_{de} = (x_{de} - x_{ad})/r_{de}$ .

Parameter values of the expression for time constant determination are calculated by formulae

$$\begin{aligned} T_{f0} &= x_f/r_f; & T_{de} &= x_{de}/r_{de}; & \sigma' &= 1 - (x'_{ad})^2/(x_f x_{ad}); \\ x'_{ad} &= x_{\sigma} x_{ad}/(x_{\sigma} + x_{ad}) = x_{\sigma} x_{ad}/x_d; \\ x'_f &= x_f - x_{ad}^2/x_d = x_{\sigma f} + x'_{ad}; \\ x'_{de} &= x_{de} - x_{ad}^2/x_d = x_{\sigma, de} + x'_{ad}; & T_{qe0} &= x_{qe}/r_{qe}. \end{aligned}$$

Total phase short circuit current at random time comprises periodic component, aperiodic component and double frequency component stipulated by both rotor asymmetry and presence of stator current aperiodic component.

For example, for phase A

$$\begin{aligned} i_A &= i_{F,A} + i_{aA} + i_{qA} + i_{F,A} - (U_{q(0)} \cos \gamma_{(0)} + U_{d(0)} \sin \gamma_{(0)})[(x''_d + x''_q) : \\ &: (2x''_d x''_q)] \exp(-t/T_a) - [U_{q(0)} \cos(2\omega t - \gamma_{(0)}) - \\ &- U_{d(0)} \sin(2\omega t + \gamma_{(0)})] \times [(x''_q - x''_d)/(2x''_d x''_q) \exp(-t/T_a)], \end{aligned} \quad (2.44)$$

where  $T_a = x_2/r$ ;  $x_2 = 2x''_d x''_q/(x''_d + x''_q)$  - negative sequence impedance.

According to the expression (2.44), the conclusion can be made that in complex power supply systems that include powerful loads in addition to power sources, it is complicated to make exact calculation of short circuit current at random time.

Therefore if high accuracy of calculation is not required approximate calculation methods are used to determine short circuit current at random time.

## 2.6. Short circuit current of emergency steady state

All free currents that had occurred damp out and voltage variation on synchronous machine clamps finishes to the beginning of steady-state operation, under the effect of automatic excitation control device. The parameters of short circuit current path in steady-state operation can be defined on the basis of both no-load and short circuit characteristics of synchronous machine, using its direct- and quadrature-axes synchronous reactances, stator leakage reactance and boundary field current  $I_{f, \lim}$ .

Direct-axis synchronous reactance is defined by expression

$$x_{*d} = c / k_c, \quad (2.45)$$

where  $c$  – is relative value of emf by unsaturated no-load characteristic when  $I_{*f} = 1$ ;  $k_c$  - short circuit ratio (steady-state current of three-phase short circuit in per unit) on generator clamps to field current in per unit that is equal to one. Non-salient pole machines have  $x_d \approx x_q$ , and salient pole machines have  $x_q \approx 0,6x_d$ . In this case  $c = 1.05 \dots 1.2$ .

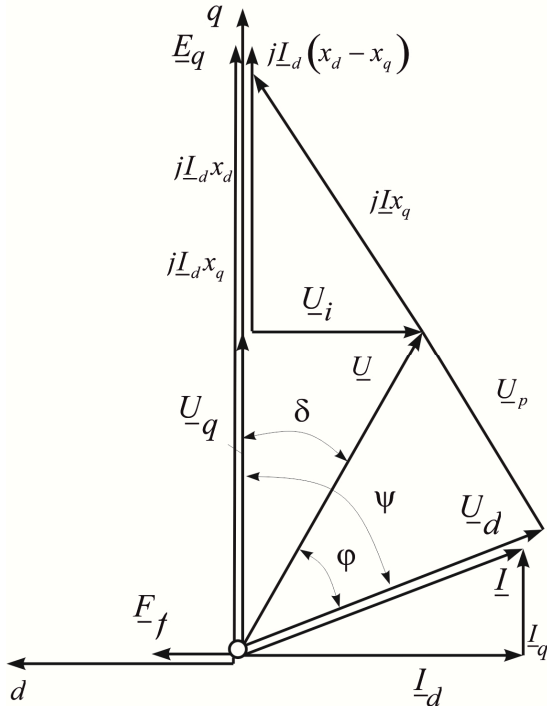


Fig. 2.15. Vector diagram of synchronous machine with salient rotor

To simplify short circuit current calculation the no-load characteristic  $E_q = f(I_f)$  is rectified at the point with coordinates  $E_{*q} = 1$ ;  $I_{*f} = 1$ . In this case

$$x_{*d} = 1 / k_c; \quad (2.46)$$

$$E_{*q} = I_{*f}. \quad (2.47)$$

Salient pole synchronous machine field current can be found by vector diagram in per unit (Fig. 2.15) for the given load, with account (2.47):

$$E_q \approx \sqrt{(U \cos \varphi)^2 + (U \sin \varphi + I x_d)^2} \quad (2.48)$$

It follows from vector diagram as well:

$$E_q = U_q + I_d x_d = I_d (x_d + x_{\text{ext}}) + I_q r_{\text{ext}};$$

$$I_q = I_d \operatorname{ctg} \varphi = I_d r_{\text{ext}} / (x_{\text{ext}} + x_q).$$

Having transformed the last expressions we obtain

$$I_d = E_q (x_q + x_{\text{ext}}) / [(x_d + x_{\text{ext}})(x_q + x_{\text{ext}}) + r_{\text{ext}}^2]; \quad (2.49)$$

$$I_q = E_q r_{\text{ext}} / [(x_d + x_{\text{ext}})(x_q + x_{\text{ext}}) + r_{\text{ext}}^2]; \quad (2.50)$$

$$\begin{aligned} I &= \sqrt{I_d^2 + I_q^2} = \\ &= E_q \sqrt{(x_q + x_{\text{ext}})^2 + r_{\text{ext}}^2} / [(x_d + x_{\text{ext}})(x_q + x_{\text{ext}}) + r_{\text{ext}}^2]. \end{aligned} \quad (2.51)$$

For nonsalient synchronous machines the expression (2.51) changes to

$$I = E_q / \sqrt{(x_d + x_{\text{ext}})^2 + r_{\text{ext}}^2}. \quad (2.52)$$

Factual short circuit current calculations demonstrate that non-salient and salient synchronous machines current differ slightly and their approximate values can be defined according to (2.52).

Accounting electrical remoteness of the short circuit point in steady-state operation of short circuit two generator operation modes are possible

1. Rated voltage operation characterized by following parameter relations

$$E_q \leq E_{q,\text{lim}}; U_G = U_{G,\text{rated}}; I_G \leq I_{\text{cr}}; x_{\text{ext}} \geq x_{\text{cr}}, \quad (2.53)$$

where  $I_{\text{cr}}$  and  $x_{\text{cr}}$  are critical current and voltage.

When short circuit occurs at the point corresponding the reactance  $x_{\text{cr}}$ , generator works with boundary excitation when voltage on its clamps is equal to rated one and current has its critical value

$$E_q = E_{q,\text{lim}}; U_G \leq U_{G,\text{rated}}; I_G \geq I_{\text{cr}}; x_{\text{ext}} \leq x_{\text{cr}}. \quad (2.54)$$

2. Boundary excitation operation, when

$$(E_{q,\text{lim}} - U_{G,\text{rated}}) / x_d = U_{G,\text{rated}} / x_{\text{cr}}, \quad (2.55)$$

Critical reactance can be found using equation (2.56)

$$E_{q,\text{lim}} - U_{G,\text{rated}} / x_d = U_{G,\text{rated}} / x_{\text{cr}}, \quad (2.56)$$

this yields in

$$x_{\text{cr}} = x_d U_{G,\text{rated}} / (E_{q,\text{lim}} - U_{G,\text{rated}}). \quad (2.57)$$

If generalized load is considered determining short circuit steady-state current the load is usually expressed in per unit at total operating power of the load and average rated voltage of supply network. Then

$$x_{*1d} = 1, 2; E_{*1d} = 0.$$

### Test questions

1. How does the total current and its components vary when short circuit occurs on the clamps of generator without automate excitation control device?
2. How does the device of generator automatic excitation control effect the short circuit current variation at three-phase short circuit?
3. What short circuit current is named a surge current? What conditions are required for the current occurrence?
4. What parameters does surge current depend on?
5. How is actual value of short circuit total current determined?
6. How does total current and its components vary when short circuit occurs at remote points of electric supply system?
7. What expressions are used to define periodic component of short circuit initial current?
8. What is the difference between transient short circuit current and subtransient short circuit current?
9. Is it possible to determine short circuit current at any instant of time analytically?
10. What is steady-state short circuit operation characterized by?
11. How is short circuit current in steady-state operation with short circuit determined?

### Topics for essay

1. Effect of generator's automatic excitation control device on short circuit current character.
2. Effect of electrical remoteness of the point of short circuit on power sources.
3. Total current and its components variation under short circuit in different points of electric power supply system.

## **CHAPTER 3: EVALUATION OF TRANSIENTS UNDER THREE-PHASE SHORT CIRCUITS**

- 3.1. Basic statements
- 3.2. Use of short circuit periodical component diagrams in network with one source
- 3.3. Use of short circuit current periodical component diagrams in a network with several sources
- 3.4. Calculation of short circuit current periodical component by rectified characteristics method
- 3.5. Short circuit currents calculation on the principle of superposition
- 3.6. Calculation of short circuit current components stipulated by load centers
- 3.7. Calculation of short circuit currents in electrical installations with voltage up to 1 kV
- 3.8. Computer-aided calculation of short circuit current
- 3.9. Errors of assessment of short circuit current values

Test questions

Topics for essay

### 3.1. Basic statements

To compose electric supply design circuit for enterprise under short circuit emergency it is equivalently represented relatively to short circuit point of two parts according to power sources:

- generators of power system;
- consumers storing energy till the instant of emergency operation start.

Due to the different character of electromagnetic processes taking place (see div. 2.1) these components of electric power supply system should be considered separately.

To make up equivalent circuits for abovementioned components of electric power supply system (network, lines) the following considerations are taken into account:

At the instant  $t = 0$  the active components (generators, motors) are replaced with the equivalent electromotive force  $E_t = const$  having internal reactance  $x_d''$  or  $x_d'$ ; passive elements have (transformers, power supply lines, chokes) only by means of electric impedances (see chapter 1);

At  $t > 0$  equivalent active elements have  $E_t = var$  and  $x_t = var$ , and those for passive components  $x_t = const$  (frequency variation under emergency is not taken into account), and  $r_t = var$  (due to current conductors heating by short circuit current to be heavily seen in electric lines with voltage up to 1 kV which impedances are mainly ohmic).

Equivalent circuits' transformation and calculation of emergency operation mode parameters are done taking into account the reaction of sources on short circuit which is estimated on their electric remoteness.

Listed peculiarities influence the option and use of ways to determine the characteristics of transients and parameters of emergency.

Highly accurate calculations of the values of characteristics and parameters for  $t > 0$  of emergency mode which occurred due to short circuit require great volume of computation. They became greatly complicated when automatic control of generators excitation, differences of synchronous machines parameters in direct and quadrature axes, uncertainty of connected motors behavior are taken into consideration. Practical problems of design and operation of electric power supply system solving do not require high accuracy of calculations that's why simplified methods to determine performances of transient are used. In a number of cases the simplest estimation is sufficient to determine only the degree of unknown quantities. It is sufficient to determine the conditions of electric power equipment operation or to ground the practical problems solving of its use.

Simplification of calculation for obtaining simple evaluation procedures, and explicit comparability and control of calculation results with physical pattern of phenomenon is based on a number of assumptions. The latter are assumed for specific conditions proceeding from a problem put by and ultimate target of calculation.

The main difference between applying practical methods of emergency mode parameters evaluations is in determination of periodical component of short circuit current depending on demands and purpose of calculations and assumptions. For example, if short circuit current calculations are done to select electric power equipment certain assumptions are used, and when problems concerning protection of components and adjustment of electric power supply system automation facilities the other assumptions providing calculations results with comparatively less error are required.

General calculation procedure for short circuit currents has been developed for **three-phase networks of electric installation which voltage is over 1 kV**. It is described in the standard [35], and covers three-phase networks of electric power installations of industrial frequency to determine currents of initial and any instants of emergency. The procedure provides calculation of short circuit current that should be known for solving the following: selection and test of electric power equipment on short circuit conditions; selection of installations and estimation of possible performances of relay protection means and automatic equipment of electric power supply system;



determination of zero sequence currents influence in power transmission lines on nearby power lines; selection of grounding devices of electric installations.

List of emergency transient performances under short circuit to be determined, and permissible calculation error and calculation method choice depend on purposes of calculations. To solve the problem of electric power equipment choice and test it is permitted to calculate short circuit current using simplified methods, if their error does not exceed 5-10 per cent. Under these assumptions the following is calculated:

- ◆ values of current periodical component and short circuit power at the initial and any instants of emergency mode including estimated time of faulted circuit breakage;
- ◆ values of short circuit current aperiodical component in the initial and any instants including estimated time of faulted circuit breakage;
- ◆ surge current of short circuit;
- ◆ distribution of currents in short circuit design circuit;
- ◆ residual voltages in centers.

To solve the problem of elements protection tuning and adjustment of automatic equipment of the electric power supply system primary data the maximum and minimum estimated values of periodical and aperiodical components of current are calculated at initial and any current instants both in the point of short circuit and in some branches of design circuit of the short circuit.

General recommendations concerning short circuit current calculation were partially stated in chapter 2.

Procedure being recommended by the standard does not cover following calculations of short circuit currents:

- under complex asymmetry in electric power supply system (for example, simultaneous short circuit and open-fault, and under repeated short circuits and short circuit in the system of electric power supply with non-linear components;
- when electric machines dynamics is taken into account under electromechanical transients;
  - under short circuits inside electric machines, transformers, and autotransformers;
  - for noncommercial values of frequencies arising under short circuits in transmission lines with 220 kV voltage and higher;
  - in electric installations with 750 kV voltage and higher.

Specific calculation methods are used to calculate the performances of listed emergency transient operation.

The calculations of short circuit currents values for any current instants in the point of short circuit and in some branches of the equivalent circuit should be considered in the frames of electromechanical transient taking place. These calculations should be accomplished using computers and methods of electric power system dynamic stability evaluation.

Calculating short circuit current in general case it should be taken into account all active elements of power system. It is permissible to make equivalent the part of power system remote from the point of short circuit - all electric power sources for which short circuit is distant. Its correspondent elements can be substituted relatively to the point of short circuit or other network center being chosen by one source of stable voltage and one impedance (such a source is called "electric system"). If either value of current of three-phase short circuit  $I_s^{(3)}$  or power of three-phase short circuit  $S_s^{(3)}$ , is known for particular center of electric power network, the equivalent inductive reactance of electric system source is determined by the expression

$$x_{GS} = U_{av, rated} / \left( \sqrt{3} I_s^{(3)} \right) = U_{av, rated}^2 / S_s^{(3)}, \quad (3.1)$$

where  $U_{av, rated}$  - is an average voltage in the center of electric system.

Value of electromotive force of electric system source should be considered as equal to rated voltage in the center of power network.

Depending on the complexity of short circuit calculations the currents are determined analytically using either analog design models of alternative and direct current or mathematic computer simulation.

Short circuit current calculation is done proceeding from the following. In one-loop patterns of calculation of short circuit the current is recommended to be calculated analytically or applying semigraphical methods using known procedures of electric circuits transformation. In multi-loop patterns of short circuit calculation it has to be done by node voltages method or the of mesh current method with the help of computers.

**For three-phase electric power networks with voltage up to 1 kV** at commercial frequency connected to power system or having self-contained power sources, short circuit currents are calculated according to the standard [36]. The standard determines general procedure of calculation currents of symmetrical and asymmetrical short circuits (maximum and minimum values) in the initial or any instants of emergency mode taking into account the parameters of synchronous and induction machines, transformers, chokes, cable and overhead lines, busways, capacitors and complex load centers.

Parameters of operation mode with short circuit are determined to obtain basic data for solve the following problems: selection and test of electric power equipment on the conditions of short circuit, choice of settings of automatic protection devices, switchgear and groundings calculation. Transient performances and the level of admissible error to calculate short circuit currents are chosen depending on calculation tasks. To choose and test the power equipment on the conditions of short circuit it is permissible to determine design performances using simplified methods of currents calculation if their error is not more than 10 per cent. Initial values of periodical and aperiodical components of short circuit current, surge current and r.m.s. values of periodical component of short circuit current in any given instant including those for estimated time of faulted circuit breaking are to be calculated.

To solve the problem concerning the design of electric power supply system elements protection both maximum and minimum values of periodical component of current in the point of short circuit in the initial and current instants including estimated time of fault circuit breaking are calculated. To select groundings of power installations the current of single-phase short circuit is calculated.

Procedure given in the standard does not cover such calculations of current as:

- under complex kinds of asymmetry in the system of electric power supply (for example, simultaneous short circuit and open-fault), under repeated short circuits and under short circuit in electric installations with non-linear components;
- when dynamics of electric machines in electromechanical transients are taken into consideration;
- under short circuits inside the electric machines and transformers.

Methods of transient performances calculation as the result of three-phase short circuit are oriented to use for determined hierarchical levels of electric power supply system (external electric power supply, internal electric power supply and electric lines up to 1 kV). Calculation methods are applied for the following specific cases:

- joint feeding by electric power system and thermal electric station;
- replenishment of short circuit point by electric motors; replenishment of short circuit point by the center of complex load; when short circuit point is fed by the sources of reactive power.

It is proved by practice that if transients caused by short circuit are calculated, the errors in the calculation of initial values of current using practical methods are within  $\pm 5\%$ . When values of currents are calculated in the branches for emergency mode random instants with the help of practical calculation methods, the errors are 10-15 per cent (it depends on short circuit remoteness and duration).

If calculations are approximate it is common thing to determine analytically only r.m.s. value of periodical component of short circuit current for the first period, and surge short circuit current. If the electric system is fed by the source of unlimited power, named as “electrical system”, the rms value of periodical component of three-phase current of short circuit for the first period

$$I_{F, t=0}^{(3)} = U_{av, rated} / (\sqrt{3} \cdot z_{res}). \quad (3.2)$$

Short circuit surge current is calculated using rms value of periodical component for the first period:

$$i_{srg}^{(3)} = \sqrt{2} \cdot k_{srg} I_{F, t=0}^{(3)}; \quad (3.3)$$

r.m.s. value

$$I_{srg} = I_{F, t=0}^{(3)} \sqrt{1 + 2(k_{srg} - 1)^2}. \quad (3.4)$$

When  $t = 0,01$  s and average value of time constant of aperiodical component damping  $T_a = 0,05$  s

$$k_{srg} = 1 + \exp(-0,01/0,05) = 1,8, \quad (3.5)$$

and current values:

$$i_{srg}^{(3)} = 2,55 I_{F, t=0}^{(3)}; \quad (3.6)$$

$$I_{srg}^{(3)} = 1,52 I_{F, t=0}^{(3)}.$$

When short circuit currents are calculated at the remote points of electric power network in which resistance of shorted circuit influences greatly, current value it is recommended to specify the values of time constant of aperiodical component damping according to the expression

$$T_a = x_{res} / (\omega r_{res}), \quad (3.7)$$

and to determine the specified value  $k_{srg}$  on (2.11). Such cases take place when short circuit occur behind low power transformers, in lengthy cable lines and in networks with voltage up to 1 kV.

Surge factor can be taken roughly  $k_{srg} = 1,3$  under approximate calculation of short circuit in lengthy cable lines or behind transformers which power is less than 1000 kV·A.

### 3.2. Use of short circuit periodical component diagrams in network with one source

Under three-phase short circuit the values of periodical current component in networks stipulated by synchronous generators for unspecified instants of fault can be calculated by means of graphic and analytical way using specially made up diagrams. The latters have specific sphere of application depending on electric power supply system structure, and power of the sources, calculation aims, and demands and terms of emergency operation mode results estimation. They are the basis of simplified methods of transients performances values in electric power supply system stipulated by three-phase short circuits calculation.

The diagrams are graphic representations of functional dependences between electromotive force of electric power source, and periodical component of short circuit currents, and time of transient, and electric remoteness of short circuit point from the electric power source. They cover a wide range of energizing sources power, and implicitly account the variations of electromotive force  $E_t$  and differ only by the availability and kinds of systems of generators excitation.

The diagrams are used to calculate values of **periodical component of current in the short circuit point** in a quick and simple way. Universality of application is the result of averaging of real generators parameters and approximate account of load influence in the system of electric power supply on current value in the point of short circuit. Accordingly, it is connected with calculations accuracy reduce. Limited field of application (they are used for current value determination only in the point of short circuit) is their disadvantage. That is, the determination of emergency operation mode in the branches of electric network of electric power supply system is not foreseen.

Earlier the diagrams named *design curves* have been used for *simplified calculations*. Then more multipurpose diagrams called *typical curves* have been developed. The latter have extended field of application in relation of electric sources power, account of kinds of generators excitation system; the parameter values of new kinds synchronous generators have been used to plot them.

*Typical curves* are series of dependences (Fig. 3.1,a)

$$\gamma_{t,G} = f_1\left(t, I_{*(\text{rated})F,t=0,G}\right), \quad (3.8)$$

and extra dependences (Fig. 3.1,b):

$$\gamma_{t,G} = f_2\left(\gamma_{t\Sigma}, C_G\right), \quad (3.9)$$

where  $\gamma_{t,G} = I_{F,t,G} / I_{F,t=0,G}$  - is relative value of short circuit current periodical component stipulated by generator for instants of emergency operation mode in the range of  $t \in [0; 3 \text{ s}]$ ;  $I_{*(\text{rated})F,t=0,G} = I_{F,t=0,G} / I_{\text{rated},G}$  - is an index by which the electric remoteness of short circuit point from the generator is specified;  $\gamma_{t\Sigma} = I_{F,t\Sigma} / I_{F,t=0,\Sigma}$  - is relative value of current periodical component in the point of short circuit;  $C_G$  - is a share (current distribution coefficient) of current stipulated by generator in total current in the point of short circuit for  $t = 0$ .

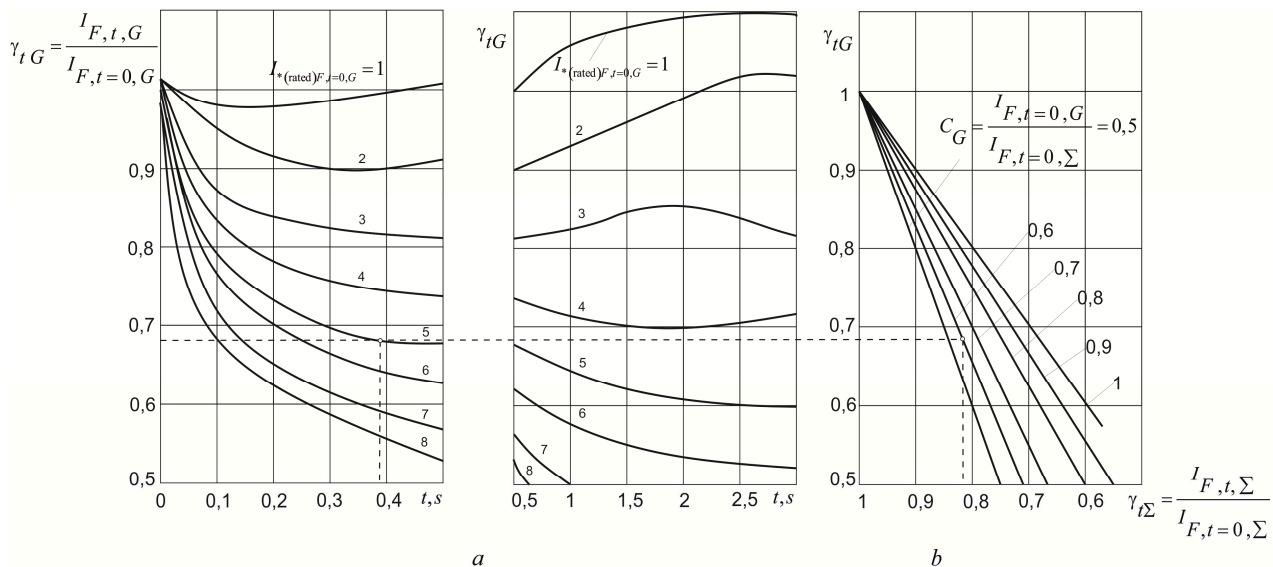


Fig. 3.1. Short current point periodical component generated by synchronous generators and compensators with a thyristor or a high-frequency excitation system depending on time and electric remoteness of the point of short circuit

Dependence (3.8) cover synchronous generator (equivalent source) operation mode parameters:  $I_{F,t,G}$ ,  $I_{F,t=0,G}$  - accordingly, r.m.s. value of periodical component of short circuit current in the generating branch in running instants  $t$  and this component initial value;  $I_{\text{rated},G}$  - value of rated current of synchronous generator reduced to voltage stage of the short circuit point.

Graphic dependences (3.9) are used in design circuits with two-side supply of short circuit point: by synchronous generator (equivalent source) and from electric system as the source of unlimited power. They connect with generator branch operation mode parameters  $I_{F,t,G}$ ,  $I_{F,t=0,G}$  with operation mode parameters in the point of short circuit ( $I_{F,t=0,\Sigma}$ ,  $I_{F,t,\Sigma}$  - are values of current periodical component at the initial and running instants of emergency operation mode  $t$  from all sources in the place of short circuit).

Data used for typical curves have determined their application point in calculations: they can be used for hydrogenerators with power 12-800 MW, for turbogenerators with power up to 500 MW and synchronous capacitors of 37.5...100 MV·A apparent power having radial structure. The curves differ from each other according to generators excitation kind; they are built for synchronous generators (capacitors) which limiting excitation voltage is not more than double rated voltage.

Typical curves: those in Fig. 3.1 are meant for calculations of r.m.s. periodical components of short circuit current caused by synchronous generators with thyristor-based or high-frequency excitation. They can also be used for calculations of shorts in synchronous capacitors; and if electrical remoteness from short circuit is not significant, i.e. when  $I_{*(rated)F,t=0,G} > 3$ , they can be used for hydrogenerators having field voltage exceeding rated one more than twice. Curves in Fig. 3.2 (a, b) can be used for calculations of short circuit currents in networks with synchronous generators with thyristor self-exciting systems, correspondingly, with and without series transformers. Curves in Fig. 3.3 are designed for synchronous generators with diode brushless excitation system.

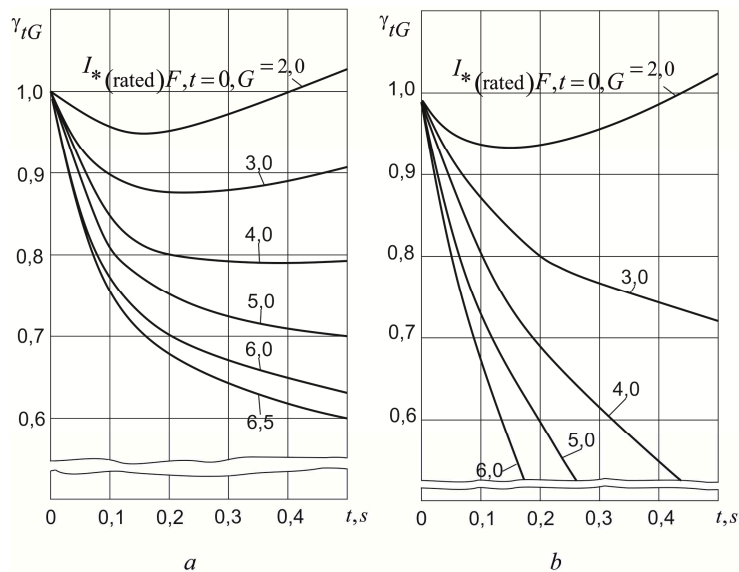


Fig. 3.2. Short circuit point current periodical component generated by synchronous generators and capacitors with thyristor system of self-excitation depending on time and electric remoteness:

**a**-with series transformers; **b**-without series transformers.

Short circuit is considered to be electrically remote if ratio of r.m.s. value of synchronous generator current periodical component at the initial instant of short circuit to its rated current is  $\gamma_{t,G} < 2$ . Periodical component of short circuit current from this generator should be considered as constant in magnitude.

For the branch of independent radial feeding of short circuit point from synchronous generator (capacitor), or some similar synchronous generators (capacitors) being under the same conditions as to the point of short circuit, the r.m.s. of their current periodical component calculation is done as following:

1) equivalent circuit is made up and resulting impedance  $Z_{res}$  is calculated from the source to the short circuit point;

2) initial value of short circuit periodical component from synchronous generator (a group of generators) is calculated, and reference value of current is determined;

3) current ratio

$$\left( I_{F, t, G} / I_{F, t=0, G} \right)_{t_i} = \gamma_{t_i, G}, \text{ determined on the}$$

curve  $\gamma_{t, G} = f_1(t, I_{*(rated)G, t=0, G})$  which corresponds to the determined value of electric remoteness for specified time instant  $t_i$ ;

4) actual r.m.s. value of periodical component of short circuit current from synchronous generator (a group of generators) is determined at the time instant  $t_i$

$$I_{F, t_i, G} = \gamma_{t_i, G} \cdot I_{F, t=0, G}. \quad (3.10)$$

Determination of short circuit current periodical component with the use of typical or rated design curves takes into consideration parameters and characteristics of feeding source generators (power, a kind of excitation system, estimated time of short circuit) and short circuit point remoteness, and a structure of enterprise external and internal electric power supply system.

### 3.3. Use of short circuit current periodical component diagrams in a network with several sources

At calculation of current in the point of short circuit in electric power supply system with some power sources, it is necessary to evaluate the possibility of their number in equivalent circuit decrease using data concerning their kind, power, excitation system and electric remoteness of the short circuit point. The difference in calculation of short circuit current periodical component by common variation from united sources, and individual variation if it is necessary to calculate values of short circuit current components from each generator.

The matter of calculation *on common variation* is to substitute groups of similar and equally electrically distant synchronous generators with equivalent ones with subsequent determination of short circuit current periodical component for equivalent generators. It is done as following:

1) first of all short circuit is made up, and then the same is done for equivalent circuit of electric power supply system to determine initial value of current periodical component in each point of short circuit;

2) values of equivalent impedances between each source and short circuit point, and electromotive force of generating branches are determined;

3) values of electric remoteness of short circuit point from each source for independent generating branches are estimated if they are not connected with short circuit point by means of common impedance;

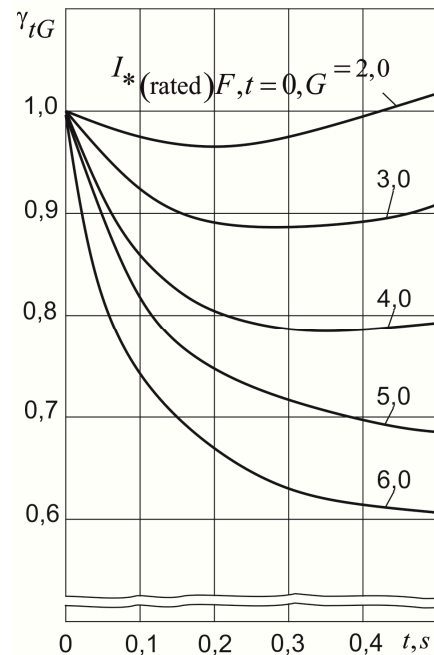


Fig. 3.3. Periodical components of SC current generated by synchronous generators with diode brushless excitation system depending on time and electric remoteness of short circuit

4) the way to calculate short circuit current is chosen. If it is done on initial data, this energizing branch of power system  $U_{GS} = const$  or equivalent source belong to the sources of unlimited power, or can belong to them under the evaluation of electric remoteness ( $I_{*(rated)F, t=0,G} < 2$  or  $x_{*(rated)des} > 3$ ), the current periodical component generated by equivalent generator is calculated by formula (3.4); for electrically close short circuits depending on equivalent generator characteristics, to calculate it either typical or design curves (see div. 3.2).

Calculation *on individual variation* consists in determination of short circuit current components from diverse synchronous generators including ones that differ with excitation system or electric power stations with different electric remoteness from the short circuit point. Periodical components of short circuit current of each separate generators or electric power stations have different time variations. So, error will be significant if not individually short circuit currents generated by diverse generators or electric power stations with different electric remoteness from short circuit are calculated.

If sources characteristics are those that to determine short circuit current periodical components the design curves can be used, first of all the equivalent circuit of electric power supply is transformed to conditional radial one where each branch corresponds to a correspondent selected source (a group of similar sources) and is connected with short circuit point via one impedance. This transformation of equivalent circuit is done using current distribution coefficient (see div.1.6).

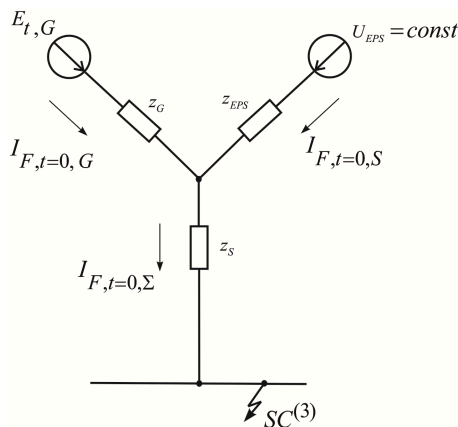


Fig. 3.4. Equivalent circuit of electric power supply system if short circuit point is fed by generator and electric system via common impedance

Values of short circuit current periodical components generated by sources are determined for equivalent circuit each branch individually, and total up in the point of short circuit.

If characteristics of sources make possible use the typical curves to calculate values of short circuit current periodical components adds, the calculation succession for independent generating branches is the same as it is done on common variation of short circuit current.

For branches of generator and electric system connected with short circuit point by means of common impedance  $Z_S$ , the calculation of current periodical component is done as following (Fig. 3.4):

1) resulting impedance  $Z_{*(b)res}$  for  $t = 0$  and total electromotive force  $E_{*(b), t=0}$  are found to calculate initial value of current periodical component in the point of short circuit on formula

$$I_{F, t=0, \Sigma} = E_{*(b), t=0, \Sigma} \cdot I_b / Z_{*(b)res};$$

2) initial value of current periodical component in the branch of generator is calculated by expression

$$I_{F, t=0, G} = \left[ E_{*(b), t=0, \Sigma} \cdot I_b - I_{F, t=0, \Sigma} \cdot Z_{*(b)s} \right] / Z_{*(b)G}; \quad (3.11)$$

3) ratios  $I_{F, t=0, G} / I_{rated, G}$  and  $I_{F, t=0, G} / I_{F, t=0, \Sigma}$  are determined (if  $C_G = I_{F, t=0, G} / I_{F, t=0, \Sigma} < 0,5$ , which corresponds to significant electric remoteness of generator from the short circuit point, or its small power, it is expedient to unite branches of generator and electric system)

4) ratio  $\gamma_{t,G}$  is determined by typical curves  $\gamma_{t,G} = f_2(\gamma_{t\Sigma}, C_G)$  under known value of  $I_{*(rated)G, t=0,G}$  for estimated instant  $t_i$  and then value  $\gamma_{t\Sigma}$  is determined on  $C_G$  value;

5) using the determined ratio  $\gamma_{t\Sigma}$  and known value of  $I_{F, t=0, \Sigma}$  the current periodical component in the point of short circuit is calculated by the expression

$$I_{F, t_i, \Sigma} = \gamma_{t_i, \Sigma} \cdot I_{F, t=0, \Sigma} \quad (3.12)$$

Total value of current periodical component in the point of short circuit being generated by several sources is calculated by formula

$$I_{F, t, (1, \dots, N)} = I_{F, t, 1} + I_{F, t, 2} + \dots + I_{F, t, N} \quad (3.13)$$

It is not expedient to separate many generating branches in equivalent circuit of electric power supply system. It is enough to reduce the equivalent circuit of any complexity to two or three branches, including to each of them generating sources (generators or stations) having approximately similar electric remoteness from the short circuit point.

### 3.4. Calculation of short circuit current periodical component by rectified characteristics method

The method of rectified characteristics is developed to calculate the short circuit current periodical component at arbitrary instant under simplified account of electromotive force  $\dot{E}_t$  variation in magnitude and phase. It is used when it is necessary to calculate refined values for  $t > 0$ . It takes into consideration damping of free current and influence of generator voltage control. The use of graphic dependences to determine design values  $E_t$  and  $x_t$  of turbogenerators with devices of automatic excitation control is its basis.

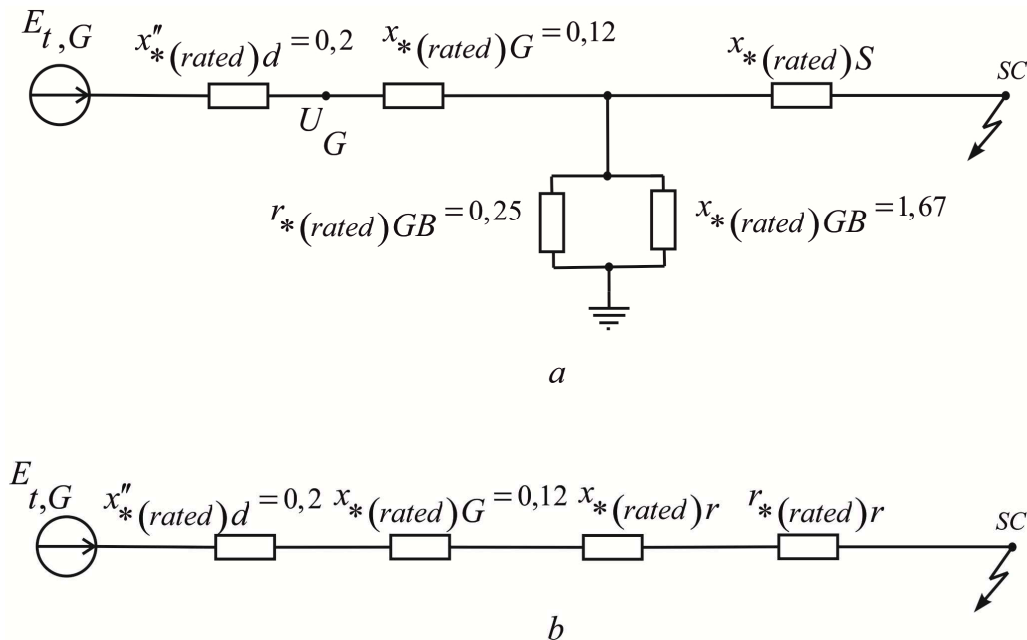


Fig. 3.5. Equivalent circuits for rectified characteristics composing: a-initial b – transformed

Here you can see the use of mentioned dependences obtained for wide-spread kinds of turbogenerators with power of 200 to 300 MW when they operate with rated load under previous mode. Composition of rectified characteristics is done for operation conditions of standard generator in radial circuit shown as equivalent circuit at Fig. 3.5, a. It is usual that standard load is equivalent by constant impedance related to terminals of transformer higher voltage, and it is possible to calculate



approximately values  $r_{*(rated)ld}$  and  $x_{*(rated)ld}$  for  $\cos \varphi_{ld} = 0,8$ . If rated load was followed by short circuit:  $r_{*(rated)ld} = 1/0,8 = 1,25$  and  $x_{*(rated)ld} = 1/0,6 = 1,67$ .

Values of standard parameters taken for turbo-generators with power of 200 to 300 MW with devices of excitation automatic control:

$x''_{*(rated)d} = 0,2$ ;  $x'_{*(rated)d} = 0,28$ ;  $x_{*(rated)d} = 1,9$ ;  $T_{fo} = 6 c$ ;  $T''_d = 0,115 c$ ;  $\cos \varphi = 0,8$ .

Plotting of rectified characteristics  $E_{*(rated)} = E_G = f_1(t)$  and  $x_{*(rated)} = x_G = f_2(t)$  is done for transformed equivalent circuit shown in Fig. 3.5,a, and in Fig. 3.5,b, where t new design values of element parameters are applied:

$$x_{*(rated)r} = \frac{r_{*(rated)ld}^2 x_{*(rated)n}}{r_{*(rated)ld}^2 + x_{*(rated)n}^2}; \tag{3.14}$$

$$r_{*(rated)r} = \frac{r_{*(rated)ld} x_{*(rated)n}^2}{r_{*(rated)ld}^2 + x_{*(rated)n}^2}, \tag{3.15}$$

where

$$x_{*(rated)n} = \frac{x_{*(rated)ld} x_{*(rated)s}}{x_{*(rated)ld} + x_{*(rated)s}}. \tag{3.16}$$

To plot rectified characteristics the estimated curves of short circuit current of turbo-generators with power of 200 to 300 MW with excitatory time constant  $T_e = 0,2...0,3$  s and  $T_e = 0...0,15$  s were used. Estimated values of electromotive force  $E_{*(rated)t}$  and reactance  $x_{*(rated)t}$  of generator for separate instants were obtained by rectification of characteristics  $E_t = f_1(I_G)$  and  $x_t = f_2(I_G)$ . Results of characteristics rectification for turbo-generators with different values of exciter time constant  $T_e = 0,2...0,3$  s and  $T_e = 0...0,15$  s practically coincided with permissible error. That's why curves  $E_t(t)$  and  $x_t(t)$  are admissible for turbo-generators with the exciter time constants  $T_e = 0$  to 0.3 s (machine and ion excitation).

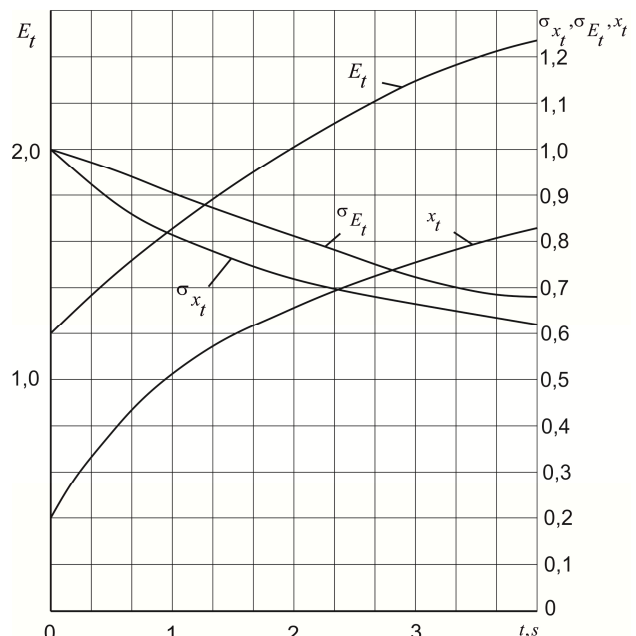


Fig. 3.6. Rectified characteristic  $E_t, x_t, \sigma_{E_t}, \sigma_{x_t}$  (per unit) of standard torbogenerators with power of 200 to 300 MW with devices of automatic control of excitation ( $T_e = 0...3$  s)

For those cases when real values of generators parameters differ from the standard generator values,  $\sigma_{Et} = f_3(t)$  and  $\sigma_{xt} = f_4(t)$  curves (Fig. 3.6) can be applied. Estimated expressions are refined taking into consideration coefficients  $\sigma_{Et}$  and  $\sigma_{xt}$  :

$$E_t = E_{q,\text{lim}} - \left( E_{q,\text{lim}} - E_G'' \right) \sigma_{Et} ; \tag{3.17}$$

$$x_t = x_d - \left( x_d - x_d'' \right) \sigma_{xt} , \tag{3.18}$$

where  $E_{q,\text{lim}}$  - is the synchronous EMF ( $t = \infty$ ), corresponding to maximum field current (excitation forcing ratio  $k_e$  is accounted)

$$E_{q,\text{lim}} = k_e \sqrt{\left( U_G \cos \varphi_G \right)^2 + \left( U_G \sin \varphi_G + I_G x_d \right)^2} ; \tag{3.19}$$

$E_G''$  - is subtransient electromotive force.

If time constant  $T_{f0}$  of generator differs from estimated value used at plotting the curves which is equal to 6 s, the same curves in Fig. 3.6 can be used with some approximation for its reduced value

$$\hat{t} = t \frac{6}{T_{f0}} . \tag{3.20}$$

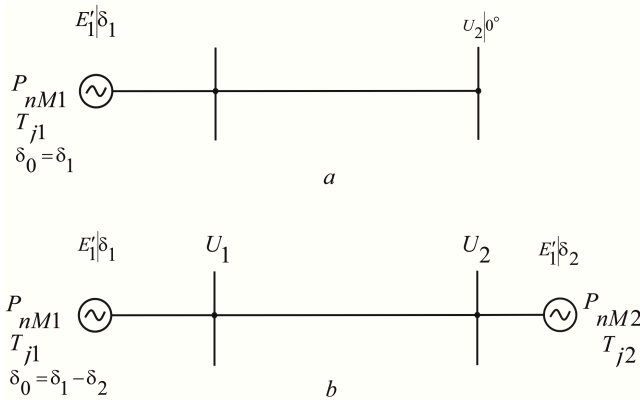
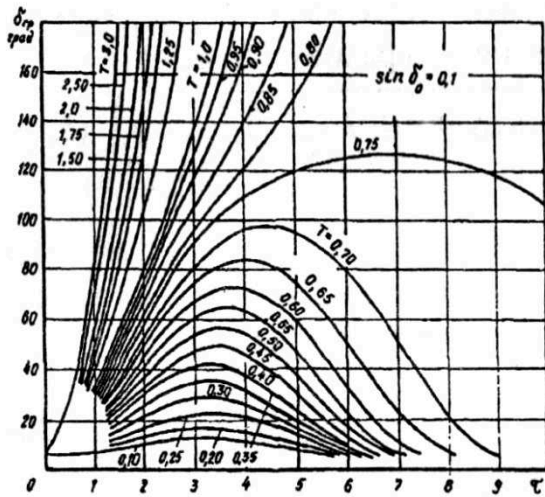


Fig. 3.7. Initial circuits to explain the method of critical curves: a- a system of unlimited power with one electric power station; b-the same with two electric power stations

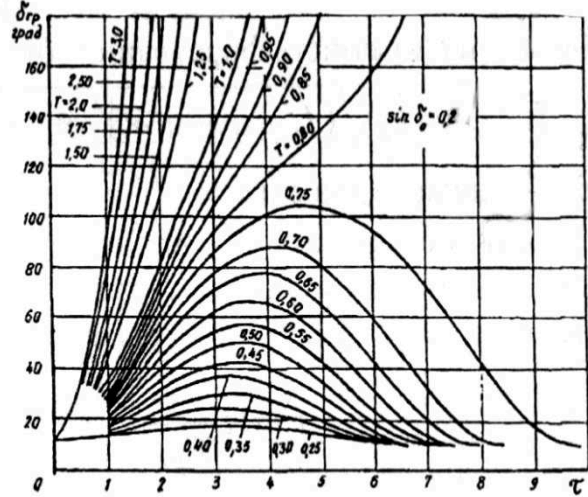
Short circuit periodical component determination includes the use of ultimate time curves method, that provides simplified account of displacement angle  $\delta$  between electromotive forces of several generators in phase. The following assumptions take place: stability of electromotive force values behind transient reactance  $x_d'$  of generator; substitution of loads with constant impedances; constant power of the prime motor. Standard curves of ultimate time (integral curves) let find the dependence of angle  $\delta$  values between electromotive force of generating station and voltage of unlimited power buses in electric system (Fig. 3.7, a), or angle  $\delta$  values between electromotive force

of two generating stations (Fig. 3.7, b) on estimated (reduced) time  $\tau$ . As for electric systems with a number of generating stations it is necessary to divide them into two groups, and in each of them its own similar time variation of electromotive force of angle  $\delta$  is assumed.

Standard curves (curves of ultimate time) of angle  $\delta_{\text{lim}}$  dependence on electromotive force of generating station behind transient impedance, or of the angle between generating system electromotive force and voltage of buses of unlimited power system on the reduced time  $\tau$  under different values of parameter  $T$  and initial angle  $\delta_0$  are shown in Fig. 3.8.

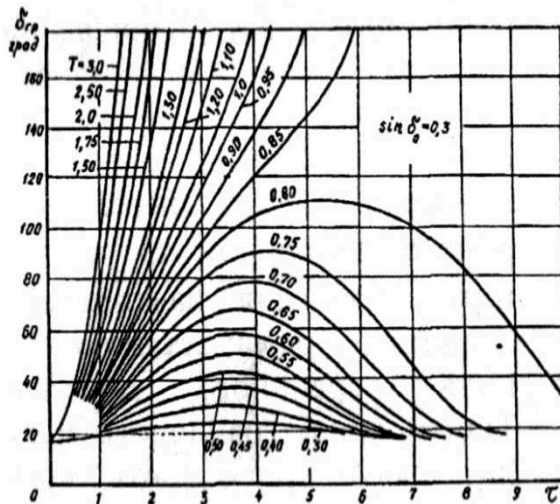


a

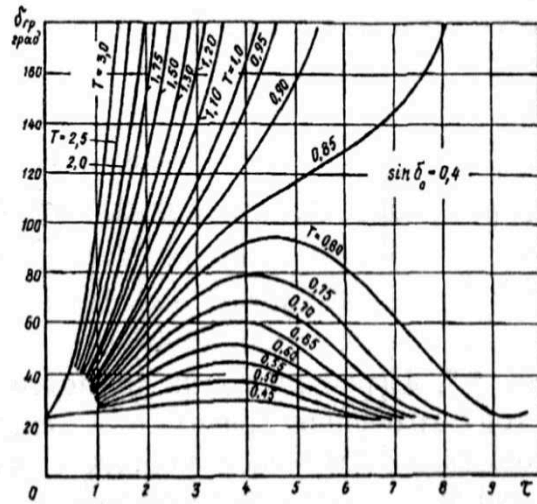


b

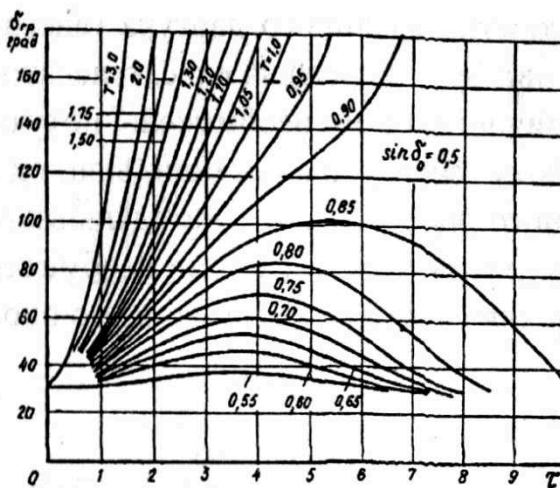
Fig. 3.8 (a, b). Curves of ultimate time for different values of  $\sin \delta_0$  (in the field of the figure):  
a – 0.1; b – 0.2



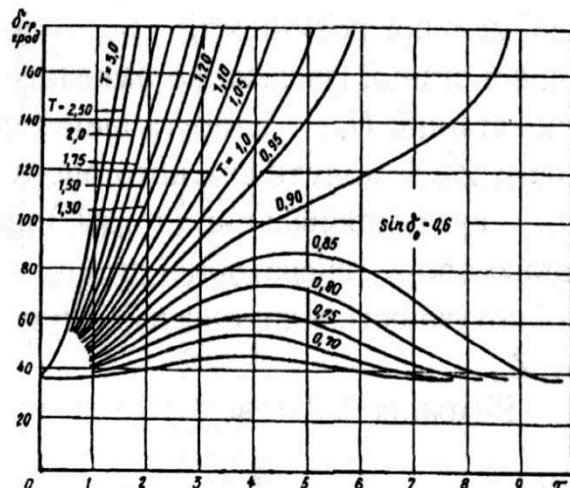
c



d



e



f

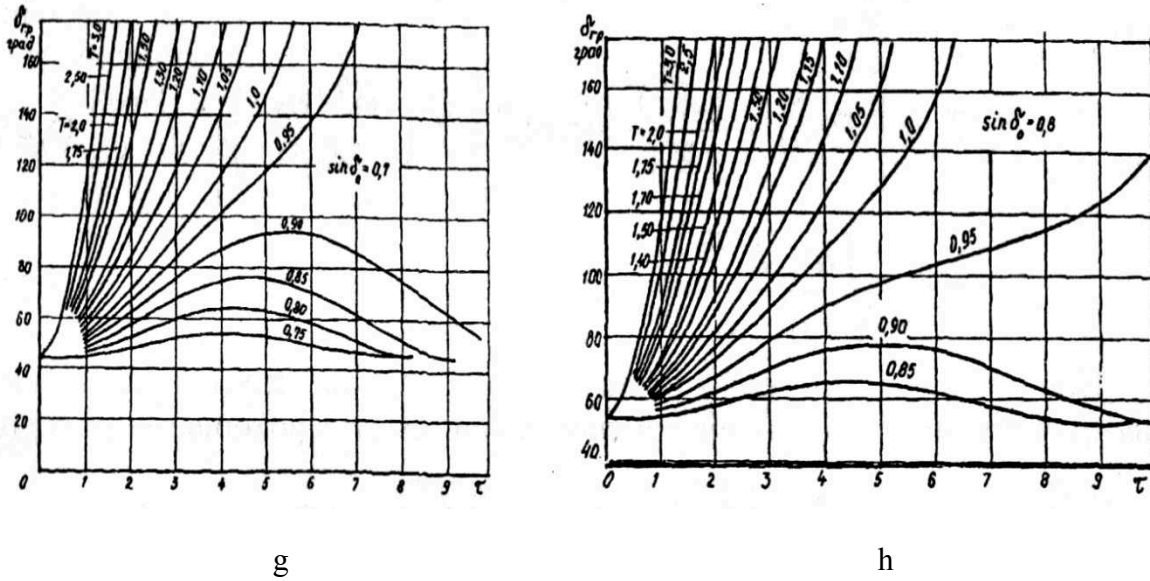


Fig. 3.8 (c, d, e, f, g, h). Curves of ultimate time for different values of  $\sin \delta_0$  (in the field of the Figure): c – 0.3; d – 0.4; e – 0.5; f – 0.6; g – 0.7; h – 0.8

These curves are built using differential equation of generators rotors motion.

If generating station works on buses of electric station of unlimited power (Fig. 3.7,a)

$$\begin{aligned} \frac{T_{J1}}{\omega_0} \frac{d^2 \delta_{12}}{dt^2} = P_{PM1} - \frac{(E'_1)^2}{Z_{11}} \sin \alpha_{11} - \\ - \frac{E'_1 U_{GS}}{Z_{12}} \sin(\delta_{12} - \alpha_{12}), \end{aligned} \quad (3.21)$$

where  $T_{J1}$  - is the generating station inertia constant reduced to base conditions;  $\delta_{12}$  - is the angle between generating station electromotive force and bus voltage of electric station with unlimited power;  $P_{PM1}$  - is the power of generating station prime mover reduced to base conditions;  $E'_1$  - is the transient electromotive force of generating station;  $U_{GS}$  - is the voltage at buses of electric system with unlimited power;  $Z_{11}$  - is the magnitude of own impedance of circuit on the side of generating station, per unit;  $\alpha_{11}$  - is a complementary angle up to  $90^\circ$  of own  $Z_{11}$  impedance;  $Z_{12}$  - is a module of mutual impedance of generating station and buses of electric system with unlimited power, per unit;  $\alpha_{12}$  - is a complementary angle up to  $90^\circ$  of mutual  $Z_{12}$  impedance.

Expression (3.20) is reduced to the following form

$$\frac{d^2 \delta}{d\tau^2} = T - \sin \delta \quad (3.22)$$

according to which, the graphic dependences in Fig. 3.8 are calculated. Here parameters are

$$T = \left[ P_{PM1} - \frac{(E'_1)^2}{Z_{11}} \sin \alpha_{11} \right] / \left( \frac{E'_1 U_{GS}}{Z_{12}} \right); \quad (3.23)$$

$$\tau = t \sqrt{\frac{\omega_0}{T_{J1}} \frac{E'_1 U_{GS}}{Z_{12}}}; \quad \delta = \delta_{12} - \alpha_{12}. \quad (3.24)$$

For the case of two generating stations connected with electric network work in parallel operation mode, the differential equation of generator rotors motion is

$$\begin{aligned} \frac{1}{\omega_0} \frac{d^2 \delta_{12}}{dt^2} = \frac{1}{T_{J1}} \left[ P_{PM1} - \frac{(E'_1)^2}{Z_{11}} \sin \alpha_{11} - \frac{E'_1 E'_2}{Z_{12}} \sin(\delta_{12} - \alpha_{12}) \right] - \\ - \frac{1}{T_{J2}} \left[ P_{PM2} - \frac{(E'_2)^2}{Z_{22}} \sin \alpha_{22} + \frac{E'_1 E'_2}{Z_{12}} \sin(\delta_{12} - \alpha_{12}) \right], \end{aligned} \quad (3.25)$$

where  $E'_2$  - is the transient electromotive force of the second generating station;  $Z_{22}$  - is a module of own circuit impedance from the side of the second generating station, per unit;  $\alpha_{22}$  - is complementary angle up to  $90^\circ$  of own  $Z_{22}$  impedance;  $P_{PM2}$  - is the power of the primary motor of the second generating station, reduced to base conditions;  $T_{J2}$  - is inertia constant of the second generating station reduced to the base conditions.

Expression (3.25) is reduced to a form of (3.22), where parameters are

$$\delta = \delta_{12} + \psi; \quad (3.26)$$

$$\psi = \arctg \left( \frac{T_{J1} - T_{J2}}{T_{J1} + T_{J2}} \operatorname{tg} \alpha_{12} \right); \quad (3.27)$$

$$T = \frac{T_{J1} \left[ P_{PM1} - \frac{(E'_1)^2}{Z_{11}} \sin \alpha_{11} \right] - T_{J2} \left[ P_{PM2} - \frac{(E'_2)^2}{Z_{22}} \sin \alpha_{22} \right]}{\frac{E'_1 E'_2}{Z_{12}} \sqrt{T_{J1}^2 + T_{J2}^2 + 2T_{J1} T_{J2} \cos(2\alpha_{12})}}; \quad (3.28)$$

$$\tau = t \sqrt{\frac{\omega_0}{T_{J1} T_{J2}} \frac{E'_1 E'_2}{Z_{12}} \sqrt{T_{J1}^2 + T_{J2}^2 + 2T_{J1} T_{J2} \cos(2\alpha_{12})}}. \quad (3.29)$$

Steps to calculate short circuit current periodical component for a given time instant with the help of rectified characteristics in combination with ultimate time curves are as following:

1. Both own and mutual impedances  $Z_{11}$ ,  $Z_{22}$  and  $Z_{12}$  are calculated. Own impedances  $Z_{11}$  and  $Z_{22}$  are determined by the current caused by the given source electromotive force if there are no electromotive forces caused by other sources:

$$\underline{Z}_{11} = \left. \frac{\dot{E}_1}{\dot{I}_1} \right|_{\dot{E}_2=0}; \quad \underline{Z}_{22} = \left. \frac{\dot{E}_2}{\dot{I}_2} \right|_{\dot{E}_1=0}.$$

Mutual impedance is determined by current of the given source stipulated by another source electromotive force,

$$\underline{Z}_{12} = \left. \frac{\dot{E}_1}{\dot{I}_2} \right|_{\dot{E}_2=0} = \left. \frac{\dot{E}_2}{\dot{I}_1} \right|_{\dot{E}_1=0}.$$

2. Parameter  $T$ , and given time  $\tau$  value, and initial angle  $\delta_0$  are calculated
3. The angle  $\delta_{lim}$  between generating stations electromotive force and bus voltage of electric system with unlimited power, or between two generating stations electromotive forces is calculated with the help of ultimate time curves using values of  $\tau$ ,  $T$  and  $\delta_0$ .
4. Values of electromotive force  $E_t$  module and reactances  $x_t$  of generators are determined for a given time instant  $t$  on the dependences of  $E_{*(rated)} = f_1(t)$  and  $x_{*(rated)} = f_2(t)$  or  $\sigma_{E_t} = f_3(t)$  and  $\sigma_{x_t} = f_4(t)$ .
5. Equivalent circuit with generator parameters on item 4 is made up; electric system of unlimited power is introduced by means of its own voltage reduced to base conditions and internal impedance equal to zero.
6. Calculation of short circuit given form for assumed time  $t$  is done for the made up the equivalent circuit.

The method of symmetrical components is used to calculate positive sequence for any kind of asymmetrical short circuit. Dependences  $E_{*(rated)} = f_1(t)$  and  $x_{*(rated)} = f_2(t)$  are used as well. To do that, generators are introduced into positive sequence equivalent circuit with electromotive force  $E_t$  values and reactance  $x_t$ , being determined according to curves in Fig. 3.6, and electric system of unlimited power with correspondent voltage and impedance equal to zero. If short circuit current calculations for the given time instant result shows that voltage value in per unit on terminals of one or several generators is more than one, the repeated calculation is done. If so, correspondent generators are introduced into equivalent circuit with electromotive force as equal to one, and impedances equal to zero (it makes possible to consider generators as those operating under the operation mode of standard voltage).

If short circuit current is calculated with the help of rectified characteristics, the loads have to be introduced in equivalent circuits of proper sequences in the points of their actual application.

### 3.5. Short circuit currents calculation on the principle of superposition

The principle of superposition reduces to the idea of actual short circuit mode representation of two superposed operation modes: the preceding operation under load, and the following operation that is a matter of fact the emergency one.

The calculation is based on application in the short circuit point of two mutually opposite voltages  $\pm U_s^{(n)}$ , equal to voltage of preceding operation mode in the point of short circuit (Fig. 3.9,a).

Electromotive force of generator in combination with voltage  $+U_s^{(n)}$  assures the conditions of preceding loading operation mode (Fig. 3.9, b). Voltage  $-U_s^{(n)} = U_{fault}$  (Fig. 3.9, c) applied in short circuit point provides the conditions of emergency operation mode.

Any circuit branch current under short circuit should be determined as following

$$\underline{I}_s = \underline{I}_{(0)} + \underline{I}_{fault}, \quad (3.30)$$

where  $\underline{I}_{(0)}$  - is circuit branch current under normal loading operation mode;  $\underline{I}_{fault}$  - is circuit branch current under fault operation mode. The current at the point of short circuit  $\underline{I}_s = \underline{I}_{fault}$  as current was not available in short circuit branch before the short circuit ( $\underline{I}_{(0)} = 0$ ).

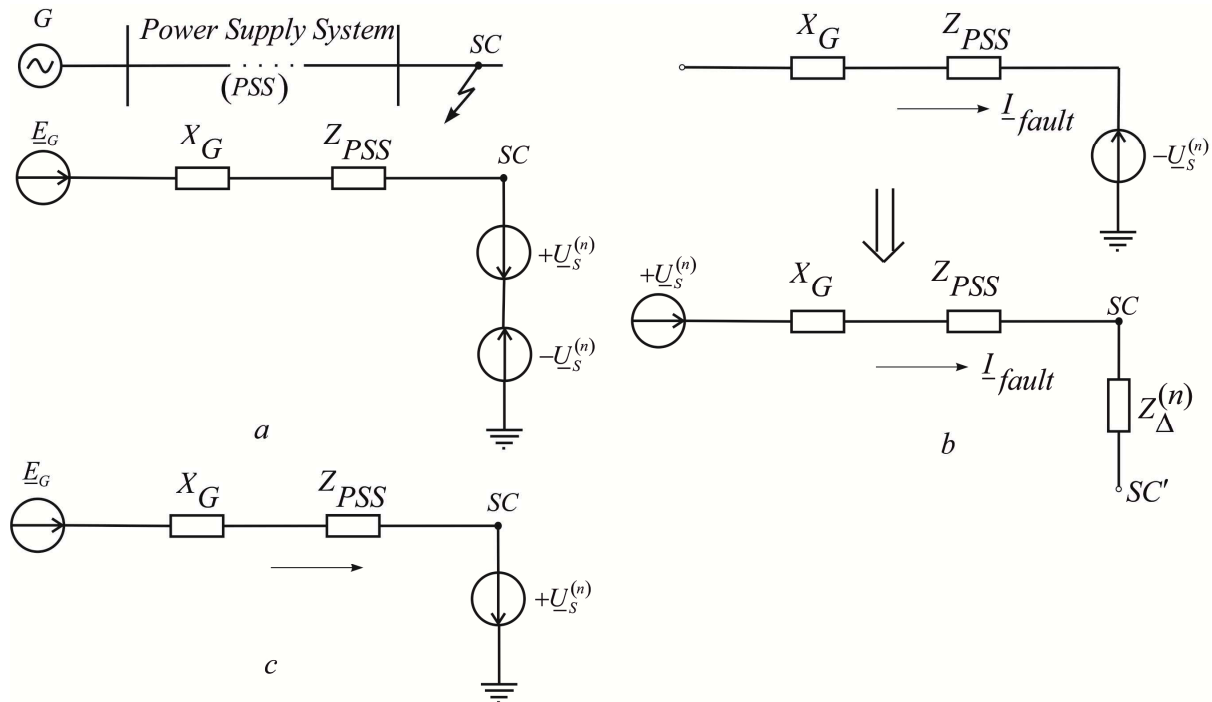


Fig. 3.9. Steps of circuits representation according to the method of superposition: a –design and equivalent circuits of actual operation with short circuit available; b - equivalent circuit of preceding operation mode under load; c-equivalent circuit of emergency operation mode

As for asymmetrical short circuit the  $\pm U_s^{(n)}$  voltages should be applied to point  $SC'$  of a dummy short circuit point removed from the impedance  $Z_{\Delta}^{(n)}$  from the actual short circuit point (Fig. 3.9,c). For this purpose the integrated equivalent circuits are used. They are obtained by means of positive sequence equivalent circuit expansion. According to the equivalency rule of positive sequence current the short circuit asymmetrical current are determined as currents of conditional three-phase short circuit at the point remote for impedance  $Z_{\Delta}^{(n)}$ . Currents and voltages of actual operation mode are determined proceeding from the following.

In positive sequence equivalent circuit the currents in the point of short circuit are equal to emergency currents, and voltages at the point of short current, and currents and voltages of other parts of circuit are expressed by means of both operation modes parameters summing up. In equivalent circuits of negative and zero sequences the currents and voltages in the point of short circuit, and in other parts are equal to emergency operation mode parameters.

In the integrated equivalent circuit of emergency operation mode the impedance  $Z_{\Delta}^{(n)}$  can be represented as complete equivalent circuits of negative and zero sequence according to expressions given in chapter 5.

Superposition method application is done to calculate short circuit currents by means of simplified methods if preceding operation mode under load currents are either known or can be approximately estimated partially in those circuit elements for which it is necessary to know both the positive sequence currents distribution and total actual phase currents. Besides it is used to determine only quantities which characterize operation mode. In particular, the use of superposition method is preferable if electromotive forces of sources to determine currents and voltages of negative and zero sequences are prescribed. With it, only components of emergency operation mode can be calculated proceeding from the coming before operation mode in the point of short circuit. As voltages on buses of substations in loading operation mode are almost equal to standard one, in the first approximation when actual voltages of preceding operation under load are unknown, it can be assumed as design voltage when simplified calculation methods are used.

### 3.6. Calculation of short circuit current components stipulated by load centers

In short circuit current calculations the kind of load in its effect at power supply system centers should be accounted. In general case industrial load has complex structure and comprises following main components: induction and synchronous motors, AC and DC traction equipment, static consumers (furnaces, lighting, welding plants, converter installations etc.), compensating and balancing devices. Total load of consumer set in power supply system is characterized by various structure, operation modes and power supply circuits affects short circuit current and accurate its account is a complicated problem.

According to the above mentioned it is permissible to consider the short circuit point infeed from industrial load centers approximately.

When short circuit occurs in the system of external power supply (feeding network with voltage over 35 kV) the load is electrically remote from the point of short circuit and its consideration is just additionally adjusting factor. In this case the load is presented as total load with the following parameters:

to consider infeed effect when  $t = 0$

$$E_{*(rated)t=0,ld} = 0,85; \quad x_{*(rated)t=0,ld} = 0,35; \quad (3.31)$$

in equivalent circuits

$$\underline{Z}_{ld} = \frac{U_{ld}^2}{S_{ld}} (\cos \varphi_{ld} + j \cdot \sin \varphi_{ld}), \quad (3.32)$$

where  $U_{ld}$  - is line-to-line voltage in the center, to which total load is connected;  $S_{ld}$  - is consumed power of the load;  $\varphi_{ld}$  - is angle of load voltage and current phase displacement.

When short circuit occurs in systems of internal power supply (distribution networks with voltage up to 10kV) local sources of short circuit point infeed should be specified such as motors, of reactive power compensating equipment.

#### 3.6.1. Calculation of short circuit current components from motors went into generating

Short circuit point in power supply system is powered not only by the sources of electrical system or local power stations but also from motors that are located nearby and went to generating mode and continue rotating by inertia because of kinetic energy saved in the operating machines. During transient current from synchronous motor reduces to stable value, which is defined by field current; current from induction motor reduces to zero. Due operating of motor in generating mode current at the point of short circuit can increase considerably if some powerful motor or group of motors are connected to the point of short circuit. It is typical for networks and electrical installations of voltage 6-10 kV and where motors of 1000 kW and more are present.

Normally motors connected to the point of short circuit directly or through power transmission line, current distributors, line choking coils or double-wound transformers with comparatively low electrical impedance are considered. Some motors are connected to healthy sectors of multiple-section distribution substations and these sectors are connected with the section with short circuit occurred through transformer split winding or *double choke*. Consideration of these motors is questionable.

Criterion of motor going to generating mode is the condition

$$U_{rs} < E_M, \quad (3.33)$$

where  $U_{rs}$  - is residual voltage at the point of motor connection, estimated using design short circuit diagram neglecting infeed from the motor;  $E_M$  - is the motor emf.



Accordingly to residual voltages of power supply system centers, a special zone where motors go to generating mode can be outlined. It improves the accuracy of short circuit design circuit by a number of infeeding sources.

Motor currents are considered when devices of switch-gears with voltage 6-10 kV and conductors are checked up on short circuit operation conditions, and for calculation of internal and external power supply installations relay protection. It requires to determine periodical components initial value of motor generated current  $I_{F,t=0,M}$ , surge current  $i_{srg,M}$ , periodical current component  $I_{F,t,M}$ , aperiodical current component  $I_{a,t,M}$  at both arbitrary instant of transient  $t$  and short circuit switching-off instant  $\tau$ .

Rated motor voltage and power are taken as base values calculating in per unit.

The algorithm of short circuit current calculation considering generating motors depends on the point of motor connection to the power supply system. Connection equivalent circuit can be transformed to any of two kinds:

- 1) radial, where each motor is connected with short circuit trough an individual impedance (Fig 3.10, a);
- 2) complex where short circuit point is situated after impedance common for motor group or feeding electric system (Fig 3.10, b, c).

Motor of different kinds, connected radially, should be considered individually while the rest of the circuit should be transferred to equivalent are concerning the point of short circuit with determination of its resulting parameters.

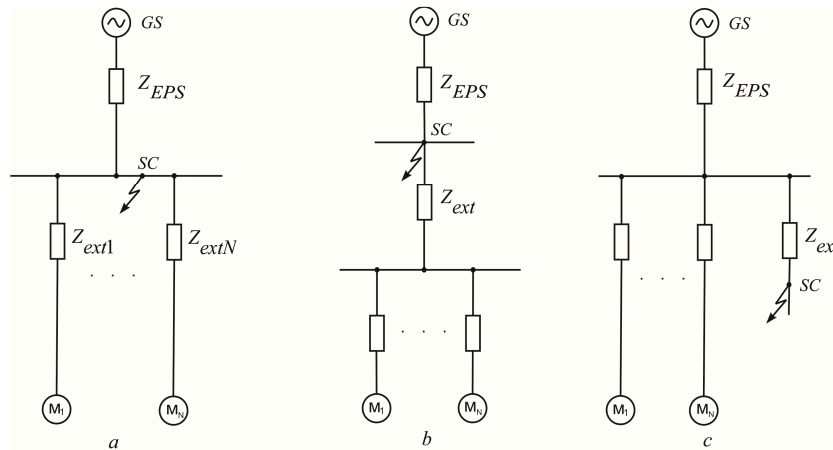


Fig. 3.10. Design circuits with motors: a – radial; b – complex with impedance common for motors; c – complex with impedance common for all sources

**Induction motors.** Initial r.m.s. value of short circuit periodical current component from induction motor considering external impedance through the motor is connected

$$I_{F,t=0,IM} = \frac{E_{*(rated)t=0} I_{rated,M}}{\sqrt{\left(x_{*(rated)IM}'' + x_{*(rated)ext}''\right)^2 + r_{*(rated)ext}^2}}, \quad (3.34)$$

where  $E_{*(rated)t=0}$  - is subtransient motor emf (when there are no apriory data it is assumed approximately that  $E_{*(rated)t=0} = 0,9$ );  $x_{*(rated)IM}''$  - is a subtransient reactance.

Total external impedance can be neglected when  $z_{*(rated)ext} \leq (0,1 \dots 0,2) \cdot x_{*(rated)IM}''$  (impedance of cables of length to 200...300 m and section not less than 50-70 mm<sup>2</sup>). Then: for induction motors

$$I_{F,t=0,IM} = k I_{*(rated)start} I_{rated,IM}, \quad (3.35)$$

where  $k = 1, 2$  - for motors of special design;  $k = 1$  - for other induction motors.

When external impedance corresponds inequality  $z_{*(rated)ext} > 0, 2x''_{*(rated)IM}$ , it should be accounted in calculation :

$$I_{F,t=0,IM} = kI_{*(rated)start} I_{rated,IM} / \left( 1 + z_{*(rated)ext} / x''_{*(rated)IM} \right). \quad (3.36)$$

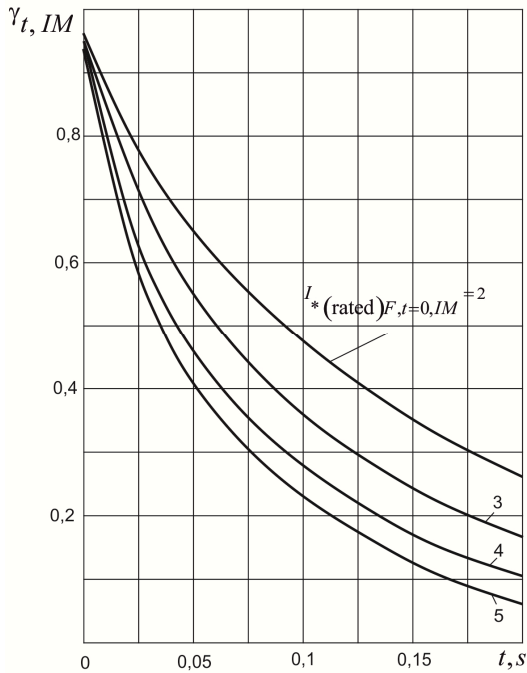


Fig. 3.11. Short circuit periodical current component generated by induction motors depending on electrical remoteness of short circuit point

R.m.s. value of short circuit periodical current component from radially connected induction motors at arbitrary instant can be determined by simplified method using typical curves for motors (Fig. 3.11). These dependences characterize periodical current component time variation (up to 0.2 s) at different electrical remoteness from the point of short circuit. Short circuit periodical current component value at arbitrary instant  $t$ , referred to initial value of this component at  $t = 0$

$$\gamma_{t,IM} = I_{F,t,IM} / I_{F,t=0,IM}. \quad (3.37)$$

Electrical remoteness of the point of short circuit from induction motor is characterized by ratio of motor periodical current component r.m.s. value at zero time of short circuit and its rated current

$$I_{*(rated)F,t=0,IM} = I_{F,t=0,IM} / I_{rated,IM}.$$

Algorithm of calculation of rms value of short circuit periodical current component from induction motor for arbitrary instant  $t$  is similar to the algorithm that provides the use of generator typical curves (see

div.3.2). Short circuit periodical current component for instant  $t$  is equal

$$I_{F,t,IM} = \gamma_{t,IM} I_{F,t=0,IM} \quad (3.39)$$

or (when calculation is made for base conditions, in per unit)

$$I_{F,t,IM} = \gamma_{t,IM} I_{*(b)F,t=0,IM} I_b. \quad (3.40)$$

**Synchronous motors.** Initial r.m.s. value of short circuit periodical current component from synchronous motor considering external impedance is determined by (3.33), as well, where  $x''_{*(rated)SM}$  is substituted for  $x''_{*(rated)IM}$ . Emf  $E_{*(rated)t=0}$  is determined with assumption that before short circuit motor was operating under rated conditions with overexcitation. If there are no a priory data, then  $E_{*(rated)t=0}$  is approximately taken  $E_{*(rated)t=0} = 1, 1$ .

R.m.s. value of short circuit periodical current component for arbitrary instant at radial connection to the point of short circuit is defined by synchronous motors typical curves (Fig. 3.12). These typical curves characterize periodical current component time variation up to 0,2 s for different electrical remoteness of short circuit from the motor. On the graphs short circuit periodical current component for arbitrary instant is divided by initial value of this component when  $t = 0$ :

$$\gamma_{t,SM} = I_{F,t,SM} / I_{F,t=0,SM}. \quad (3.41)$$

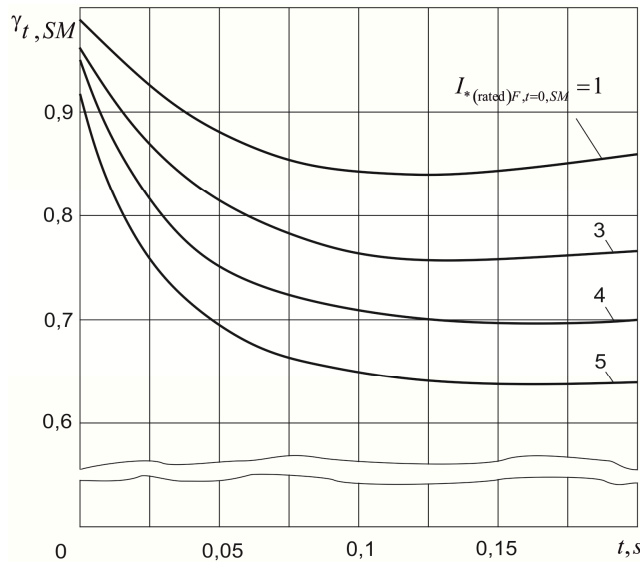


Fig. 3.12. Short circuit periodical current component generated by synchronous motors depending on electrical remoteness of short circuit point

Electrical remoteness of short circuit point from synchronous motor is characterized by ratio of motor periodical current component initial value at  $t = 0$  and to its rated current

$$I_{*(rated)F,t=0,SM} = I_{F,t=0,SM} / I_{rated,SM} \cdot \tag{3.42}$$

Algorithm of calculation of r.m.s. value of short circuit periodical current component from synchronous motor for arbitrary instant  $t$  is similar to the succession given for generators and induction motors. Value of this current component at the instant  $t$  is equal to

$$I_{F,t,SM} = \gamma_{t,SM} I_{F,t=0,SM} = \gamma_{t,SM} I_{*(b)F,t=0,SM} I_b \cdot \tag{3.43}$$

Improved methods include calculation of short circuit periodical current component from either synchronous or induction motors for arbitrary instant solving correspondent differential equation sets of the motors transients with the use of computer.

Short circuit aperiodical current component produced by synchronous or induction motor for arbitrary instant is calculated by formula

$$i_{a,t,m} = \sqrt{2} I_{F,t=0,m} \exp(-t / T_{a,m}), \tag{3.44}$$

where  $T_{a,m}$  - is damping time constant of motor short circuit aperiodical current component given in Table 3.1 (induction motors) or by curves in Fig. 3.13 (synchronous motors).

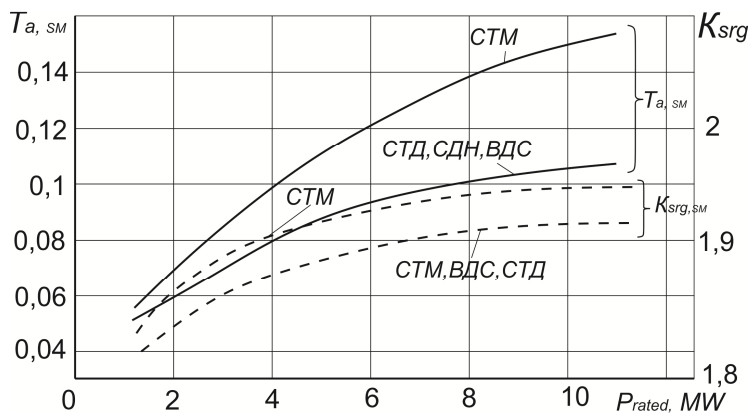


Fig. 3.13. Curves for determination of surge coefficient and damping time constant for short circuit aperiodical current component produced by synchronous motors depending on the power and kind of motor

Table 3.1  
Values of time constant and surge ratio for short circuit at clamps of motors

Parameter	Motor series						
	A	АО	ДА30	АТД	АТМ	ВДД, ДВДА	ДАМСО
$T'_{IM,des}, S$	0.04	0.04	0.09	0.06/S <sub>rated</sub>	0.075	0.06	0.044
$T_{a,M}, S$	0.04	0.03	0.02	0.058	0.043	0.05	0.035
$k_{srg,M}$	1.56	1.49	1.5	1.6	1.67	1.66	1.55

Considering external impedance time constant,  $T_{a,M}$  is calculated by formula:  
for induction motors

$$T_{a,IM,des} = \frac{x''_{*(rated)IM} + x''_{*(rated)ext}}{\omega \left[ x''_{*(rated)IM} / (\omega T_{a,IM}) + r''_{*(rated)ext} \right]}; \quad (3.45)$$

for synchronous motors

$$T_{a,SM,des} = \frac{x''_{*(rated)SM} + x''_{*(rated)ext}}{\omega \left[ x''_{*(rated)SM} / (\omega T_{a,SM}) + r''_{*(rated)ext} \right]}. \quad (3.46)$$

Short circuit motor surge current

$$i_{srg,m} = \sqrt{2} \cdot k_{srg,m} I_{F,t=0,m}^{(3)}. \quad (3.47)$$

Here  $k_{srg,m}$  - is coefficient determined considering external impedance:  
for induction motors

$$k_{srg,IM} = \exp(-0,01/T_{IM,des}) + \exp(-0,01/T_{a,IM,des}); \quad (3.48)$$

for synchronous motors

$$k_{srg,SM} = 1 + \exp(-0,01/T_{a,SM,des}), \quad (3.49)$$

where  $T_{IM,des}$  - is damping time constant of induction motor current periodical component, calculated analogously to (3.45) considering network external impedance (Table 3.1).

When external impedance is not considered  $Z''_{*(rated)ext} < (0,1 \dots 0,2)x''_{*(rated)m}$ , surge coefficient is determined by Table 3.1 (induction motors) or by curves of Fig. 3.13 (synchronous motors).

Total current at the point of short circuit in radial connection of  $N$  motors is found by summation of all power sources of short circuit point by periodical and aperiodical components

$$I_{F,t} = I_{F,t,GS} + \sum_{i=1}^N I_{F,t,m,i};$$

$$i_{a,t} = \sqrt{2} I_{F,t=0,GS} \exp(-t/T_{a,GS}) + \sqrt{2} \sum_{i=1}^N I_{F,t=0,m,i} \exp(-t/T_{a,m,i}). \quad (3.50)$$

Surge current at the point of short circuit is defined by expression

$$i_{ext,m} = \sqrt{2} \cdot (k_{ext,GS} I_{F,t=0,GS}^{(3)} + \sum_{i=1}^N k_{ext,m,i} I_{F,t=0,m,i}^{(3)}) \quad (3.51)$$

When infeed is calculated for  $t < 0,2$  s induction ( $m$ ) and synchronous ( $n$ ) motors can be substituted by equivalent motor and initial total value of periodical current components of motor group is determined as

$$I_{F,t=0,m,\Sigma} = \sum_{i=1}^m I_{F,t=0,IM,i} + \sum_{j=1}^n I_{F,t=0,SM,j} \quad (3.52)$$

For equivalent motor should be calculated as well:  
total rated current

$$I_{rated,m,eq} = \sum_{i=1}^m I_{rated,IM,i} + 1,2 \sum_{j=1}^n I_{rated,SM,j}; \quad (3.53)$$

starting current ratio

$$I_{*(rated)start,eq} = I_{F,t=0,\Sigma} / I_{rated,m,eq}; \quad (3.54)$$

subtransient reactance

$$x_{*(rated)eq}'' = 1 / I_{*(rated)start,eq}; \quad (3.55)$$

damping time constant of short circuit periodical current component

$$T_{F,eq} = \sum_{i=1}^m T_{F,IM,des,i} I_{F,t=0,IM,i} / \sum_{i=1}^m I_{F,t=0,IM,i}; \quad (3.56)$$

damping time constant of short circuit aperiodical current component

$$T_{a,eq} = \left( \sum_{i=1}^m T_{a,IM,des,i} I_{F,t=0,IM,i} + \sum_{j=1}^n T_{a,SM,des,j} I_{F,t=0,SM,j} \right) / I_{F,t=0,m,\Sigma}. \quad (3.57)$$

Here  $I_{rated,IM,i}$ ,  $I_{rated,SM,j}$  - are rated currents of induction motors groups  $\{1; m\}$  and synchronous motors groups  $\{1; n\}$   $T_{F,IM,des,i}$  - are design constant of periodical current component damping time in the group of induction motor;  $T_{a,IM,i}$ ,  $T_{a,IM,j}$  - are damping time constant of aperiodical current component in the groups of induction and synchronous motor.

When short circuit occurs behind common external impedance initial value of short circuit periodical current component from equivalent motor

$$I_{F,t=0,m,eq} \approx I_{*(rated)start,m,eq} I_{rated,m,eq} \left( 1 + \frac{Z_{*(rated)ext}}{x_{*(rated)eq}} \right). \quad (3.58)$$

Short circuit current components from equivalent motor are calculated by formulae:  
periodical component

$$I_{F,t,eq} = I_{F,t=0,m,eq} \exp(-t/T_{F,\Sigma}), \quad (3.59)$$

where  $T_{F,\Sigma} = T_{F,eq} \left( 1 + \frac{Z_{*(rated)ext}}{x_{*(rated)eq}} \right)$ ;

aperiodical component

$$i_{a,t,\Sigma} = \sqrt{2} I_{F,t=0,m,eq} \exp(-t/T_{a,\Sigma}), \quad (3.60)$$

where  $T_{a,\Sigma} = \frac{x_{*(rated)eq} + x_{*(rated)ext}}{\omega \left[ x_{*(rated)eq} / (\omega T_{a,eq}) + I_{*(rated)ext} \right]}$ .

Short circuit surge current generated by equivalent motor is defined by the expression

$$i_{srg,\Sigma} = \sqrt{2} \cdot k_{srg,\Sigma} I_{F,t=0,m,eq}^{(3)}, \quad (3.61)$$

where  $k_{srg,\Sigma} = \exp(-0,01/T_{F,\Sigma}) + \exp(-0,01/T_{a,\Sigma})$ .

When the point of short circuit is situated behind external impedance common for a group of motors and electricity system and  $t > 0,2$  s, short circuit currents are calculated according to the recommendations given for use of (Fig. 3.10,b,c) and for correspondent steps of short circuit current calculation.

### 3.6.2. Peculiarities of consideration of the short circuit point infeed current from the auxiliary motors of thermal power stations

Estimating emergency operations that occur due to short circuit it is necessary to consider currents generated to the point of short circuit by motors of auxiliary electrical installations of thermal power stations. Group consideration of these motors is recommended for currents determination on the tap of a switch-gear section. Motors that take part in the infeed of short circuit point are substituted by equivalent motor with total power  $P_{rated}$  and the following average values of calculated parameters:

power factor $\cos \varphi_{eq}$ .....	0.87
efficiency $\eta_{eq}$ .....	0.94
starting-to-rated current ratio $I_{*(rated)start,eq}$ .....	5.6
periodical current component	
damping time constant $T_{F,eq}$ , S .....	0.07
aperiodical current component	
damping time constant $T_{a,eq}$ , S .....	0.04
surge coefficient $k_{srg}$ .....	1.65

Design model of the auxiliary installation at short circuit should be chosen considering the plan of operational and reserve power supply. When the reserve is clearly defined such the conditions when the switch-gear section is powered by circuit with the lower the electric impedance and all section motors take part in short circuit point infeed should be assumed. When the reserve is hidden it is assumed that one of transformers operating for own needs is switched off and motors of two sections connected by reserve current distributor take part in the short circuit point infeed.

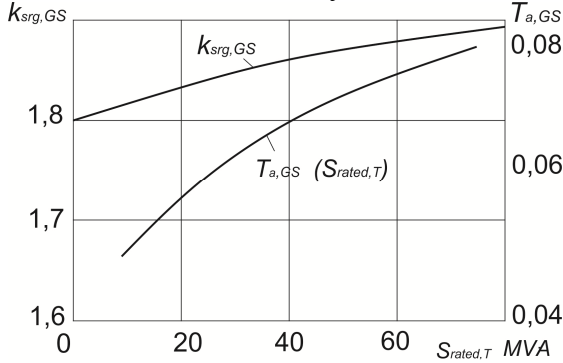


Fig. 3.14. Curves for determination of damping time constant of aperiodical SC current component and surge coefficient depending on power of own needs section transformer of thermal power station

Calculation of short circuit currents in auxiliary electric installations made for selection or check up of their conductors and equipment includes determination of a number of parameters. First of all, on the basis of equivalent circuit for auxiliary section of electric power supply system, short circuit periodical current components from the sources of electric circuit (GS) is determined. Damping time constant of aperiodical current component from electricity system  $T_{a,GS}$  can be defined by the curve (Fig. 3.14), plotted in accordance with transformer rated power  $S_{rated,T}$  (when the transformer has split secondary windings it is taken rated power of the winding to which the section for own needs is connected). If transformer is connected to the section through the extent current distributor time constant  $T_{a,GS}$  is calculated with account the distributor impedance.

The further calculation includes:

- short circuit periodical current component total value generated by section motors or two sections when there is hidden reservation

$$I_{F,t=0,m,eq} = I_{*(rated)start,eq} P_{rated,\Sigma} / \left( \sqrt{3} \eta_{eq} U_{rated} \cos \varphi_{rated} \right), \quad (3.62)$$

where  $U_{rated}$  - is section motors rated voltage;

- short circuit periodical current component total initial value.

$$I_{F,t=0,\Sigma} = I_{F,t=0,GS} + I_{F,t=0,m,eq}; \quad (3.63)$$

- total value of short circuit periodical current component at time  $\tau$

$$\begin{aligned} I_{F,\tau,\Sigma} &= I_{F,\tau,GS} + I_{F,t=0,m,eq} \cdot \exp(-\tau/T_{F,eq}) = \\ &= I_{F,\tau,GS} + \gamma_{\tau,eq} I_{F,t=0,m,eq}, \end{aligned} \quad (3.64)$$

where  $\gamma_{\tau,eq}$  - is damping coefficient of short circuit periodical current component (Fig. 3.15);

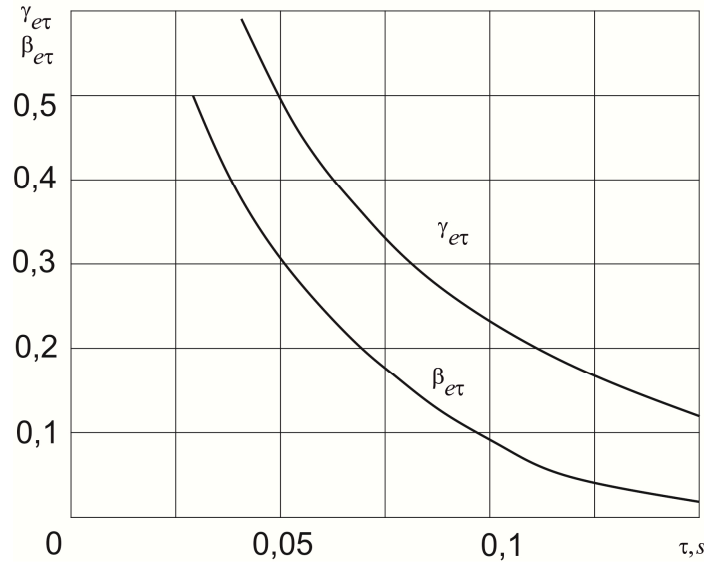


Fig. 3.15. Curves for defining of time dependences of short circuit both periodical and aperiodical current components coefficients when short circuit is generated by equivalent auxiliary motor

- total value of short circuit aperiodical current component at the instant  $\tau$

$$\begin{aligned}
 i_{a,\tau,\Sigma} &= \sqrt{2}I_{F,t=0,GS} \cdot \exp(-\tau / T_{a,GS}) + \\
 &+ \sqrt{2}I_{F,t=0,m,eq} \cdot \exp(-\tau / T_{a,eq}) = \\
 &= \sqrt{2}I_{F,t=0,GS} \cdot \exp(-\tau / T_{a,GS}) + \\
 &+ \sqrt{2}\beta_{\tau,eq}I_{F,t=0,m,eq},
 \end{aligned} \tag{3.65}$$

where  $\beta_{\tau,eq}$  - is short circuit aperiodical current component damping coefficient (Fig. 3.15);

- total short circuit surge current

$$i_{srg,\Sigma} = \sqrt{2} \cdot \left( k_{srg,GS} I_{F,t=0,GS} + k_{srg,eq} I_{F,t=0,m,eq} \right). \tag{3.66}$$

value of  $k_{srg,GS}$  is defined by Fig. 3.14, if current distributor impedance in the connection circuit of transformer for own needs is neglected.

### 3.6.3. Consideration of short circuit point infed from complex load

In calculations of short circuit currents the effect of other complex load components should be considered beside the motors, if current fraction of this load at the point of short circuit accounts for more than 5% short circuit current, determined without complex load consideration.

In general case short circuit current from complex load is defined as geometric sum of its separate components currents. Simplified calculation method allows presenting complex load as an equivalent source of emf with internal impedance.

Current components of short circuit from capacitor batteries of lateral compensation can be defined by expression

$$I_{max,CB} = U_{(0)} / \left[ \sqrt{3}\omega_0 (L_{CB} + L_{ext}) \right], \tag{3.67}$$



where  $U_{(0)}$  - is average rated voltage on capacitor battery before short circuit, kV;  $L_{CB}$  and  $L_{ext}$  - are inductances of capacitor battery and circuit between the battery and the point of short circuit correspondently, H;

$\omega_0$  - is angular frequency of free oscillations of short circuit contour with capacitors battery,

$\omega_0 = 1/\sqrt{(L_{CB} + L_{ext})C}$ ;  $C$  - is capacitor battery capacitance, F.

Free current component of short circuit from capacitor battery

$$i_{CB} = I_{max,CB} \exp(-bt) \cdot \sin(\omega_0 t + \pi), \quad (3.68)$$

where  $b = (R_{(\omega_0)CB} + R_{ext}) / [2(L_{cb} + L_{ext})]$  - is damping coefficient, Ohm/H;  $R_{(\omega_0)CB}$  is an active resistance of capacitor battery at frequency  $\omega_0$ , Ohm;  $R_{ext}$  is resistance of external circuits part referring to capacitor battery, Ohm.

When a static controlled capacitor batteries are available in electric networks, they are introduced into equivalent circuit by a proper combination of capacitive and inductive reactances depending on their design.

### 3.7. Calculation of short circuit currents in electrical installations with voltage up to 1 kV

In electrical installations with voltage up to 1 kV of the most of power supply systems of industrial enterprises the point of short circuit is characterized by the great electrical remoteness from the sources of power supply system. Their power usually exceeds the consumed one. If the source power exceeds power of a step-down transformer more than 25 times at the voltage up to 1 kV, the magnitude of short circuit periodical current component on the side of low voltage received from the electric system can be considered constant. It validates the assumption that electrical enterprises electrical installations with voltage up to 1 kV are connected to unlimited power source. It is characterized by emf equal to the voltage in the place of connection and reactance of the sources coupling with the point of connection. The value of the coupling reactance is reduced to the stage of low voltage by formula, MOhm

$$x_{GS} = U_{av,LV}^2 / (\sqrt{3} I_{F,HV} U_{av,HV}) = U_{av,LV}^2 \cdot 10^{-3} / S_s \quad (3.69)$$

or

$$x_{GS} = U_{av,LV}^2 / (\sqrt{3} I_{br, rated} U_{av,LV}), \quad (3.70)$$

where  $U_{av,LV}$  - is the average rated voltage of the mains connected with the transformer low-voltage winding, V;  $U_{av,LV}$  - is average rated voltage of the mains connected with the transformer high-voltage winding, V;  $S_s$  - is the short circuit power at the terminals of transformer high-voltage winding, MV·A;  $I_{F,HV}$  is r.m.s. value of the three-phase short circuit current periodic component at the terminals of transformer high-voltage winding, kA;  $I_{br, rated}$  is the rated current of switching off for breaker mounted in the subcircuit were the step-down transformer is connected, kA.

If the step-down transformer is connected to the network of power system by choke, overhead or cable transmission line (that's longer than 1 km) inductive reactance and resistance of these components should be taken into account.

Calculating short circuit current in electrical installation with independent power sources it is required to find parameter values of independent electric system elements including independent

sources (synchronous generators) distribution network and consumers. It is also required to consider:

- shorted circuit conductor resistance variation caused by their heating at short circuit;
- complex load (electric motors, converters, thermal installations, incandescent lamps) effect on short circuit current value.
- if complex load rated current does not exceed 10% of short circuit periodical current component initial value when short circuit is calculated with neglect of load;
- affect the capacitor batteries when short circuit current is calculated to choose fuses.

In this case it is allowed:

- to simplify and make equivalent all external network relative to the point of short circuit and consider separately only independent power sources and electric motor immediately adjacent to the point of short circuit;
- the effect of induction electric motors can be neglected if their total rated current does not exceed 10 per cent of initial value of periodical current component at the point of short circuit, obtained without electric motors account.

It is convenient to made calculations of short circuit current in electrical installations with voltage up to 1 kV in concrete units. When equivalent circuits are made, the design circuit component parameters of the initial equivalent circuit are reduced to the voltage step at which the point of short circuit is situated, resistance and reactance of the components of all equivalents in the circuit are stated in milliohms.

Certainty of short circuit current calculation results depends on accurate estimation and full consideration of elements and their impedances under short circuit. In electrical installations with voltage up to 1 kV essential influence on short circuit current values have resistance of the circuit. Their values are comparable to and sometimes even exceed inductive reactance component values. Therefore, determining resulting impedance for short circuit current, impedances of collective buses, bus ways and distribution bus ducts, automatic circuit breakers and relays of current coil, current transformer coils, switching unit contacts, contacts in the network and switch-gears and also arc resistance at the point of short circuit are considered.

Determination of shorted circuit element parameters.

Power transformers. Step-down transformer impedance full value as well as its active and inductive components reduced to the step of low voltage (in milliohms) are calculated by formulae

$$Z_T = u_s U_{\text{rated,LV}}^2 \cdot 10^4 / S_T; \quad (3.71)$$

$$r_T = p_s U_{\text{rated,LV}}^2 \cdot 10^6 / S_T^2; \quad (3.72)$$

$$x_T = \sqrt{u_s^2 - (100 p_s / S_T)^2} \cdot U_{\text{rated,LV}}^2 \cdot 10^4 / S_T, \quad (3.73)$$

where  $S_T$  - transformer rated power, kV A;  $U_{\text{rated,LV}}$  - rated line voltage of transformer low voltage winding, kV;  $p_s$  - short circuit test loss in the transformer, kW;  $u_s$  - transformer short circuit test voltage, %.

For step-down transformers with  $\Delta/Y_0$  winding connection when short circuit occurs in low-voltage network active and inductive reactance components of zero sequence should be assumed equal to active and inductive impedance components of positive sequence. For transformers with different connection pattern active and reactive components of zero sequence should be assumed according to those specified by manufacturer.

Buses and busways. Their impedances are determined using active and reactive impedance components referred to length. These values are given for flat buses in Table 3.2 and for completed bus ducts in Table 3.3.

Overhead transmission lines and cable lines

Transmission line resistance and inductive reactance values are calculated using tabulations given in reference literature. Inductive reactance is approximately equal 0.4 mOhm/m for overhead lines and 0,08 mOhm/m for cable lines.

Table 3.2  
Resistance and inductive reactance of flat buses

Bus cross section area, mm <sup>2</sup>	Impedance components, mOhm/m					
	Resistance at 65°C for material		Reactance at average gheometric remoteness between phases, mm			
	copper	aluminium	100	150	200	300
25×3	0.268	0.457	0.179	0.200	0.295	0.244
30×3	0.223	0.394	0.163	0.189	0.206	0.235
30×4	0.167	0.296	0.163	0.189	0.206	0.235
40×4	0.125	0.222	0.145	0.170	0.189	0.214
40×5	0.100	0.177	0.145	0.170	0.189	0.214
50×5	0.080	0.142	0.137	0.156	0.180	0.200
50×6	0.067	0.118	0.137	0.156	0.180	0.200
60×6	0.056	0.099	0.119	0.145	0.163	0.189
60×8	0.042	0.074	0.119	0.145	0.163	0.189
80×8	0.031	0.055	0.102	0.126	0.145	0.170
80×10	0.025	0.044	0.102	0.126	0.145	0,170
100×10	0.020	0.035	0.090	0.113	0.133	0.157
2(60×8)	0.0209	0.037	0.120	0.145	0.163	0.189
2(80×8)	0.0157	0.0277	-	0.126	0.145	0.170
2(80×10)	0.0125	0.0222	-	0.126	0.145	0.170
2(100×10)	0.010	0.0178	-	-	0.133	0.157

Completed bus ducts parameters

Bus ducts kind	Rated current, A	Specific impedance components, mOhm/m	
		resistance	reactance
ШМА73	1600	0.031	0.017
ШМА68H	2500	0.027	0.023
ШМА68H	4000	0.013	0.020
ШЗМ16	1600	0.017	0.014
ШПА73	250	0.200	0.100
ШПА73	400	0.130	0.100
ШПА73	630	0.085	0.075

Chokes. Resistance of a current limiting reactors (chokes), in milliohms,

$$r_{ch} = \Delta p_{rated,ch} \cdot 10^3 / I_{rated,ch}^2 \tag{3.74}$$

where  $\Delta p_{rated,ch}$  - is active power loss in choke phase at its rated current, A.

Choke inductive reactance is taken as indicated by manufacturer or calculated by formula, in milliohms

$$x_{ch} = \omega(L - M) \cdot 10^3, \quad (3.75)$$

where  $\omega$  - is angular frequency of mains voltage, rad/s;  $L$  - is three-phase choke coil inductance, H;  $M$  - is mutual inductance of choke branches, H.

Current transformers, switching units and relays

Their impedance values depend on the rated current. For primary windings of all multiturn current transformers, their rated data or average values are given in Table 3.4.

Table 3.4  
Current transformer winding impedance components

Transformation coefficient of a current transformer	Current transformer class of accuracy			
	first		second	
	$x$	$r$	$x$	$r$
20/5	67	42	17	19
30/5	30	20	8	8.2
40/5	17	11	4.2	4.8
50/5	11	7	2.8	3
75/5	4.8	3	1.2	1.3
100/5	2.7	1.7	0.7	0.75
150/5	1.2	0.75	0.3	0.33
200/5	0.67	0.42	0.17	0.19
300/5	0.3	0.2	0.08	0.088
400/5	0.17	0.17	0.04	0.05
500/5	0.07	0.07	0.02	0.02

Note: single-turn current transformer impedances for currents more than 500 A can be neglected.

For coils of automatic breakers trip it is allowed to use trip coil impedance values and movable contact intermediate impedance values given in Table 3.5 if there are no manufacturer data on inductive reactance and resistances. The indicated resultant impedance values are given for automatic circuit breaker coils and contacts (A3700, "Электрон" and BA). They depend on rated current and do not depend on a breaker kind.

Contact transition in electrical network. Contact resistance values in cable connections, plug-and-socket of switching units and bus ducts are given respectively in Table 3.6-3.8. Conductor contact connection resistances are defined for the most typical connection positions: bus duct- bus duct; plug-and-socket; bus duct automatic breaker; cable-automatic breaker

Table 3.5  
Electrical resistances values of automatic breaker contacts and coils.

Rated current , A	Impedance components, mOhm	
	Resistance	Reactance
50	7	4.5
70	3.5	2.0
100	2.15	1.2
140	1.3	0.7
200	1.1	0.5
400	0.65	0.17
600	0.41	0.13
1000	0.25	0.1
1600	0.14	0.08
2500	0.13	0.07
4000	0.1	0.05

Table 3.6  
Electrical resistances values of aluminum cable contact connections

Cross section area, mm <sup>2</sup>	16	25	35	70	95	120	150	240
Resistance, mOhm	0.085	0.064	0.056	0.043	0.029	0.027	0.021	0.12

Table 3.7  
Electrical resistance values of bus ducts contact connections

Kind	ИИПА-73	ИИПА-73	ИИПА-73	ИИПА-73	ИИМА-68H	ИИМА-68H
Rated current, A	250	400	630	1600	2500	4000
Resistance, mOhm	0.009	0.006	0.004	0.003	0.002	0.001

Arc at the point of short circuit

To consider arc in calculation it is recommended to use approximate active resistance values given in Table 3.9. Arc resistance is neglected in calculation of maximum short circuit current values.

Table 3.8  
Approximate values of electrical resistances of switching unit plugs-and-sockets with voltage up to 1 kV, mOhm

Rated current of switching unit, A	Resistance values		
	Automatic breaker	Knife-switch	Disconnecter
50	1.3	-	-
70	1.0	-	-
100	0.75	0.5	-
150	0.65	-	-
200	0.6	0.4	-
400	0.4	0.2	0.2
600	0.25	0.15	0.15
1000	0.12	0.08	0.08
3000	-	-	0.02

Table 3.9  
Electrical arc resistance values

Short circuit rated conditions	Resistance values, mOhm, when short circuit occurs behind transformers of power, kV A					
	250	400	630	1000	1600	2500
1) Transformer secondary voltage in cable pothead; in bus duct IIIA in bus duct IIIPA	15	10	7	5	4	3
2) Short circuit at the end of bus duct IIIPA at the remoteness of 100-150 m;	-	-	-	6	4	3
3) SC at the end of bus duct IIIA at the remoteness of 100-150 m;	-	18	15	10	7	5
4) In cable line of cross section 25-240 mm <sup>2</sup> at the remoteness of 100 m	-	30-45	25-45	20-40	15-30	20-40
	-	-	-	6-8	5-7	4-6
	-	45-16	45-15	45-12	45-11	40-10

Resistances of electrical equipment and appliances, contacts, and arc at the point of short circuit can be determined as a part of resulting electrical transient resistance

$$r_{tr} = r_{cont} + r_{CB} + r_{CT} + r_{arc}. \quad (3.76)$$

Here  $r_{cont}$  - is resistance of contacts in network elements connection;  $r_{CB}$  - is resistance of automatic circuit breaker made up of active resistance of trip current coils and resistance of contacts;  $r_{CT}$  - is resistance of primary winding of current transformer;  $r_{arc}$  - is resistance of arc at the point of short circuit.

Resulting resistance  $r_{tr}$  depends on the capacity of step-down transformer of a completed substation, electrical remoteness of the point of short circuit according to stages of electrical power distribution (short circuit step) and minimum phase remoteness at the point of short circuit values of  $r_{tr}$ . at short circuit that occurred on secondary side of completed transformer station are given below:

<i>transformer power, kV·A,</i>	400	630	1000	1600	2500
<i>contact resistance, mOhm,</i>	9.21	8.02	8.41	5.51	5.12

Considering factors given in [5] adequate estimation of resulting contact resistance has been get [3.76] for those short circuit points in the network that are situated behind completed transformer substation:

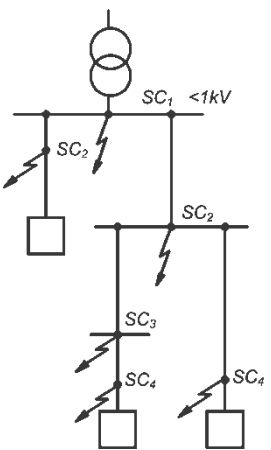
$$r_T = \left( 2,5\sqrt{S_T} k_s^3 + 320 \cdot a \right) / S_T, \quad (3.77)$$

where  $k_s$  - is short circuit step coefficient, determined in accordance with design circuit of the network by Table 3.10; phase conductors distance between network of the point at the short circuit (resistance at the point of short circuit depends on it), its values, in mm are given below for different network components:

<i>Power of transformer, kV·A</i>		
400 .....	60	
630 .....	60	<i>Section of cable, mm<sup>2</sup></i>
1000 .....	70	2,5-10 .....
1600 .....	120	16-35 .....
2500 .....	180	50-95 .....
<i>Bus duct</i>		120-150 .....
III MA .....	10	240 .....
III PA .....	45	4.8

Table 3.10

Short circuit stage coefficient values

Design network circuit with points of short circuit	Short circuit step characteristics	Values
	Switch-gear at power stations and substations	Are determined for the point SC <sub>1</sub> by given above data
	Primary department distribution centers: equipment terminals, radial feeder from substation board or main trunk	2
	Secondary department distribution centers: equipment terminals that is powered from primary distribution centers	3
	Equipment located near using equipment powered from secondary distribution centers	4

Independent electric power sources and synchronous electric motors for instant  $t = 0$  are considered as electromotive force source with subtransient reactance of synchronous machine on direct axis  $x''_d$ . Simplified calculations assume that:  $x''_{*(rated)d} = 0,15$ ;  $x''_{*(rated)2} = x''_{*(rated)d}$ ;  $r_{*(rated)SM} = 0,15 x''_{*(rated)d}$ .

Induction electric motors for time  $t = 0$  should be introduced into the equivalent circuit as emf source with subtransient inductive reactance  $x''_{IM}$ . When short circuit current calculation is distinguished induction electric motors are introduced into equivalent circuit as emf sources with subtransient inductive and active components of stator impedance. Total resistance that characterizes induction electric motor at the point of short circuit, in milliohms,

$$r_{IM} = r_1 + 0,96\hat{r}_2, \tag{3.78}$$

where  $r_1$  - is stator ohmic resistance, mOhm;

$\hat{r}_2$  - is rotor active resistance, reduced to stator, and defined by the expression, mOhm:

$$\hat{r}_2 = 0,36M_{*(rated)start} (P_{rated} + \Delta p_{mech}) \cdot 10^6 / \left[ I_{*(rated)start}^2 (1 - s_{rated}) I_{rated}^2 \right], \tag{3.79}$$

where  $M_{*(rated)start}$  - is electric motor starting torque-to-nominal torque ratio;  $P_{rated}$  and  $I_{rated}$  - are rated power (kW) and current (A) of electric motor;  $\Delta p_{mech}$  - is mechanical power loss in

electric motor including additional losses, kW;  $\Delta p_{\text{mech}} = 0,01P_{\text{rated}}$ ;  $I_{*(\text{rated})\text{start}}$  - is ratio of starting motor current to rated motor current;  $s_{\text{rated}}$  - is rated slip, in per unit.

Electrical stator resistance is calculated by the following formulae if it is not specified by manufacturer:

For motors with phase-wound rotor and simple squirrel-cage rotor,

$$r_1 = 0,52\hat{r}_2 M_{*(\text{rated})\text{start}} \left( 1/s_{\text{cr}} + s_{\text{cr}} - 2M_{*(\text{rated})\text{max}} / M_{*(\text{rated})\text{start}} \right) / \left[ s_{\text{cr}} \left( M_{*(\text{rated})\text{max}} - M_{*(\text{rated})\text{start}} \right) \right], \quad (3.80)$$

where  $s_{\text{cr}}$  - is the critical slip of electric motor, its value is equal to

$$s_{\text{cr}} = M_{*(\text{rated})\text{max}} s_{\text{rated}} \left( 1 - M_{*(\text{rated})\text{start}} \right) + \frac{\sqrt{s_{\text{rated}} M_{*(\text{rated})\text{start}} \left( M_{*(\text{rated})\text{max}} - 1 \right) \left( M_{*(\text{rated})\text{max}} - M_{*(\text{rated})\text{start}} \right)}}{M_{*(\text{rated})\text{max}} - M_{*(\text{rated})\text{rated}}}; \quad (3.81)$$

For motors which rotor electrical parameters depends on slip (motors with deep slots or with two cages),

$$r_1 = k_m P_{\text{rated}} (1 - \eta_{\text{rated}}) \cdot 10^6 / (3I_{\text{rated}}^2 \eta_{\text{rated}}), \quad (3.82)$$

where  $k_m$  - is ratio of the stator winding power loss to the total loss in the motor when it operates under rating conditions (normally  $k_m = 0,35$ );  $\eta_{\text{rated}}$  - is the rated motors efficiency.

Subtransient inductive reactance of induction motor, mOhm,

$$x_{\text{IM}}'' = \sqrt{\left[ U_{\text{ph,rated}} \cdot 10^3 / \left( I_{*(\text{rated})\text{start}} I_{\text{rated}} \right) \right]^2 - r_{\text{IM}}^2}, \quad (3.83)$$

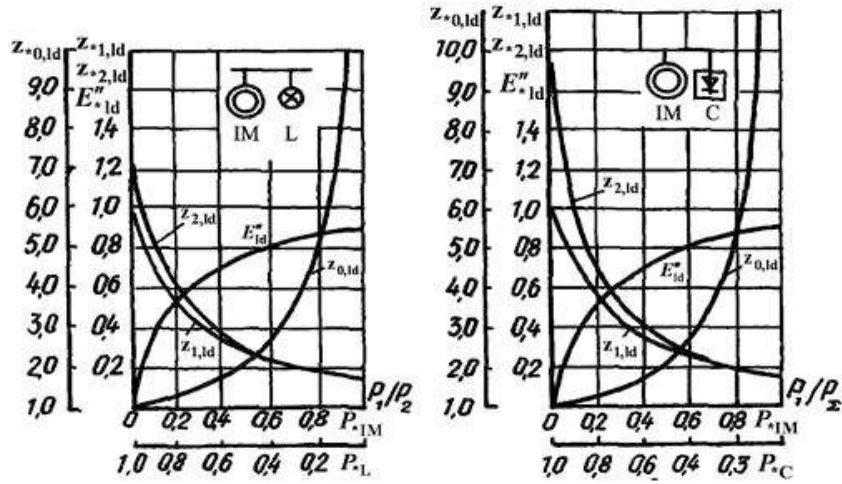
where  $U_{\text{ph,rated}}$  - is the rated phase voltage of the motor, V.

When the calculation is simplified:  $x_{*(\text{rated})\text{IM}} = 0,18$ ;  $r_{*(\text{rated})\text{IM}} = 0,36$ .

At short circuit current calculations complex load accounted by positive, negative and zero sequence parameters. Recommended values of impedances of positive and negative sequences of complex load of separate components are given in Table 1.3 (chapter 1).

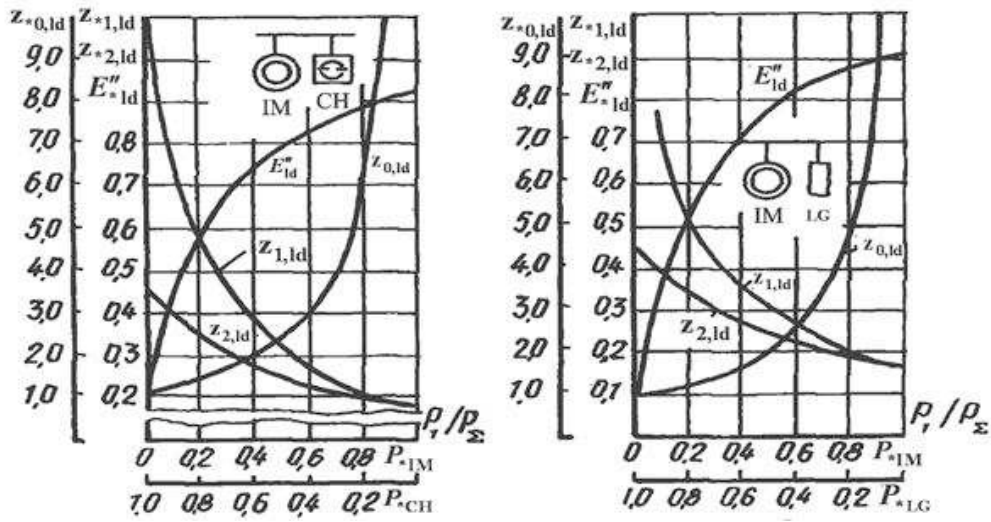
Impedance magnitude values of positive  $Z_{*(\text{rated})1,\text{ld}}$ , negative  $Z_{*(\text{rated})2,\text{ld}}$  and zero  $Z_{*(\text{rated})0,\text{ld}}$  sequences as well as complex load emf  $E_{*(\text{rated})t=0,\text{ld}}$  in per unit are defined by curves given in Fig. 3.16 accordingly to load center consumer relative structure  $P_i / P_{\Sigma}$ , where  $P_{\Sigma}$  - total rated load power, kW,  $P_i$  - is rated power of  $i$ -th load consumer, kW; ( $P_{\text{IM}}$  - for induction motors;  $P_{\text{SM}}$  - for synchronous motors;  $P_{\text{DL}}$  - for gas discharge lamp power;  $P_{\text{conv}}$  - for converters;  $P_{\text{TE}}$  - for electric thermal installations).



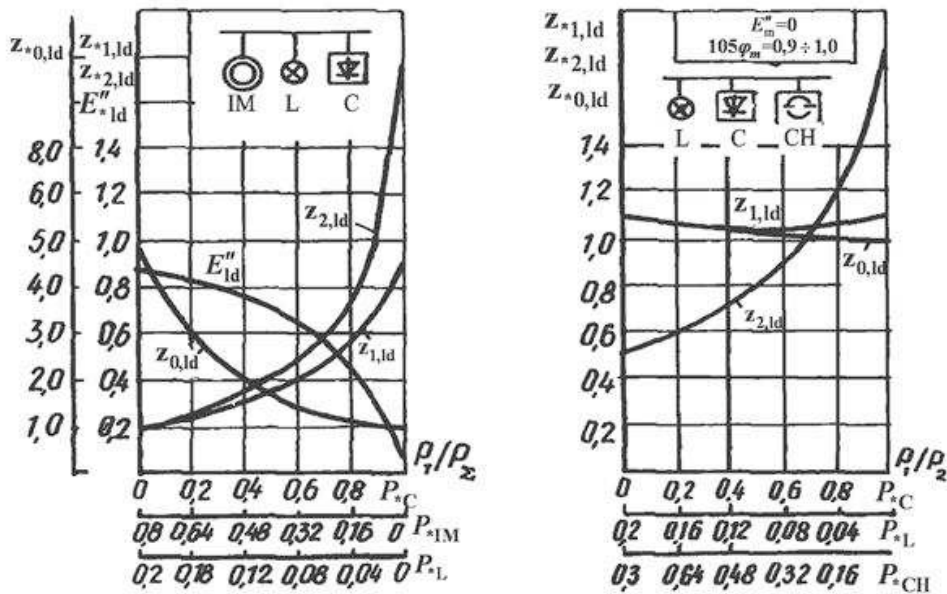


a b

Fig. 3.16 (a,b). Dependence of complex load parameters on its components



c d



e f

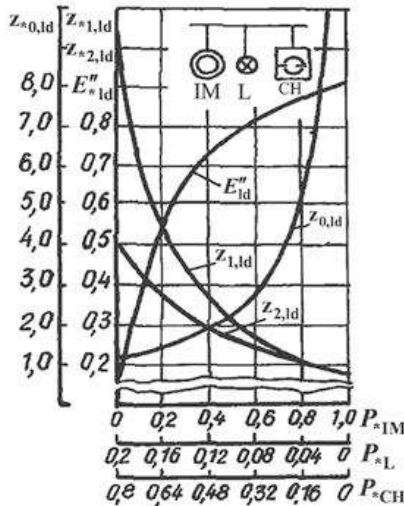


Fig. 3.16 (c,d,e,f,g). Dependencies of complex load parameters on its components

g

When the calculation is simplified it is allowed to take into account complex load at the point of short circuit by impedances magnitude values:

$$Z_{*(rated)1,ld} = Z_{*(rated)2,ld} = 0,4$$

Banks of Capacitors. Resistance, inductive and capacitive reactances of the capacitors bank are assigned according to those given by manufacturer. Resulting components of short circuit positive sequence impedance  $\tilde{o}_{1,res}$  and  $r_{1,res}$  are determined by equivalent circuit transformation according to chapter 1 recommendations considering contact resistance. When resulting components have been found, short circuit current is determined.

$$Z_{*(rated)0,ld} = 0,3.$$

If there are synchronous or induction motors or other complex load components near the point of short circuit, initial r.m.s. values of short circuit periodical current component considering infeed should be defined as a sum of power system currents (independent sources) and motor currents or currents of other components of the complex load.

Initial periodical component r.m.s. value of the current from power system at three-phase short circuit is calculated by formula, kA:

$$I_{F,t=0,GS} = U_{av,rated} / \left( \sqrt{3} \sqrt{r_{1,res,GS}^2 + x_{1,res,GS}^2} \right). \quad (3.84)$$

Short circuit periodical current component initial r.m.s. value in electrical installations with independent sources is calculated by formula, kA:

$$I_{F,t=0,G} = E_{ph,t=0,G} / \left( \sqrt{r_{1,res,G}^2 + x_{1,res,G}^2} \right), \quad (3.85)$$

where  $E_{ph,t=0,G}$  - is equivalent subtransient emf of independent sources, V, (emf value is calculated in the same way as for synchronous motors).

Initial periodical component r.m.s. value of the short circuit current from synchronous motors is calculated by expression, kA:

$$I_{F,t=0,SM} = E_{ph,t=0,SM} / \left( \sqrt{r_{1,res,SM}^2 + x_{1,res,SM}^2} \right), \quad (3.86)$$

where  $E_{ph,t=0,SM}$  - is phase value of synchronous motor subtransient emf, V;

$$E_{ph,t=0,SM} = \sqrt{\left(U_{ph(0)} \pm I_{(0)} x_d'' \sin \varphi_{(0)}\right)^2 + \left(I_{(0)} x_d'' \cos \varphi_{(0)}\right)^2}, \quad (3.87)$$

where plus sign "+" means overexcitation mode; minus sign "-" means underexcitation;  $U_{(0)}$ ,  $I_{(0)}$ ,  $\varphi_{(0)}$  - are correspondingly phase voltage on motor terminals, stator current and angle of voltage and current phase displacement at the instant preceding short circuit (generally they are taken equal to rated values).

Starting rms value of periodical current component of short circuit from induction motors is calculated by formula, kA:

$$I_{F,t=0,IM} = E_{ph,t=0,IM} / \left(\sqrt{r_{1,res,IM}^2 + x_{1,res,IM}^2}\right), \quad (3.88)$$

where  $E_{ph,t=0,IM}$  - is phase value of induction motor subtransient emf, V,

$$E_{ph,t=0,IM} = \sqrt{\left(U_{ph(0)} \cos \varphi_{(0)} - I_{(0)} r_{IM}\right)^2 + \left(U_{ph(0)} \sin \varphi_{(0)} - I_{(0)} x_{IM}''\right)^2}. \quad (3.89)$$

Estimation of complex load effect on total short circuit current is made in accordance with consumer structure in complex load center and short circuit point position (Fig 3.17). In radial equivalent circuit (Fig. 3.17,a) it is allowed to neglect static consumers effect (converters, electric thermal installations, electric lighting). Initial value of short circuit periodical current components, surge current and periodical current components of short circuit from motors at arbitrary instant are calculated accordingly to the recommendation given above.

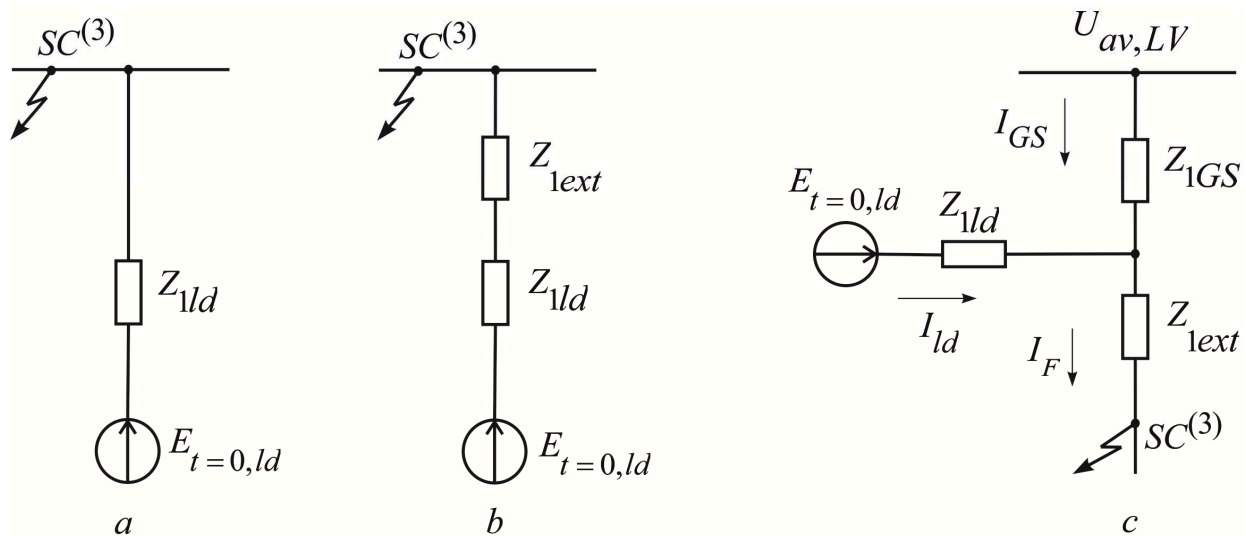


Fig. 3.17. Complex load equivalent circuits:

a – radial; b – radial considering external impedance; c – integrated with common impedance for all sources

When short circuit occurs behind the mutual for load center impedance (Fig. 3.17,b), the initial value of three-phase short circuit periodical current component is defined considering effect of motors and static load, kA:

$$I_{F,t=0,ld} = E_{*(rated)t=0,ld} U_{av,LV} / \left(\sqrt{3} z_{res,ld}\right), \quad (3.90)$$

where

$$Z_{\text{res,ld}} = \sqrt{\left( \frac{Z_{*(\text{rated})1,\text{ld}} \cos \varphi_{\text{ld}} U_{\text{av,LV}}^2}{S_{\text{rated,ld}}} + r_{\text{1ext}} \right)^2 + \left( \frac{Z_{*(\text{rated})1,\text{HR}} \sin \varphi_{\text{ld}} U_{\text{cp,LV}}^2}{S_{\text{rated,ld}}} + x_{1,\text{ext}} \right)^2}.$$

In these equations  $E_{*(\text{rated})t=0,\text{ld}}$  and  $Z_{*(\text{rated})1,\text{ld}}$  - are the load current equivalent emf and positive sequence impedance, their values in per unit are defined by dependences given in Fig. 3.16, according to the consumer structure;  $S_{\text{rated,ld}}$  - is total rated load apparent power, kV·A.

When short circuit occurs after the impedance common for load and power system (Fig. 3.17,a) and close values of equivalent circuit branches component  $x, r$  initial value of short circuit positive current component is calculated by formula

$$I_{F,t=0,\text{ld}} = \frac{U_{\text{av,LV}} Z_{1,\text{ld}} + E_{*(\text{rated})1,\text{ld}} U_{\text{av,LV}} Z_{\text{GS}}}{Z_{\text{GS}} Z_{1,\text{ld}} + Z_{\text{GS}} Z_{1,\text{ext}} + Z_{1,\text{ext}} Z_{1,\text{ext}}}. \quad (3.91)$$

When there is a capacitors battery in the load center (3.66) and (3.67) are used to determine maximum magnitude of short circuit high-frequency current component and free current component of short circuit. Aperiodical current component in general case is taken equal to periodical current component magnitude at zero time of short circuit i.e.  $i_{a,t=0} = \sqrt{2} I_{F,t=0}$ .

In radial circuits, aperiodical current component for arbitrary instant is calculated by formula (3.44).

If radial branches independent from each other converge at the point of short circuit, then SC aperiodical current component is defined as sum of separate branches aperiodical current components

$$i_{a,t,\Sigma} = \sum_{j=1}^m i_{a,t=0,j} \cdot \exp(-t/T_{a,j}), \quad (3.92)$$

where  $m$  is the number of independent circuit branches,  $i_{a,t=0,j}$  is the initial value of aperiodical current component in  $j$ -branch, kA.

Surge current of three-phase short circuit in electrical installations with one power source (supply from power system or independent source) is defined by expression

$$i_{\text{srg}} = \sqrt{2} \cdot k_{\text{srg}} I_{F,t=0}, \quad (3.93)$$

where  $k_{\text{srg}} = 1 + \sin \varphi_s \cdot \exp(-t_{\text{srg}}/T_a)$  - is surge factor that can be determined according to dependences shown at Fig. 3.18;  $\varphi_s = \arctg(x_{1,\text{res}}/r_{1,\text{res}})$  - is angle of the phase displacement of the source voltage or emf and short circuit periodical current component;  $t_{\text{srg}}$  - is surge current occurrence instant of time, s,

$$t_{\text{srg}} = 0,01 \left( \frac{\pi}{2} + \varphi_s \right) / \pi. \quad (3.94)$$

Calculating surge current of short circuit on independent sources terminals and synchronous and induction motor terminals it is allowed to suppose surge current occurrence in 0,01 s after short circuit. SC periodical current component maximum value at  $t = 0,01$  s is equal to this component maximum value at the initial instant of short circuit occurrence.

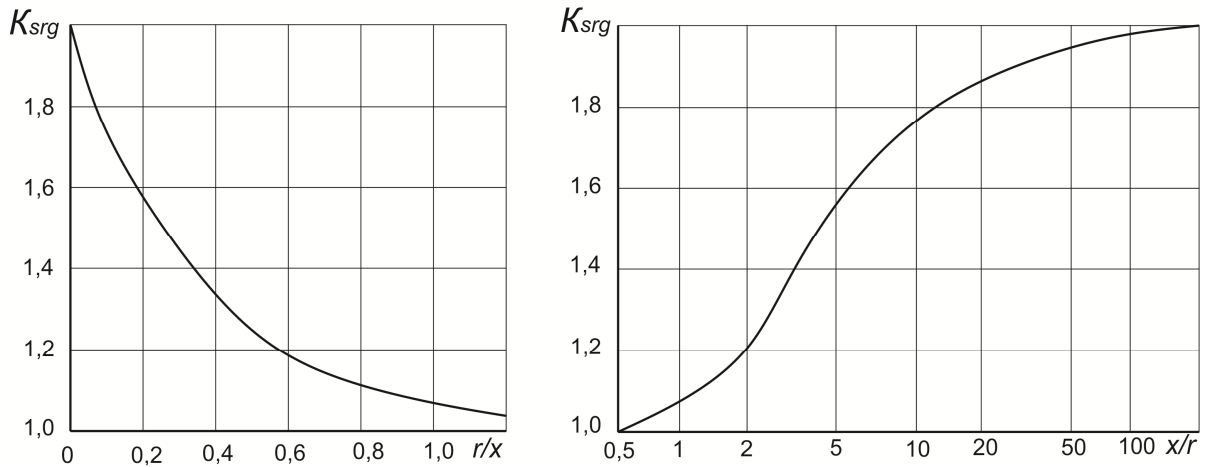


Fig. 3.18. Dependence of surge coefficient on  $r/x$  (a) and  $x/r$  (b)

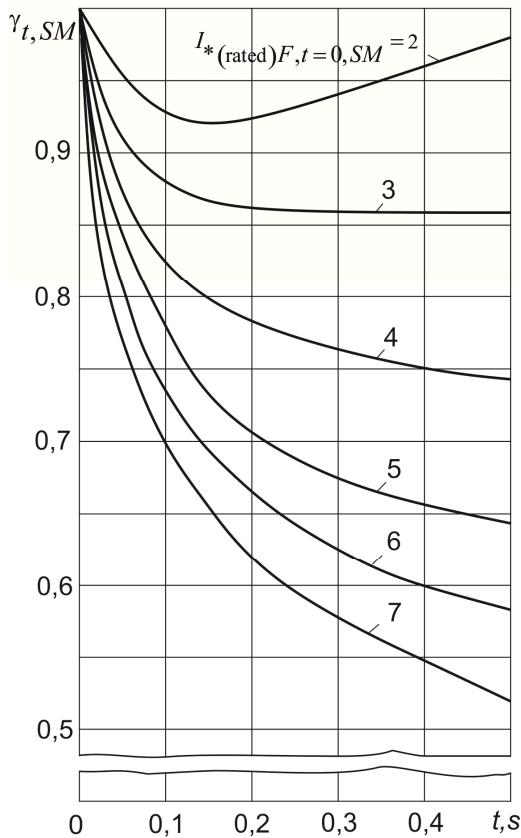


Fig. 3.19. Typical curves for determination of short circuit current periodic component produced by synchronous machines according to time and electrical remoteness of short circuit

Surge current from induction motor is calculated by formulae (3.46) and (3.47) considering aperiodical and periodical short circuit current components maximum value dying out.

Before these calculations it is necessary to determine the following:

$$T_{ch} = (x''_{IM} + x_{1,ext}) / (\omega \hat{r}_2);$$

$$T_a = (x''_{IM} + x_{1,ext}) / (\omega (r_1 + r_{1,ext})),$$

where  $r_1, \hat{r}_2$  - are correspondently stator resistance and rotor active resistance, reduced to stator and calculated by expression (3.79).

Short circuit surge current is defined as a sum of separate branches surge components if radial independent from each other branches converge at the point of short circuit

$$i_{srg,\Sigma} = \sqrt{2} \cdot \sum_{i=1}^m k_{srg,i} I_{F,t=0,i} \quad (3.95)$$

Calculation of periodical current component of short circuit from independent power sources with radial connection and in synchronous motors as well for an arbitrary instant is made using design curves (Fig. 3.19). Design curves characterize this component relative values time variation at different electrical remoteness from the point of short circuit. Values of short circuit periodical current component for arbitrary instant are referred to initial value of this component  $\gamma_{t,SM} = I_{F,t,SM} / I_{F,t=0,SM}$ . Electrical remoteness of synchronous machine from the point of short circuit is characterized by ratio of periodical current component r.m.s. value at initial instant of short circuit and its rated current  $I_{*(rated)F,t=0,SM} = I_{F,t=0,SM} / I_{rated,SM}$ .

R.m.s. value of short circuit periodical current component from a synchronous machine for arbitrary instant (or when there are several synchronous machines of the same type in similar conditions concerning the point of short circuit) is defined by formulae

$$I_{F,t,SM} = \gamma_{t,SM} I_{*(rated)F,t=0,SM} \cdot I_{rated,SM} \quad (3.96)$$

Typical curves, shown at Fig. 3.20,  $\gamma_{t,IM} = I_{F,t,IM} / I_{F,t=0,IM}$  are used for simplified calculation of periodical current component value of short circuit from induction motors in radial connection for arbitrary instant.

The curves are plotted for different electrical remoteness from the point of short circuit that is defined by ratio

$$I_{*(rated)F,t=0,IM} = I_{F,t=0,IM} / I_{rated,IM}$$

Then for induction motor (or several induction motors in similar conditions concerning the point of short circuit) the following expression is true:

$$I_{F,t,IM} = \gamma_{t,IM} I_{*(rated)F,t=0,IM} \cdot I_{rated,IM} \quad (3.97)$$

To provide more precise calculation of short circuit current and its components calculation, it is necessary to consider variation of conductors and circuit elements reactance due to heating by short circuit current.

Cable transmission lines. Cable resistance at heating by short circuit current is calculated by formula

$$r_{CL} = c_v r_{(v=+20^\circ C)} \quad (3.98)$$

where  $c_v$  - is coefficient accounting cable resistance component increase (the value of  $c_v$  is allowed to take approximately equal to 1.5; at precise calculation  $c_v$  should be defined by graphs given in [36], in accordance with cable material, cable cross section area, connected transformer volt-amperes, and short circuit current duration;  $r_{(U=+20^\circ C)}$  is resistance of copper/aluminum cable strand at  $+20^\circ C$ .

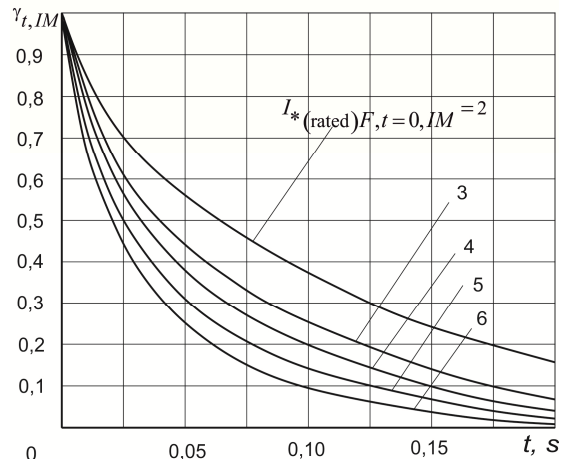


Fig. 3.20. Typical curves for determination of periodical current component value of short circuit caused by induction motors as function of the electrical remoteness and time of short circuit

Overhead transmission lines  
Phase resistance, mOhm,

$$r_{AL} = k_{\nu} k_{tw} k_{se} \rho_{\nu, norm} l / s, \quad (3.99)$$

where  $k_{\nu}$  - is coefficient considering resistance value increase with temperature rise

$$k_{\nu} = 1 + 0,004(\nu - 20 \text{ }^{\circ}\text{C}); \quad (3.100)$$

$k_{tw}$  is the coefficient considering the conductor resistance value increase as a result of wires twisting (for multiwire strands  $k_{tw} = 1.2$ ; for single-wire strands  $k_{tw} = 1.0$ );  $k_{se}$  - is coefficient considering resistance variation under skin effect (for copper and aluminum conductors  $k_{se} = 1$ );  $\rho_{\nu, norm}$  - is cable specific resistance at  $\nu = +20^{\circ}\text{C}$ ;  $l, s$  - are cable length and cross section area.

Design (allowable) temperature for conductors with rubber and plastic insulation is assumed to be  $\nu = +65^{\circ}\text{C}$ .

Bus ducts. Resistance of a bus duct phase at temperature  $\nu$  is, mOhm,

$$r_{bus} = \rho_{\nu, norm} \cdot \frac{T + \nu}{T + \nu_{norm}} k_{ad} \cdot 10^3 l / s, \quad (3.101)$$

where  $\nu_{norm}$  - is temperature for which specific resistance  $\rho_{\nu, norm}$  is given;  $l$  - is bus length;  $s$  - is bus the phase cross section area;  $T$  - constant dependent on conductor material (for hard-drawn copper  $T = 242^{\circ}\text{C}$ ; for annealed copper  $T = 234^{\circ}\text{C}$ ; for aluminum  $T = 236^{\circ}\text{C}$ );  $k_{ad}$ ,  $k_{\nu}$ ,  $k_{se}$ ,  $k_N$  - are coefficients of additional losses caused by bus temperature increase (3.100), skin effect; of proximity effect. The coefficient values  $k_{se}$ ,  $k_N$  for copper and aluminum buses depend on size, position and number of buses.

Coefficient value  $k_{ad}$  for aluminum buses with cross section  $100 \times 10 \text{ mm}^2$  according to the number of buses in a phase :

$n = 1$	1.18;
$n = 2$	1.25;
$n = 3$	1.6;
$n = 4$	1.72.

When a bus duct is laid in the gallery or tunnel the coefficient of additional losses increase in 25 % comparing with the conditions of duct in the open air.

For design (allowed) temperature of bus duct heating in continuous operation it is assumed:  $\nu = 70^{\circ}\text{C}$ .

### 3.8. Computer-aided calculations of short circuit current

Nowadays computers are widely used for analysis of operation conditions and designing of power supply systems for major enterprises, infrastructure of cities and agrarian regions. It is stipulated by necessity of more complete transient description according to main parameters and both power supply and power consumption characteristics. The calculation of power supply system operation becomes more complex as there are more factors considered and they are to be repeated for many times as primary data vary.

Complex circuits of enterprise electricity network, deep lead-in usage with voltage more than 110 kV, network heterogeneity, significant increase of power system sources power, combination of different kinds of sources feeding short circuit in the power supply system, complex structure, availability of sharp load changes and a number of other factors predetermine the necessity of rising short circuit current calculation accuracy when calculation is automated.

Determination of short circuit current in systems like that is complicated problem, its solution can be aided by application of digital and analog calculating machines. The use of computers allows avoiding many assumptions designing equivalent circuit of electric power network. It allows to present load and generators in design circuit more accurately, to consider components of the load mode. Great advantage of computers use for electromagnetic transient calculation is the possibility to “monitor” in large scale system elements parameters and consideration not only their kind, but also specific peculiarities. As a result calculation accuracy increases, the authenticity of the results obtained is granted at the given primary parameters for many variants of power supply system circuit designs and short circuit kinds.

The accuracy of computer-aided calculations depends on the precision of the approach used for a program and authenticity of primary data. The main requirement to the program of calculation of short circuit currents in complex electrical system is that multivariant calculations at different points of the electric system with possibility of changes (switching from one operation mode to another, branch connection and disconnection, emergency and operational commutation, etc.) should not result on calculation time duration increase. The program should be universal, that would allow transient calculation for any practically possible circuit. There are too common requirements to primary data preparation, simplicity of obtained results processing, compactness and simplicity of calculation algorithm. Characteristics of the program for short circuit current calculation are determined by the chosen method of calculation and the way of method implementation. Calculation of major circuit parameters should be performed by standard programs.

Mathematical description of complex power supply system with all assumptions can be reduced to generation of linear algebra equations set. Normally, assumption means the neglect of transformers and chokes saturation, simulation of loads by constant impedances, representation of synchronous generators by the sources with emf constant in magnitude and correspondent power impedance. Electric network can be described with mesh current equations, nodal voltage equations, or their combination.

The method of nodal voltage is mostly used for emergency operation calculation. Mesh current method is used more rarely, as it is hard to implement. But it is more convenient when considering mutual inductance of transmission lines in zero sequence system

In networks design and evaluations of short circuit currents, developed by the institute “Energoset’proekt” and “The Institute of Electrodynamics of the National Academy of Sciences of Ukraine” are widely used the programs performing calculations considering active network components impedance and given divergence of emf in magnitude and phase at faults of any kind in complex networks. Gauss method and Z-transform are used to solve linear algebraic equations in these programs.

To make calculations of three-phase short circuit current with the use of computers the iteration method is offered. It is based on presentation of electric network state at short circuit by direct form of nodal voltage equation set. Coefficient of speedup is used to accelerate convergence of iteration process of the nodal voltages computation. Iteration method of short circuit current calculation does not require much memory of computer. Therefore it should be used for calculation of electrical network having a great number of nodes and branches. It is more efficient to use the direct method of short circuit current calculation that involves matrix of circuit center impedances. It has the following advantages before iteration method: absence difficulties by computation processes convergence; calculation time reduction; high accuracy of calculations. In this case electric state of the network under short circuit is presented by inversed form of node voltage equation set. There is an algorithm of calculation of short circuit current in networks, developed on the base of this method.

Primary data for three-phase short circuit calculation are the equivalent circuit of the network with branch impedances reduced to base voltage step. The network is represented by complex branch impedance. In equivalent circuit the generators are presented by active branches with electromotive forces that are connected after transient source impedance. At zero instant of transient



the generator emf does not vary its value. Circuit nodes are denoted by members  $1, 2, \dots, n$ , and the “grounded” node with voltage  $v=0$  is denoted by number  $n+1$ .

Primary information about electrical network comprises the following data: for each branch should be given a pair of nodes  $i$  and  $j$ , that are the branch ends, real and imaginary parts of complex impedance ( $Z'_{ij}$  and  $Z''_{ij}$ ); for each generator connection - the node number  $i$ , transient reactance  $x_i$ , real and imaginary parts of complex generator emf ( $E'_i$  и  $E''_i$ ); circuit nodes numbers  $k$ , where short circuits are considered.

As the first stage primary equivalent circuit should be transformed substituting the given active branches with reactances  $x_i$  and generator electromotive forces  $E'_i$  and  $E''_i$ , by passive branches with the same impedances and equivalent current sources. Equivalent source current is defined by expression

$$\dot{I}_i = \frac{E'_i + jE''_i}{jx_i}. \quad (3.102)$$

Normally node currents of transformed equivalent circuit sources are oriented in the same coordinate system where transient generator electromotive forces for working steady-state operation were set before.

Electrical state of the network before short circuit occurrence can be described by equation

$$\sum_{j=1}^n \underline{Z}_{ij} \dot{I}_j = \dot{U}_i^{(n)}, \quad (3.103)$$

where  $\underline{Z}_{ij}$  - is the component of network node impedance matrix;  $\dot{I}_j$  - is the node current (for the nodes to which generators are connected, it is equal to current of the equivalent source, for other nodes it is equal to zero);  $\dot{U}_i^{(n)}$  is the voltage of the node in normal network mode.

When three-phase short circuit occurs in  $k$ -th node, voltage in this node  $U_k = 0$ . In the  $k$ -th node additional current appears that is directed from the node and equal to short circuit current  $\dot{I}_s$ . As voltage in the  $k$ -th node is known, the equation correspondent to this node allows determination of short circuit current at the point of fault. Electric state of the network at three-phase short circuit in the  $k$ -th node can be described by linear algebraic equation set :

$$\left. \begin{aligned} \sum_{j=1}^n \underline{Z}_{kj} \dot{I}_j - \underline{Z}_{kk} \dot{I}_s &= 0; \\ \sum_{j=1}^n \underline{Z}_{ij} \dot{I}_j - \underline{Z}_{ik} \dot{I}_s &= \dot{U}_i^{(n)}, \end{aligned} \right\} \quad (3.104)$$

where  $i = 1, 2, \dots, n; i \neq k$ .

From equation set (3.103) short circuit current and residual voltage in the network nodes can be determined

$$\dot{I}_s^{(3)} = \sum_{j=1}^n \underline{Z}_{kj} \dot{I}_j / \underline{Z}_{kk} = \dot{U}_i^{(n)} / \underline{Z}_{kk}; \quad (3.105)$$

$$\dot{U}_i = \sum_{j=1}^n \underline{Z}_{ij} \dot{I}_j - \dot{I}_k \underline{Z}_{ik} = \dot{U}_i^{(H)} - \dot{I}_k \underline{Z}_{ik}, \quad (3.106)$$

where  $i = 1, 2, \dots, n; i \neq k$ .

With the known residual voltage in circuit nodes, the short circuit operation parameters are determined:

current in the branches of the transformed network circuit

$$\dot{I}_{ij} = (\dot{U}_i - \dot{U}_j) / \underline{Z}_{ij}; \quad (3.107)$$

current produced by each generator in the primary network circuit

$$\dot{I}_{G,i} = (\dot{E}_i - \dot{U}_i) / \underline{Z}_{ij}; \quad (3.108)$$

input impedances of the primary network circuit

$$\underline{Z}_{inp,i} = \dot{E}_i / \dot{I}_{G,i}. \quad (3.109)$$

Dividing complex value into real and imaginary parts

$$\begin{aligned} \dot{E}_i &= E_i' + jE_i''; \dot{U}_i = U_i' + jU_i''; \\ \dot{I}_i &= I_i' + jI_i''; \underline{Z}_{ij} = Z_{ij}' + jZ_{ij}'' \end{aligned}$$

and substituting them into expressions (3.101), (3.102) and (3.104)-(3.108), we get expressions convenient for programming and computer calculation.

Computers are widely used for solution of wide range of research and calculation problems concerning electromagnetic parameters of emergency operation in power systems and major power supply systems.

These problems are:

determination of resistance and capacitive susceptance of transmission line with voltage 110-750 kV influence on short circuit power in;

calculation of relay protection settings (for distance protection, ground short circuit protection, line-to-line short circuit current protection);

calculation of single-phase currents of short circuit in the networks with voltage 500-750 kV considering additional factors (before fault conditions, power electric mains capacitive susceptance, etc.);

calculation of emergency open-phase operation in complex electrical network;

research and calculation of short circuit current, produced by a group of induction and synchronous motors (determination of short circuit periodical current component initial value, periodical and aperiodical component dying out rate);

analyzing and obtaining design curves for power AC generators.

The set of calculating programs is provided with common information basis that comprises electric network circuit. The set is based on carrying dialog with user. They provide execution as operational, so and scheduled calculations. In general it predetermines development and implementation of automated layout of industrial enterprises power supply systems.

The examples of elaborated algorithms and software for short circuit current in power supply system calculations are given in [33]. The development needs use of mathematical simulation of all power supply system elements and couplings between them. It is a time-consuming problem.

The special features of electric power supply networks designed for power distribution are: presence of load nodes, which can be fed by a group of different motors in short circuit mode; importance of short circuit current in electric power distribution node calculation.

Equivalent circuit of the typical complex load node of industrial enterprise is given at Fig. 3.21. Primary electric supply system is easily reduced to the given form analytically or using calculated models. Calculation of short circuit current values in this equivalent circuit for multiple repetitions can be made by computers. As initial data for these computations, the statistical data about complex load node state and parameters of complex load node of different kind or parameters of equivalent models of actual electric power supply system reduced to the node could be considered.

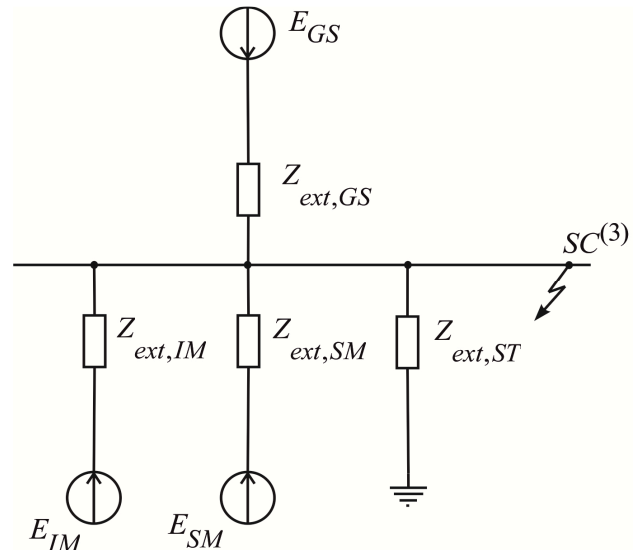


Fig. 3.21. Equivalent circuit of the node with complex load in the electric power supply system with voltage over 1 kV

### 3.9. Errors of assessment of short circuit current values

In most cases high accuracy of short-circuit currents assessment cannot be obtained due to incomplete and incorrect output information as well as due to inaccuracy of calculation methods. Meanwhile practice demonstrates application of complex and cumbersome calculation methods to improve accuracy of calculations that not always brings the desirable effect and is necessary.

Consider in more details a problem of possible accuracy of short-circuit currents calculation.

Accuracy of short-circuit currents assessment depends on the errors at which rated parameters of electric equipment and electric networks are determined as well as incompleteness or uncertainty data of the equipment and its operating conditions. That is important for nonlinear loads (for example, valve inverters, electro-heating installations, powerful welding facilities etc.) which parameters are functions of load and other characteristics of operating conditions. Errors in rated data of certain types of electric equipment and components of electric networks are given below.

Thus, difference in electric machines impedance of the rated values is within  $\pm 5\%$ ; transformer impedance voltage values are in the same range is analogous. Deviation of reactors rated inductance is of the order of  $\pm 10\%$ , and a capacitor battery capacitance deviation is in the range of  $(-5 \div +10)\%$ .

Errors of impedance equivalent at short-circuit currents calculation without taking into account the power system cannot in principle exceed the largest of the errors of the total impedance. Equivalenting comprises mainly adding series and parallel impedances.

Usually, such an error is not more than  $5\%$ . However, if nonlinear devices are available at the network the error can be much greater. The same is true for impedances of positive and negative sequences which are used in calculation short-circuit currents in the case of the voltage unbalance.

Negative-sequence impedance of electric power system is assumed equal the short circuit impedance at the considered network node and is in the range of  $3 \div 13\%$ .

Now methods of measurement of short circuit currents at the initial instant having the error not more than  $3\%$  ( and some greater at further time instants) are developed. A large number of calculations and relevant experiments have been made permit to come to conclusion that short circuit currents are determined by calculations with the accuracy of  $\pm 7\%$ , and with use of the simplified method - till  $\pm 12\%$ .

**Example**

It is required to calculate initial value of current periodic component of three-phase metal short circuit at point K of the circuit shown in Fig. 3.22.

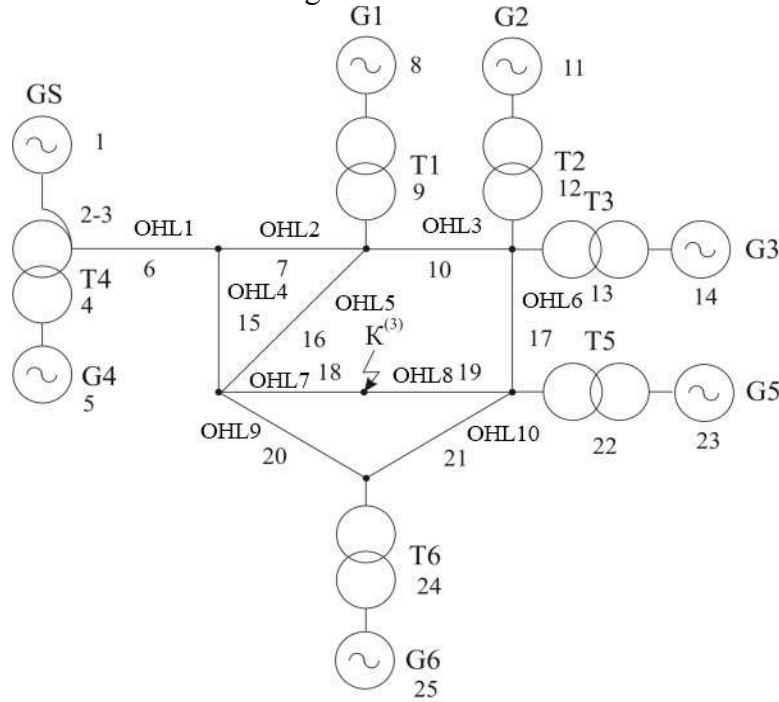


Fig. 3.22. Circuit of the network

*Initial data:*

GS system:  $S_{SC} = 2,000 \text{ MVA}$ ;

Autotransformer and transformers:

T1:  $S_r = 63 \text{ MVA}$ ,  $u_{imp} = 10.5\%$ ; T2:  $S_r = 80 \text{ MVA}$ ,  $u_{imp} = 10.5\%$ ; T3:  $S_r = 80 \text{ MVA}$ ,  $u_{imp} = 10.5\%$ ; T4:  $S_r = 125 \text{ MVA}$ ,  $u_{imp \text{ HV-MV}} = 11\%$ ,  $u_{imp \text{ HV-LV}} = 45\%$ ,  $u_{imp \text{ MV-LV}} = 28\%$ ; T5:  $S_r = 63 \text{ MVA}$ ,  $u_{imp} = 10.5\%$ ; T6:  $S_r = 40 \text{ MVA}$ ,  $u_{imp} = 10.5\%$ .

Synchronous generators:

G1: $S_r = 37.5 \text{ MVA}$ ,	$x''_{*d} = 0.15$ ,	$E''_* = 1.08$ ;
G2: $S_r = 40 \text{ MVA}$ ,	$x''_{*d} = 0.15$ ,	$E''_* = 1.08$ ;
G3: $S_r = 78 \text{ MVA}$ ,	$x''_{*d} = 0.18$ ,	$E''_* = 1.08$ ;
G4: $S_r = 78 \text{ MVA}$ ,	$x''_{*d} = 0.18$ ,	$E''_* = 1.08$ ;
G5: $S_r = 40 \text{ MVA}$ ,	$x''_{*d} = 0.15$ ,	$E''_* = 1.08$ ;
G6: $S_r = 25 \text{ MVA}$ ,	$x''_{*d} = 0.14$ ,	$E''_* = 1.08$ .

Overhead lines:

OHL1:  $l = 80 \text{ km}$ ; OHL2:  $l = 50 \text{ km}$ ; OHL3:  $l = 40 \text{ km}$ ;  
 OHL4:  $l = 100 \text{ km}$ ; OHL5:  $l = 120 \text{ km}$ ; OHL6:  $l = 100 \text{ km}$ ;  
 OHL7:  $l = 50 \text{ km}$ ; OHL8:  $l = 50 \text{ km}$ ; OHL9:  $l = 80 \text{ km}$ ;  
 OHL10:  $l = 70 \text{ km}$ .

For all the lines  $x_0 = 0.4 \text{ Ohm/km}$ , and specific resistance is not more than 30 % of  $x_0$ .

Rated voltage at point K of the network is 110 kV.

*Solution*

The node method may be applied in calculation short circuit operating condition of complex electric networks. In this case, equations set of node potentials is constituted. In matrix form its solution is represented as

$$U = -(AYA^t)^{-1}A(J + YE) \quad (3.110)$$

where  $U$  is the column matrix of node potentials,  $A$  is the incidence matrix,  $Y$  is the diagonal matrix of the network branch admittance,  $J$  is the column matrix of source currents,  $E$  is the column matrix of source voltages..

The incidence matrix is a rectangular matrix which lines correspond to nodes without one (reference) node; its columns correspond to arcs of the network directed graph. Components of the matrix are equal to: zero if the ark is not connected with the node; unit if the ark is directed from the node; and negative unit if it is directed to the node.

The diagonal matrix elements are admittances of corresponding network branches; non-diagonal elements equal zero. Equivalent circuits used in calculations of short circuit currents, as a rule, are not present, and matrix  $J$  is a zero matrix. Elements of matrix  $E$  are source voltages of corresponding network branches or equal zero if the source voltage is not available at the branch.

The equivalent circuit is shown in Fig. 3.23.

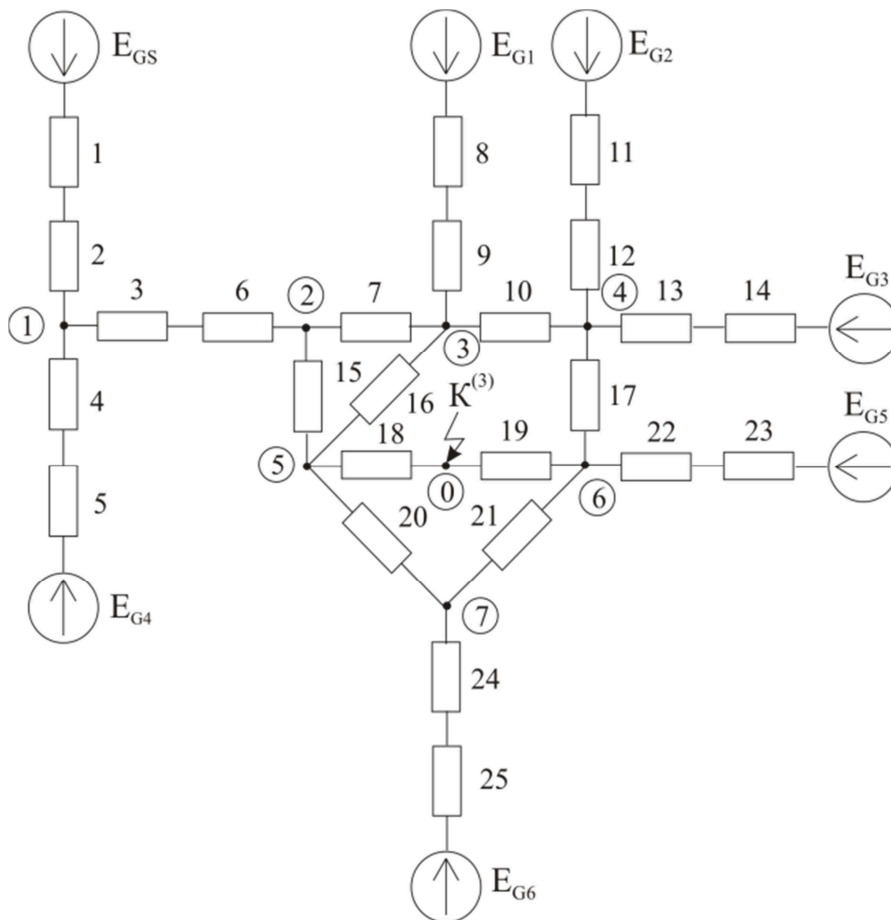


Fig. 3.23. Equivalent circuit

Perform the calculations using approximate method. Basic conditions are:

$$S_b = 100 \text{ MVA}, U_{b1} = 115 \text{ kV}, I_{b1} = \frac{S_b}{\sqrt{3}U_{b1}} = \frac{100}{\sqrt{3} \cdot 115} = 0.502 \text{ kA}.$$

As for all network elements resistance is essentially lower to compare with reactance, the resistance is neglected in calculations.

Determine reactance of the equivalent circuit.

*Reactance of the system*

$$x_{*1} = \frac{S_b}{S_{SC}} = \frac{100}{2000} = 0.05.$$

Reactance of the transformers:

$$x_{*9} = x_{*22} = \frac{u_{imp} S_b}{100 S_r} = 10.5 \cdot \frac{100}{100 \cdot 63} = 0.167,$$

$$x_{*12} = x_{*13} = \frac{u_{imp} S_b}{100 S_r} = 10.5 \cdot \frac{100}{100 \cdot 80} = 0.131,$$

$$x_{*24} = \frac{u_{imp} S_b}{100 S_r} = 10.5 \cdot \frac{100}{100 \cdot 40} = 0.263.$$

Reactance of the autotransformers:

$$x_{*2} = 0.005(u_{imp HV-MV} + u_{imp HV-LV} - u_{imp MV-LV}) S_b / S_r = 0.005(11 + 45 - 28) 100 / 125 = 0.112,$$

$$x_{*3} = 0.005(u_{imp HV-MV} + u_{imp MV-LV} - u_{imp HV-LV}) S_b / S_r = 0.005(11 + 28 - 45) 100 / 125 = -0.024,$$

$$x_{*4} = 0.005(u_{imp HV-LV} + u_{imp MV-LV} - u_{imp HV-MV}) S_b / S_r = 0.005(45 + 28 - 11) 100 / 125 = 0.248$$

Reactance of the synchronous generators:

$$x_{*8} = x''_{*d} S_b / S_r = 0.15 \cdot 100 / 37.5 = 0.4,$$

$$x_{*11} = x_{*23} = x''_{*d} S_b / S_r = 0.15 \cdot 100 / 40 = 0.375,$$

$$x_{*14} = x_{*5} = x''_{*d} S_b / S_r = 0.18 \cdot 100 / 78 = 0.231,$$

$$x_{*25} = x''_{*d} S_b / S_r = 0.14 \cdot 100 / 25 = 0.56.$$

Reactance of the overhead lines:

$$x_{*6} = x_{*20} = x_0 L S_b / U_b^2 = 0.4 \cdot 80 \cdot 100 / 115^2 = 0.242,$$

$$x_{*7} = x_{*18} = x_{*19} = x_0 L S_b / U_b^2 = 0.4 \cdot 50 \cdot 100 / 115^2 = 0.151,$$

$$x_{*10} = x_0 L S_b / U_b^2 = 0.4 \cdot 40 \cdot 100 / 115^2 = 0.121,$$

$$x_{*15} = x_{*17} = x_0 L S_b / U_b^2 = 0.4 \cdot 100 \cdot 100 / 115^2 = 0.302,$$

$$x_{*16} = x_0 L S_b / U_b^2 = 0.4 \cdot 120 \cdot 100 / 115^2 = 0.363,$$

$$x_{*21} = x_0 L S_b / U_b^2 = 0.4 \cdot 70 \cdot 100 / 115^2 = 0.212.$$

The system source voltages are:  $E_{*GS}=1$ ,  $E_{*G1}=E_{*G2}=E_{*G3}=E_{*G4}=E_{*G5}=E_{*G6}=1.08$ .

Apply the node method to calculate operating condition of the network.

Number the circuit nodes, assigning zero number to the reference node (Fig. 3.23).

Draw up the network graph (Fig. 3.24). Assume that arcs with source voltages are directed from the reference node to a node with non-zero potential. For other arcs directions are taken arbitrarily.

Reactance of the circuit branches:

$$\begin{aligned}
 X_{*1} &= x_{*3} + x_{*6} = -0.024 + 0.242 = 0.218, \\
 X_{*2} &= x_{*7} = 0.0.151, \\
 X_{*3} &= x_{*10} = 0.121, \\
 X_{*4} &= x_{*1} + x_{*2} = 0.05 + 0.112 = 0.0.162, \\
 X_{*5} &= x_{*4} + x_{*5} = 0.248 + 0.231 = 0.0.479, \\
 X_{*6} &= x_{*15} = 0.302
 \end{aligned}$$

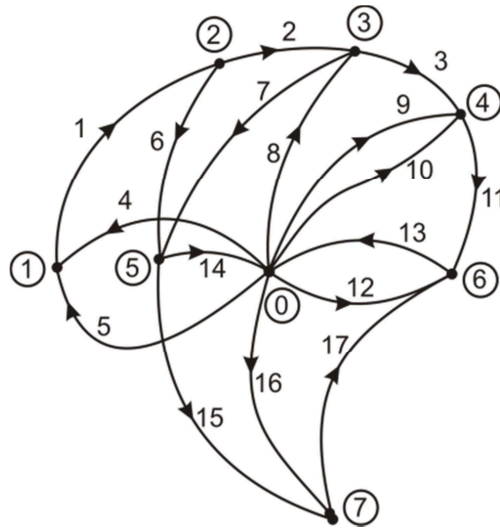


Fig.3.24. The network graph

The diagonal matrix of the graph arcs reactance is

$$\begin{aligned}
 X &= \text{diag}(X_{*1} X_{*2} X_{*3} X_{*4} X_{*5} X_{*6} X_{*7} X_{*8} X_{*9} X_{*10} X_{*11} X_{*12} \\
 &X_{*13} X_{*14} X_{*15} X_{*16} X_{*17}) = \text{diag}(0.218 \ 0.151 \ 0.121 \ 0.162 \\
 &0.479 \ 0.302 \ 0.363 \ 0.567 \ 0.506 \ 0.362 \ 0.302 \ 0.542 \ 0.151 \\
 &0.151 \ 0.242 \ 0.822 \ 0.212).
 \end{aligned}$$

The diagonal matrix of the graph arcs susceptance is

$$B = X^{-1} = \text{diag}(4,59 \ 6,61 \ 8,27 \ 6,17 \ 2,09 \ 3,31 \ 2,76 \ 1,77 \ 1,98 \ 2,76 \ 3,31 \ 1,85 \ 6,61 \ 6,61 \ 4,13 \ 1,22 \ 4,72).$$

The incidence matrix for the network graph represented in Fig 5.30 is:

$$A = \begin{vmatrix}
 1 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 -1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & -1 & 1 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 1 & 0 & 0 & 0 & -1 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 1
 \end{vmatrix}$$

The matrix of arcs source voltages is

$$\begin{aligned}
 E^t &= \parallel 0 \ 0 \ 0 \ E_{*GS} \ E_{*G4} \ 0 \ 0 \ E_{*G1} \ E_{*G2} \ E_{*G3} \ 0 \ E_{*G5} \ 0 \ 0 \ 0 \ E_{*G6} \ 0 \parallel = \\
 &= \parallel 0 \ 0 \ 0 \ 1 \ 1.08 \ 0 \ 0 \ 1.08 \ 1.08 \ 1.08 \ 0 \ 1.08 \ 0 \ 0 \ 0 \ 1.08 \ 0 \parallel
 \end{aligned}$$

Taking into account that current sources are not available in the circuit, matrix of node voltages can be determined as follows:

$$U = -(ABA^t)^{-1}A(BE) = \begin{pmatrix} 0.903 \\ 0.693 \\ 0.709 \\ 0.756 \\ 0.369 \\ 0.408 \\ 0.473 \end{pmatrix}.$$

Accordingly, voltages in nodes 5 and 6 are  $U_{*5} = 0.369$  and  $U_{*6} = 0.408$ .

Currents which flow at short circuit through branches 14 (overhead line OHL7) and 13 (overhead line OHL8) can be determined as follows

$$I_{14} = \frac{U_{*5} - U_{*0}}{X_{*14}} I_{b1} = \frac{0.369 - 0}{0.151} 0.502 = 1.22 \text{ kA},$$

$$I_{13} = \frac{U_{*6} - U_{*0}}{X_{*13}} I_{b1} = \frac{0.408 - 0}{0.151} 0.502 = 1.35 \text{ kA}.$$

where  $U_{*0}$  is nodal voltage of the reference node (the point of short circuit).

Initial value of periodical component of the short circuit current equals

$$I_{p0} = I_{14} + I_{13} = 1.22 + 1.35 = 2.57 \text{ kA}.$$

### Test questions

1. How is initial value of short circuit periodical current component caused by unlimited power generator, motor or total load determined?
2. What is the design curves method based on? What is the area of its application?
3. What rated conditions of short circuit current determination are typical curves used for?
4. How is short circuit periodical current component at any instant determined by rated and typical curves?
5. Is it possible to unite into one equivalent source the branch of power electric system with the branches powered from unlimited power generators?
6. When is it possible to make short circuit current calculations by general variation?
7. When is it required to make short circuit current calculation by individual variation?
8. How is short circuit current calculation made when the point of short circuit is infed from synchronous and induction motors?
9. What are peculiarities of short circuit current calculation in electric network with voltage up to 1 kV?
10. How is transient resistance determined when short circuit occurs at different stages of electric energy distribution in the network with voltage up to 1 kV?

### Topics for essay

1. Sources feeding the short circuit point and determination of short circuit currents generated by them.
2. Comparison of short circuit current calculation results in the case of general and individual calculation for a specific circuit of power supply system.
3. Features of short circuit current calculation for electric power supply systems of industrial enterprises.
4. Algorithm and program design for short circuit current calculation in the network with voltage up to 1 kV for typical electric power supply system.
5. Electric supply system models application for calculation of its operation under short circuit.



## **CHAPTER 4: TRANSIENTS UNDER ASYMMETRY IN THREE-PHASE NETWORK**

- 4.1. General information
- 4.2. Generation of higher harmonics by synchronous alternator
- 4.3. Method of symmetrical components
- 4.4. Relationships between symmetrical components of current and voltage
- 4.5. Resistances of negative and zero sequences of circuit elements
- 4.6. Drawing up equivalent circuits for symmetrical components
- 4.7. Resulting electromotive forces and impedances of equivalent circuits for separate sequences

Test questions

Topics for essay

#### 4.1. General information

All the phases are in equal conditions under three-phase short circuit in power network. Periodic components vectors of the phase currents and voltages are equal as for maximum values and shifted relative to each other at angles of  $120^\circ$ , forming balanced systems. That is why under three-phase short circuit the equivalent circuit of one phase is used for calculations.

When asymmetrical faults (short circuit, break of a phase, asymmetrical phase load) arise, the modules of phase currents and voltages, and angles of their shift will have different values. In common case vectors of phase currents and voltages form asymmetrical unbalanced systems. In such a case calculations should be made with use of equivalent circuits for all three phases taking into account mutual induction between them. The needed number of equations increases considerably and complexity and amount of calculations rises even if electric power supply circuit is a simple one.

Below transients caused by single asymmetry are considered. That means that an asymmetrical fault occurs only in one point of network while other its part stays to be symmetrical one. Such a single asymmetry can be lateral, i.e. any kind of asymmetrical short circuits, and longitudinal, i.e. fault of one or two-phases or non-equal impedances of three-phase network phases load.

In the networks with synchronous machines, analysis of transients under asymmetrical faults is rather difficult due to pulsation of the rotor magnetic field that results in full range of higher harmonics. Transition from phase variables to variables on  $d, q$  coordinates does not liberate the equations describing these processes from periodic coefficients. Mostly only fundamental harmonics of the quantities, being determined are found. Under such restriction, it is possible to use the method of symmetrical components to analyze networks with synchronous machines. Calculations are performed for three one-phase mutually independent equivalent circuits for positive, negative, and zero sequences.

#### 4.2. Generation of higher harmonics by synchronous alternator

Fig. 4.1 demonstrates a model of a salient-pole synchronous generator without damper winding. Suppose that one phase is clamped to ground at the generator no-load operation. The neutral of stator winding is grounded. Short-circuit current flows through this phase. The current angular frequency equals to synchronous angular frequency  $\omega_s = \omega_r$ . Magnetic flux  $\Phi_a(\omega_s)$  produced by the short-circuit current (Fig. 4.1, b) pulsates with frequency  $\omega_s$  and remains its spatial position unchanged.

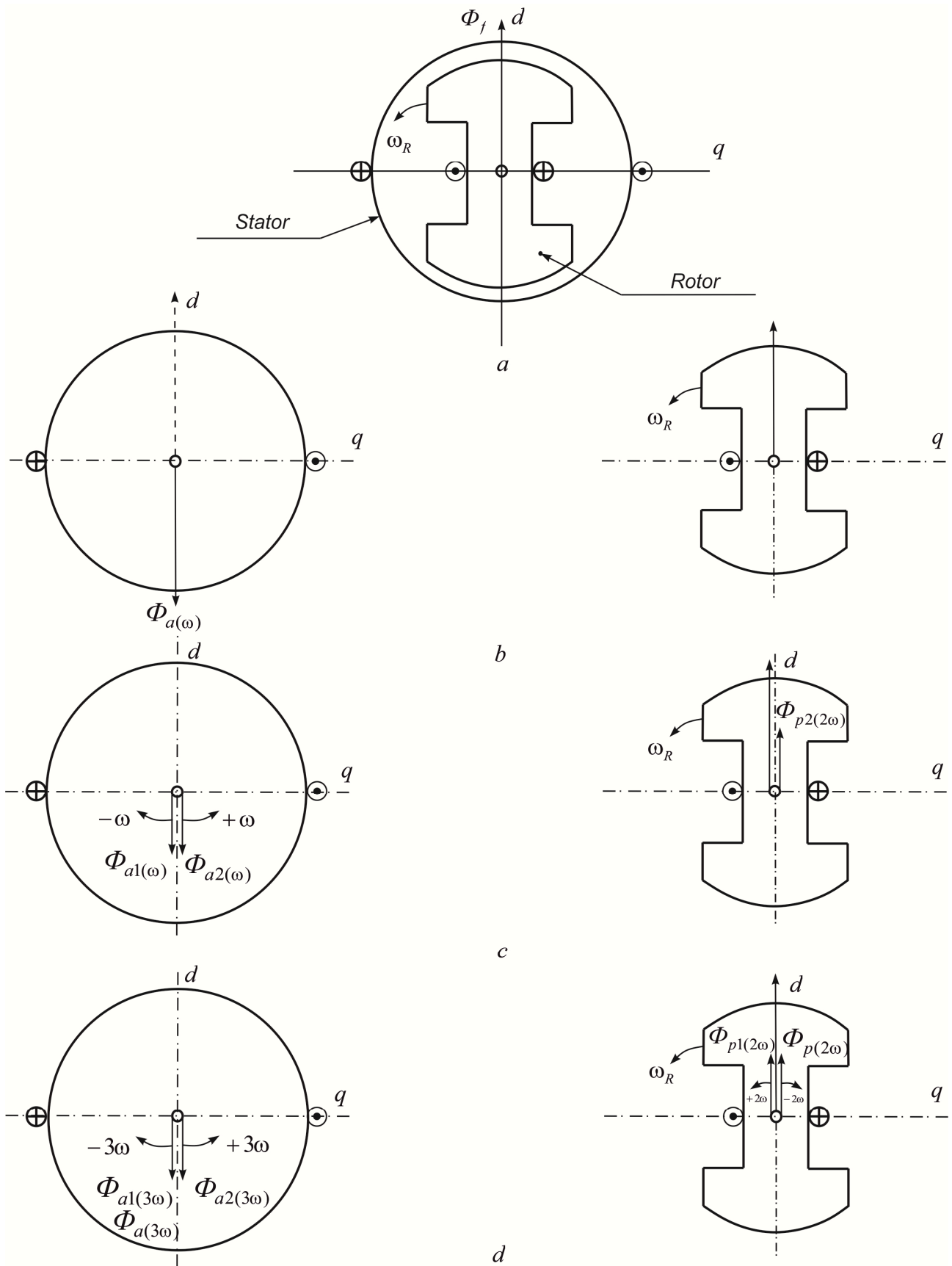


Fig. 4.1. Diagrams showing pulsating flux creation in the generator having salient pole rotor under one-phase short circuit:  
 a – generator design model; b, c, d – diagrams of sequent progress.

To analyze influence of this pulsating flux on rotor, resolve it into two components (Fig. 4.1, c), with synchronous angular frequency  $\pm\omega$  ("+" sign corresponds to counter-clockwise rotation, "-" to clockwise rotation). Flux component  $\Phi_{a1}(\omega_s)$  rotates according to the rotor and interacts with excitation winding flux  $\Phi_f$  as the usual armature reaction flux at normal operation. Flux component  $\Phi_{a2}(\omega_s)$  that rotates towards the rotor crosses its winding conductors with double angle frequency  $\omega_s + \omega_r = 2\omega_s$ . Electromotive force of doubled frequency is induced in the rotor winding. This EMF causes a current of the same frequency. It generates pulsating rotor magnetic flux  $\Phi_r$  which pulsation frequency is  $2\omega_s$  (Fig. 4.1, c). The flux rotates with rotor in space. This flux can also be shown as two flux components rotating in opposite directions with angular frequency of  $\pm 2\omega_s$  past the rotor (Fig. 4.1, d). Flux component  $\Phi_{r2}$ , which rotates opposite the rotor, has rotation frequency relative to the stator  $-2\omega_s + \omega_r = -\omega_s$ , that is tends to compensate flux of stator  $\Phi_{a1}(\omega_s)$ . Another component of rotor flux  $\Phi_{r1}$  rotates regarding to stator with frequency of  $\omega_r + 2\omega_s = +3\omega_s$  and forms electromotive force in fault phase of stator winding which causes triple frequency current. This current produces magnetic flux of stator  $\Phi_a(3\omega_s)$  with pulsating frequency of  $3\omega_s$  (Fig. 4.1, d), that can also be resolved into two flux components  $\Phi_{a1}(3\omega_s)$  and  $\Phi_{a2}(3\omega_s)$ , having opposite rotation directions relative to the stator and angular frequency of  $\pm 3\omega_s$ .

If such judgment is continued, it is made sure that each odd harmonic of one-phase short circuit current in stator winding causes next even harmonic in rotor winding and each even current harmonic in rotor winding in its turn produces next in succession odd harmonic in stator winding.

By analogy it can be determined that current of invariable direction and even harmonics of current in stator winding generate fundamental and odd harmonics of current in rotor winding and vice versa - odd harmonics of current in rotor winding stipulate even harmonics of current in stator winding.

Thus, full range of higher harmonics of current arises in stator winding of generator. If there is no capacitance in the stator winding circuit magnitudes of current harmonics decrease as their ordinal number increases. Theoretically this endless number of current harmonics arises because under asymmetrical short-circuit forms non-rotating, but pulsating magnetic field.

Electromagnetic influence of damper windings is the same as to that of excitation windings. Quadrature damper winding, and windings placed along direct axis of synchronous motor rotor are shifted in the space by 90 electric degrees, or by quarter of the period in time. As a result of their mutual interaction rotating circular field is formed which stays immovable and has the opposite direction relative to the causing it stator magnetic field. Thus, when rotor is symmetrical in quadrature and direct axes, pulsating magnetic field is not generated. But factually rotor of synchronous machine has not such a symmetry. So, under any kind of asymmetrical load higher harmonics of currents are generated in the windings which magnitude is the higher the more rotor asymmetry is. Owing to it, the presence of higher harmonics at water-wheel type generators is evident, and as for turbo-generators they are not practically shown.

### 4.3. Method of symmetrical components

Method of symmetrical components permits to present any asymmetrical system consisting of three phasors as three symmetric systems of phasors: positive, negative and zero sequences. For every of these sequences phasor system, phenomena in phases are similar and it helps to use *one* phase equivalent circuits for each sequence, and make calculations for *one* phase. Phase being under

specific conditions in comparison with other is chosen. It is called a *special* phase. This method key postulates are as follows:

Any of three phasors of symmetrical system can be represented by similar phasor of another phase using turning operator  $a$  which expressed by means of complex number

$$a = -\frac{1}{2} + j\frac{\sqrt{3}}{2} = e^{j120^\circ}$$

The following equalities are valid:

$$\left. \begin{aligned} a^2 &= -\frac{1}{2} - j\frac{\sqrt{3}}{2} = e^{-j120^\circ} = e^{j240^\circ}; \\ a^3 &= e^{j360^\circ} = 1; \\ a^4 &= a; \\ a^2 + a + 1 &= 0; \\ a^2 - a &= \sqrt{3} e^{-j90^\circ} = -j\sqrt{3}; \\ a - a^2 &= j\sqrt{3}; \\ 1 - a &= \sqrt{3} e^{-j30^\circ} = 1\frac{1}{2} - j\frac{\sqrt{3}}{2}; \\ 1 - a^2 &= \sqrt{3} e^{j30^\circ} = 1\frac{1}{2} + j\frac{\sqrt{3}}{2}. \end{aligned} \right\} \quad (4.1)$$

In symmetrical three-phase phasor system (Fig. 4. 2) each of the phase phasors can be presented as follows:

$$\left. \begin{aligned} \underline{N}_A &= a\underline{N}_B = a^2\underline{N}_C; \\ \underline{N}_B &= a\underline{N}_C = a^2\underline{N}_A; \\ \underline{N}_C &= a\underline{N}_A = a^2\underline{N}_B. \end{aligned} \right\} \quad (4.2)$$

Multiplication of a vector by the operator  $a$  means its turn through  $120^\circ$  in positive direction (counterclockwise).

A vector multiplication by  $a^2$  turns it by  $240^\circ$  in the same direction or by  $120^\circ$  in an opposite one.

Any asymmetrical system of three phasors can be divided into 3 symmetrical phasor systems: positive sequence, negative sequence and zero one.

Phasor system of positive sequence (having index "1") consists of 3 equal in magnitude phasors displaced to each other by  $120^\circ$  and having the same sequence as the original phasors set (Fig. 4. 13, a).

Phasor system of negative sequence (having index "2") also consist of 3 equal in magnitude phasors displaced to each other by  $120^\circ$  but having the sequence opposite to that of the original set (Fig. 4. 3, b).

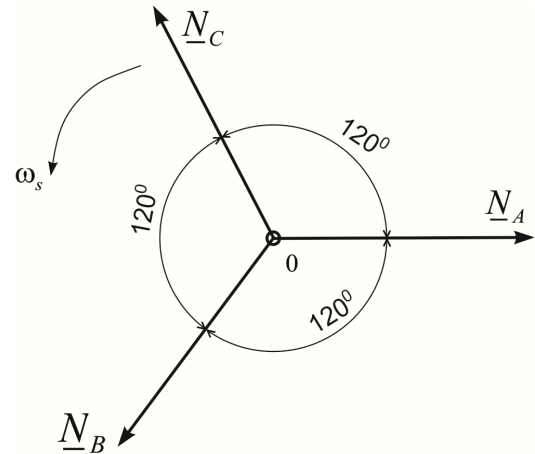


Fig. 4.2. Symmetric three-phase system of phase vectors of operating condition

As phasors sum of positive (negative) sequence system is zero, these phasor system are balanced:

$$\underline{N}_{A1} + \underline{N}_{B1} + \underline{N}_{C1} = \underline{N}_{A1} (1 + a^2 + a) = 0;$$

$$\underline{N}_{A2} + \underline{N}_{B2} + \underline{N}_{C2} = \underline{N}_{A2} (1 + a + a^2) = 0.$$

Phasor system of zero sequence consists of three phasors all equal in magnitude and in phase (Fig. 4. 3, c). Phasors have symbol “0”. Phasor system of zero sequence is symmetrical but not balanced:

$$\underline{N}_{A0} + \underline{N}_{B0} + \underline{N}_{C0} = 3\underline{N}_{A0} \neq 0.$$

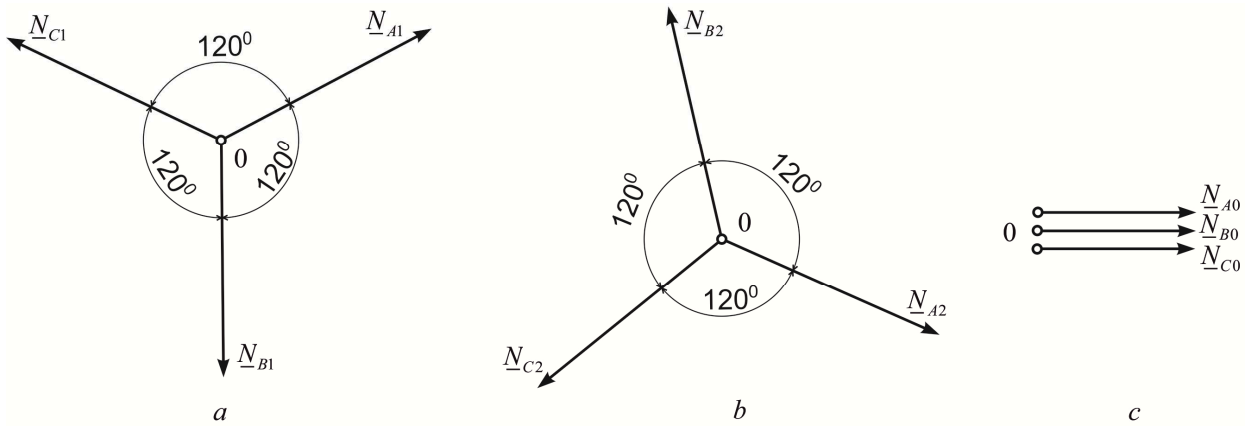


Fig. 4.3. Symmetrical sequences of asymmetrical vector set: a-positive; b-negative; c-zero.

Using components (phasors of positive, negative and zero sequences) it is possible to restore the initial asymmetrical phasor system:

$$\left. \begin{aligned} \underline{N}_A &= \underline{N}_{A1} + \underline{N}_{A2} + \underline{N}_{A0}; \\ \underline{N}_B &= \underline{N}_{B1} + \underline{N}_{B2} + \underline{N}_{B0}; \\ \underline{N}_C &= \underline{N}_{C1} + \underline{N}_{C2} + \underline{N}_{C0}. \end{aligned} \right\} \quad (4.3)$$

Table 4.1  
Coefficients for selection of symmetrical components

Equations of asymmetrical phasor system	Sequence		
	positive	negative	zero
$\underline{N}_A = \underline{N}_{A1} + \underline{N}_{A2} + \underline{N}_{A0}$	1	1	1
$\underline{N}_B = a^2 \underline{N}_{A1} + \underline{N}_{A2} + \underline{N}_{A0}$	1	$a^2$	1
$\underline{N}_C = a \underline{N}_{A1} + a^2 \underline{N}_{A2} + \underline{N}_{A0}$	$a^2$	$a$	1

If we recognize A as a special phase then taking into consideration (4.2) and Fig. 4. 3 the equation system (4.3) can be expressed by means of special phase vectors symmetrical components as follows:

$$\left. \begin{aligned} \underline{N}_A &= \underline{N}_{A1} + \underline{N}_{A2} + \underline{N}_{A0}; \\ \underline{N}_B &= a^2 \underline{N}_{A1} + \underline{N}_{A2} + \underline{N}_{A0}; \\ \underline{N}_C &= a \underline{N}_{A1} + a^2 \underline{N}_{A2} + \underline{N}_{A0}. \end{aligned} \right\} \quad (4.4)$$

An equation (4. 4) allows making selection of an asymmetric system phasor symmetric component. To do that it is necessary to add all the three equations having previously made equal the coefficients at the desired component (Table 4.1). For example, to select phasor of positive sequence, it is enough to multiply the equations (4.4) accordingly by coefficients  $1, a, a^2$ , and add then. Phasors components  $\underline{N}_{A1}, \underline{N}_{A2}, \underline{N}_{A0}$  will be expressed by means of phase vectors  $\underline{N}_A, \underline{N}_B, \underline{N}_C$  as follows:

$$\left. \begin{aligned} \underline{N}_{A1} &= \frac{1}{3}(\underline{N}_A + a\underline{N}_B + a^2\underline{N}_C); \\ \underline{N}_{A2} &= \frac{1}{3}(\underline{N}_A + a^2\underline{N}_B + a\underline{N}_C); \\ \underline{N}_{A0} &= \frac{1}{3}(\underline{N}_A + \underline{N}_B + \underline{N}_C). \end{aligned} \right\} \quad (4.5)$$

These are formulae of decomposition of asymmetrical phasor system  $\underline{N}_A, \underline{N}_B, \underline{N}_C$  in three symmetric components of special phase  $\underline{N}_{A1}, \underline{N}_{A2}, \underline{N}_{A0}$ .

Fig. 4.4, a graphically shows components  $\underline{N}_{A1}, \underline{N}_{A2}, \underline{N}_{A0}$  of the phasor system  $\underline{N}_A, \underline{N}_B, \underline{N}_C$  according to (4.5) and in Fig. 4. 4, b initial phase vector set  $\underline{N}_A, \underline{N}_B, \underline{N}_C$  are obtained again with use of the found components in accordance with (4. 3).

Equation d (4. 5) can be expressed in a matrix form

$$\vec{N} = S \cdot \vec{N}_{AS}; \quad \vec{N}_{AS} = S^{-1} \cdot \vec{N}, \quad (4.6)$$

$\vec{N} = |\underline{N}_A \ \underline{N}_B \ \underline{N}_C|^T$ ;  $\vec{N}_{AS} = |\underline{N}_{A1} \ \underline{N}_{A2} \ \underline{N}_{A0}|^T$ ;  $T$  – sign of vector transposition;  $S$  – matrix of symmetrical coordinates transformation in the phase coordinates

$$S = \begin{vmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{vmatrix}; \quad S^{-1} = \frac{1}{3} \cdot \begin{vmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{vmatrix}. \quad (4.7)$$

In abovementioned expressions phasor  $\underline{N}$  can mean phasor of current or voltage.

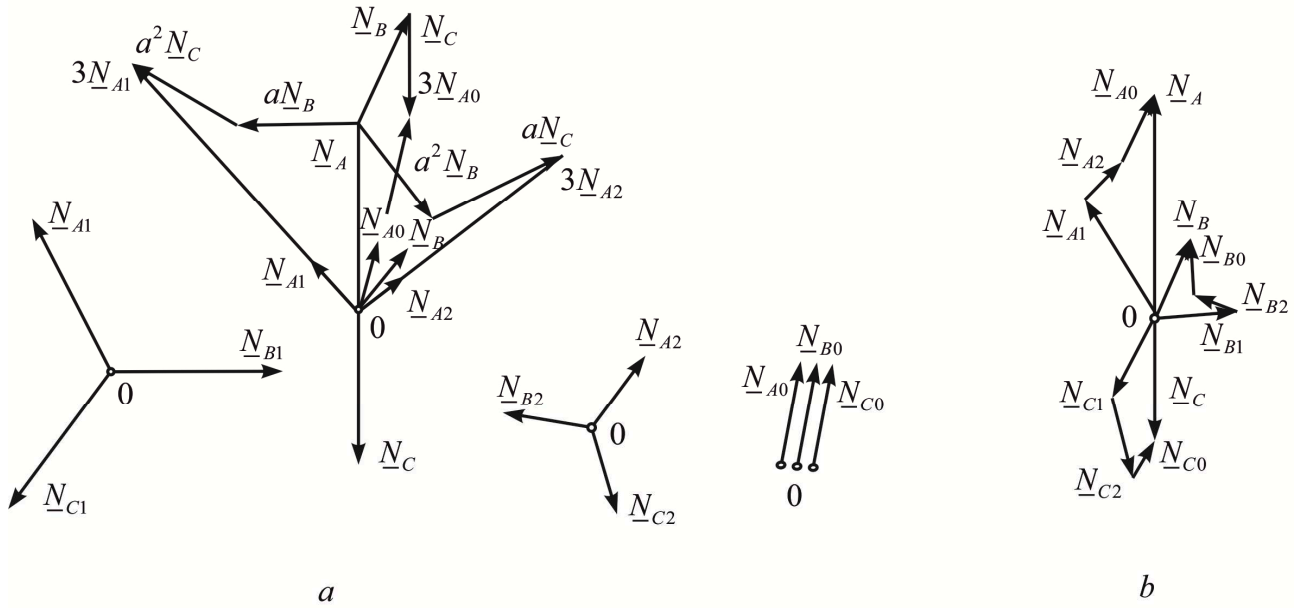


Fig. 4.4 Decomposition of asymmetric three-phase vector system: a- symmetrical components; b - phase vectors initial asymmetric system obtaining.

#### 4.4. Relationships between symmetrical components of current and voltage

Method of symmetrical components is used assuming that electric power supply system elements are symmetrical in different phases. If any element is symmetrical at all the phases and, for instance, for special phase A has impedances for symmetrical components of positive, negative and zero sequences  $\underline{Z}_1$ ,  $\underline{Z}_2$  and  $\underline{Z}_0$ , symmetrical components of voltage drop in this element are described by Ohm's law applied to each of the symmetrical components:

$$\left. \begin{aligned} \Delta \underline{U}_{A1} &= \underline{Z}_1 \underline{I}_{A1}; \\ \Delta \underline{U}_{A2} &= \underline{Z}_2 \underline{I}_{A2}; \\ \Delta \underline{U}_{A0} &= \underline{Z}_0 \underline{I}_{A0}. \end{aligned} \right\} \quad (4.8)$$

Some difficulties concerned the presence of synchronous machines in the system of electric power supply arise. As it is shown in the division 4.2, under asymmetrical short circuit, synchronous generator produces the full range of current higher harmonics. Current systems of positive and negative sequences turn out to be mutually interacted that makes additional conditions and demands using the method of symmetric components. Special equivalent chain circuit of synchronous machines are used to calculate higher harmonics of current. This factor complicates the calculations. Generally when practical calculations are performed the higher harmonics of current are neglected, and only fundamental harmonics of currents and voltages are taken into consideration, and inductive reactance  $x_2$  is used to characterize the synchronous machine negative sequence equivalent circuit. Assumption concerning absence of currents and voltages of different sequences interaction makes possible to use the method of symmetrical components for circuits with synchronous generators and calculate three single-phase mutually independent equivalent circuits composed for each of the sequences. Obviously error in calculations will be the least in the circuits with turbo-generators which practically have not higher harmonics of current. As for circuits with water-wheel generators, higher harmonics of current lead to error even if damper winding is present.

Current of positive, negative and zero sequences generate magnetic fluxes of the same sequences and the letters generate correspondent electromotive forces in the windings of stator. It is not expedient to take those electromotive forces into considerations while calculating as they are



proportional (when motors magnetic system saturation is neglected) to the currents of separate sequences which values can't be determined yet. That's why only those electromotive forces are taken into consideration which are either known or do not depend on the external conditions of stator: circuit initial values of the transient  $E'$  and subtransient  $E''$ , the synchronous  $E$  (under known field current  $i_f$ ) electromotive forces, design electromotive  $E_t$  for any arbitrary instant according to method of rectified characteristics. Those electromotive forces are of positive sequence in the case of symmetrical stator winding. As for electromotive forces stipulated by reaction of currents of positive, negative and zero sequences, they are taken into account as opposite in sign voltage drop on corresponding inductive reactance of synchronous machine.

Besides, it is assumed that devices of automatic excitation control mounted on synchronous machines, despite their type, respond only to voltage deviation of the positive sequence trying to keep the voltage equal to the rated value for each the machine.

Fig. 4.5 shows general view of electrical circuit where one-phase short circuit occurred in a  $SC^{(1)}$  point on a junction line. The current flows under affect of generators electromotive forces; asymmetrical voltage system (Fig. 4.5, b), being formed in the point o short circuit, can be resolved into three symmetrical systems - positive, negative and zero sequences (Fig. 4.5, c). Selecting symmetrical voltage components of special phase  $A$ , separate equivalent circuits for positive, negative and zero sequences are obtained (Fig. 4.5, d). According to the second Kirchhoff's law for each of these equivalent circuits, the following equality in general form can be stated:

$$\vec{U}_{s,A,S} = \vec{E}_{A,\Sigma,S} - \underline{Z}_{s,res} \vec{I}_{s,A,S}, \quad s = 1; 2; 0. \tag{4.9}$$

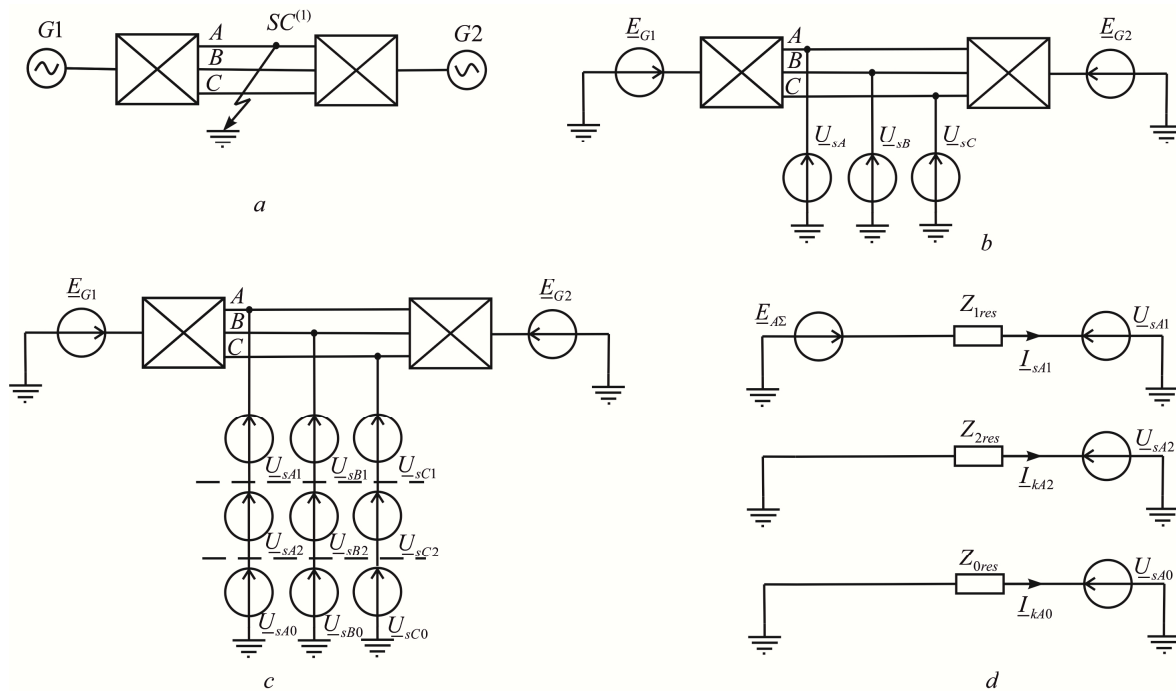


Fig. 4.5 Study single-phase short circuit in three-phase network using symmetrical components of operation phase parameters: a-circuit design; b-asymmetrical system of voltage at the point of short circuit; c-decomposition into three symmetrical component; d- equivalent circuits for positive, negative and zero sequences

Equation system (4.9) is given for special phase A. If resistance of components is neglected while determining short circuit current values in networks over 1kV, equation (4.9) in expanded form looks as follows:

$$\left. \begin{aligned} \underline{U}_{s,A1} &= \underline{E}_{A,\Sigma} - jx_{1,\text{res}}\underline{I}_{s,A1}; \\ \underline{U}_{s,A2} &= -jx_{2,\text{res}}\underline{I}_{s,A2}; \\ \underline{U}_{s,A0} &= -jx_{0,\text{res}}\underline{I}_{s,A0}. \end{aligned} \right\} \quad (4.10)$$

Under selected way to account electromotive forces caused by the current reaction of each of the sequences, the currents of negative and zero sequences formation can be linked to the negative, and zero voltage sequences that appear at the point of short circuit.

Under longitudinal asymmetry resulted from open-circuit and changes in phase resistance values (see chapter 6). The main equations for each the sequence are similar, but instead of  $\underline{U}_{s,A1}$ ,  $\underline{U}_{s,A2}$ ,  $\underline{U}_{s,A0}$  difference of phase voltage of correspondent sequences at the ends points of local asymmetry should be included, and  $\underline{Z}_{1\text{res}}$ ,  $\underline{Z}_{2\text{res}}$ ,  $\underline{Z}_{0\text{res}}$  and resistances have to be resulting resistances of correspondent sequences equivalent circuits as to the point of longitudinal asymmetry being under consideration.

#### 4.5. Resistances of negative and zero sequences of circuit elements

All resistances which are used to characterize components in ordinary symmetric mode are resistances of positive sequence. For an element which magnetically coupled circuits are stationary to each other, positive and negative sequences resistance are equal in value because mutual inductance between phase elements remains invariable at phase sequence change. Thus,  $r_2 = r_1$ ,  $x_2 = x_1$  for transformers, autotransformers, overhead lines, cable lines, and reactors.

Reactance of negative sequence of components with magnetically coupled circuits (synchronous generators; compensators, motors) depends on machine design (symmetry of the rotor). Negative sequence current produces magnetic flux which moves relative to rotor with double synchronous speed in the opposite direction. While moving that flux faces comes into coincidence in turn either with direct axis or quadrature one.

If the air gap is constant along the machine bore circumference, and rotor is symmetrical, the negative sequence magnetic flux of faces equal resistance in any direction as the positive sequence magnetic flux. For motors with non-salient pole rotor (turbo-generators) that have  $x'_d = x'_q$  and  $x''_d = x''_q$ , the negative sequence resistance does not differ much from the positive sequence resistance:

$$x_2 \cong x'_d \text{ or } x_2 \cong x''_d. \quad (4.11)$$

The first expression is valid for machines without damper windings, and the second one-for motors with damper windings. Similarly, for motors with salient pole rotor which are fed of a power supply of infinite power, the resistance of negative sequence counts equal to:

if third current harmonic is taken into consideration

$$x_2 \approx \frac{2x'_d x_q}{x'_d + x_q} \text{ or } x_2 \approx \frac{2x''_d x''_q}{x''_d + x''_q}; \quad (4.12)$$

if third current harmonic is neglected

$$x_2 \approx \frac{x'_d + x_q}{2} \text{ or } x_2 \approx \frac{x''_d + x''_q}{2}. \quad (4.13)$$

Generally magnetic field of negative sequence current generates the whole range of odd harmonics. Then machines resistances have to be determined on the expression:

$$x_2 \approx \sqrt{x'_d x'_q} \text{ or } x_2 \approx \sqrt{x''_d x''_q} \tag{4.14}$$

Usually, values of  $x''_d$  and  $x''_q$  are closer to each other than  $x'_d$  and  $x'_q$ , values. That's why difference between values reactances obtained by different expressions (4.11)-(4.14) for machines with complete damping is very small. As expressions (4.12)-(4.14) expressions are equivalent in most cases of practical computations it is expedient to assume reactance for synchronous machines on the simplest expression (4.13). When it is necessary to take into account higher harmonics the more exact expression (4.14) should be used.

As approximate equalities are assumed accordingly:

$$x_2 \approx 1,45x'_d \text{ or } x_2 \approx 1,45x''_d \tag{4.15}$$

or, making further simplifying, expressions (4.11) are used.

Current of zero sequence produce only magnetic leakage fluxes of stator winding which are less than those caused by currents of positive or negative sequence, and this decrease depends much on the type of winding. That's why value  $x_0$  for synchronous machines is changed greatly:

$$x_0 = (0,15...0,6)x'_d \text{ or } x_0 = (0,15...0,6)x'_q \tag{4.16}$$

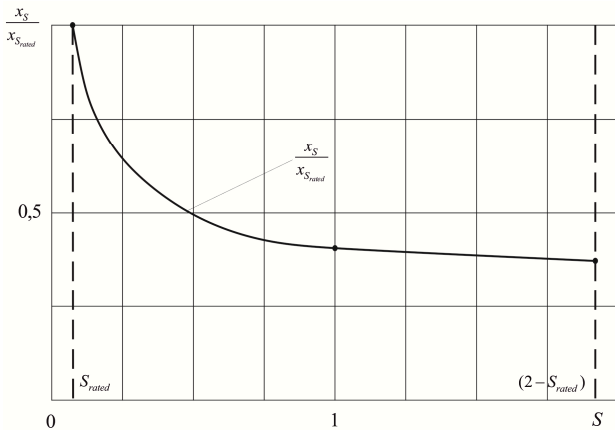


Fig. 4.6. Dependence of inductive reactance of induction motor on slip

Induction motor having slip  $s$  under ordinary mode as for magnetic flux of negative sequence has  $2-s$  slip. hence reactance of negative sequence  $x_2$  is its reactance under  $2-s$  slip. The slip influences the motor's reactance value (Fig. 4.6). When slip value increase the values of motors reactance decrease is quite insignificant. In this case motor negative sequence reactance is practically equal to short circuit reactance (if rotor is fixed) which value in per unit

$$x_{2-s} \approx x_{s=1} = x_s \tag{4.17}$$

is close to  $x_{*2} \approx 1/I_{*start}$ , where  $I_{*start}$  - is motor start-to-rated current ratio.

Zero sequence reactance of induction motor is determined by stator winding leakage field and depends greatly on the type and design of the latter. It is determined according to data of manufacturer or experimentally.

Negative sequence reactance of generalized load depends on features of electric power consumers and specific share of each of them in the load formation. Under known mix of consumers in the node, the reactance of negative sequence of combined load equipments has to be considered according to Table 1.3 data. For average ordinary industrial load one can suppose that its main part consists of induction motors which reactance of negative sequence is practically equal to that one at the initial instant of sudden operation breakdown, that is  $x_{*2} = x_{*1} = 0,35$ , considering it to be referenced to total power of load and average rated voltage of the stage it is connected.

Zero sequence reactance of generalized load is determined by reactance of the network zero sequence and step-down transformers connected to network of 110 kV and over and having grounded neutral. Main consumers, as a rule, have insulated neutrals, so paths for circulating the zero sequence currents are not available here. That's why we can consider that reactance of zero sequence of load is equal to  $x_0 \rightarrow +\infty$ . Zero sequence reactance of transformers is determined by their design, the windings connection, and type of neutral. Considering basic circuits of transformer windings connection we suppose that zero sequence voltage is applied to the winding  $I$ , depending on winding connection circuit can either produce or not produce zero sequence current in them. Zero sequence reactance of transformers on the side of winding connected in delta or in star with ungrounded neutral (Fig. 4.7,a) is infinitely large ( $x_0 \equiv \infty$ ) despite the circuit of other windings connection. If so, the ability for zero sequence current circulation is excluded.

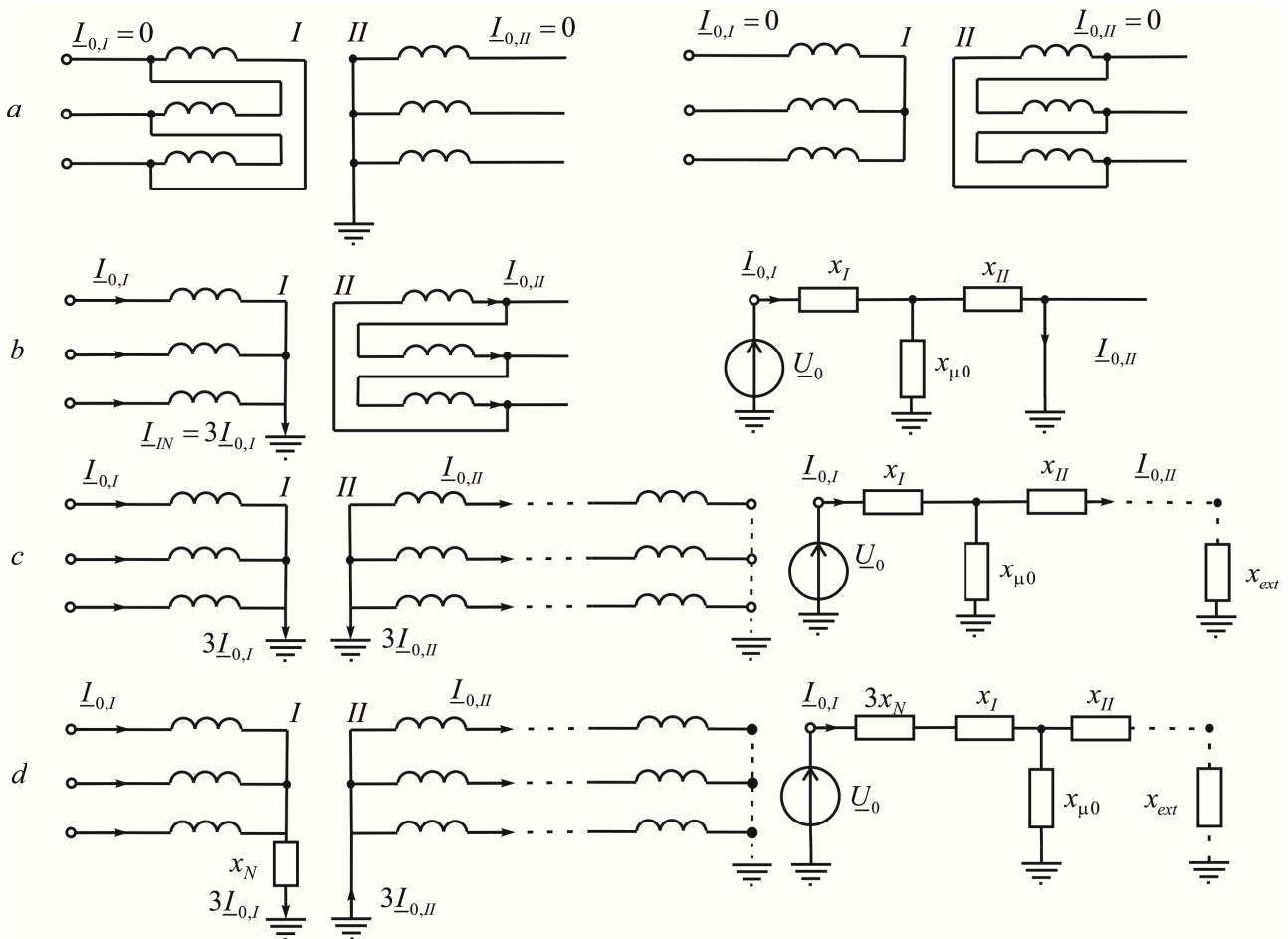


Fig. 4.7. Types of transformer windings connection and their equivalent circuits for zero sequence current

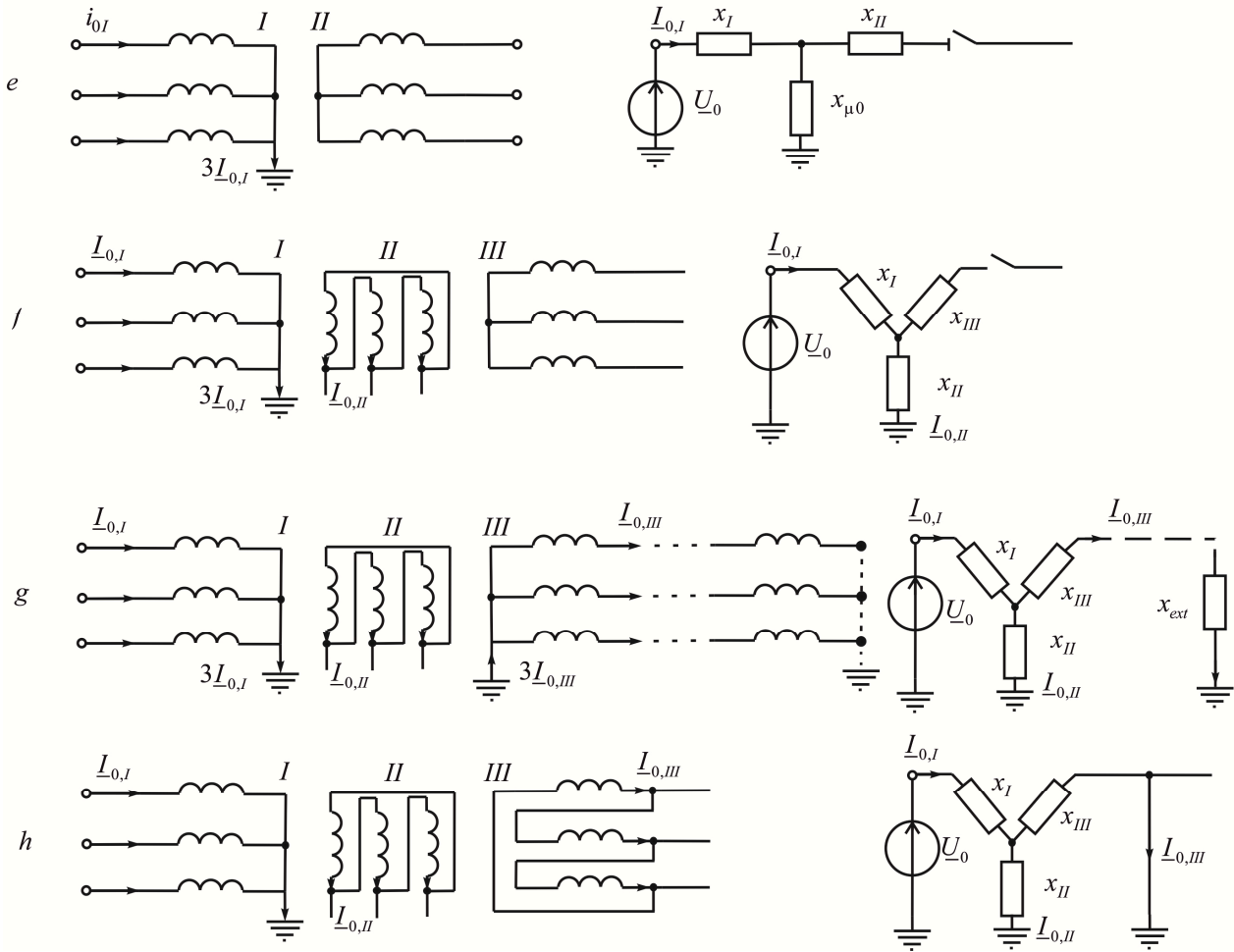


Fig. 4.7. (Continuad)

Zero sequence reactance of transformer will be finite ( $x_0 \neq \infty$ ) only on the side of its winding connected in star with grounded neutral (Fig. 4.7 b-i), and its value depends on other windings connection and on presence of loop for zero sequence current in them. For each type of windings circuit connection at Fig. 4.7, b-I, equivalent circuits for zero sequence currents are shown ( $x_I, x_{II}, x_{III}$  are leakage reactances of windings  $I, II, III$ , reduced to the step of  $I$  winding voltage:  $x_{\mu 0}$  - magnetization reactance of zero sequence (resistances of windings are neglected).

When windings of two-winding transformer are connected according to  $Y_0/\Delta$  circuit (Fig. 4.7 b) zero sequence current also flows in secondary winding  $II$ , which are delta connected. But that current (like the current harmonic) does not flow out these windings. That's why (Fig. 4.7, b)  $T$ -form equivalent circuit for zero sequence current includes  $x_I, x_{II}$  and  $x_{\mu 0}$  impedances. Zero sequence current  $I_{0,II}$  is within the winding, and it is shown as short-circuiting of the branch with  $x_{II}$ . Equal to zero potential, at the end of  $x_{II}$  the equivalent circuit does not mean artificial transfer of neutral ground as it is sometimes misunderstood: it just matches the condition that given branch of transformer equivalent circuit is the end of the path for zero sequence current circulation. Zero sequence impedance for the circuit in Fig. 4.7,b is:

$$x_0 = x_I + x_{II} x_{\mu 0} / (x_{II} + x_{\mu 0}) \tag{4.18}$$

The current in transformer neutral is tripled value of the zero sequence current which flows in winding with grounded neutral. For example, for circuit in Fig. 4.7:

$$\underline{I}_{I,N} = 3\underline{I}_{0,I} \tag{4.19}$$

When windings are connected by  $Y_0/Y_0$  according to equivalent circuit (Fig. 4.6,c) it is supposed that on the side of winding  $II$  the circuit for zero sequence current circulation is provided, that is at least one more grounded neutral is present in circuit of winding (look dash line). Then the equivalent circuit and reactance will include external circuit reactance as well:

$$x_0 = x_I + x_{\mu 0} (x_{II} + x_{ext}) / (x_{II} + x_{ext} + x_{\mu 0}) \tag{4.20}$$

Reactance  $x_N$  is introduced to the neutral of primary winding of the circuit (Fig. 4.7,d). As tripled current value of zero sequence flows in the transformer neutral, voltage drop across  $x_N$  is  $3jx_N I_{0,I}$ . Reactance  $x_N$  is introduced into equivalent circuit in series with that winding reactance to which neutral it is connected. To make the design circuit equivalent to the equivalent circuit that tripled value  $3x_N$  reactance is introduced into it. If in secondary circuit under windings being connected in  $Y_0/Y_0$ , the path for zero sequence current is not provided, the equivalent circuit will be similar to  $Y_0/Y$  (Fig. 4.7,e). It is in accordance with the transformer no-load mode, under which:

$$x_0 = x_I + x_{\mu 0} \tag{4.21}$$

Value  $x_{\mu 0}$  depends on transformers design (Fig. 4.8). For a group of three single-phase transformers, and three-phase four-and five core-type (shell-type) transformers, zero sequence magnetization current does not depend on the voltage sequence applied to the transformer as zero sequence magnetic flux has the same path as that the positive sequence magnetic flux. As current  $I_{\mu 0}$  is very low, it is admitted that the reactance  $x_{\mu 0} \equiv \infty$  is as for positive equivalent circuits.

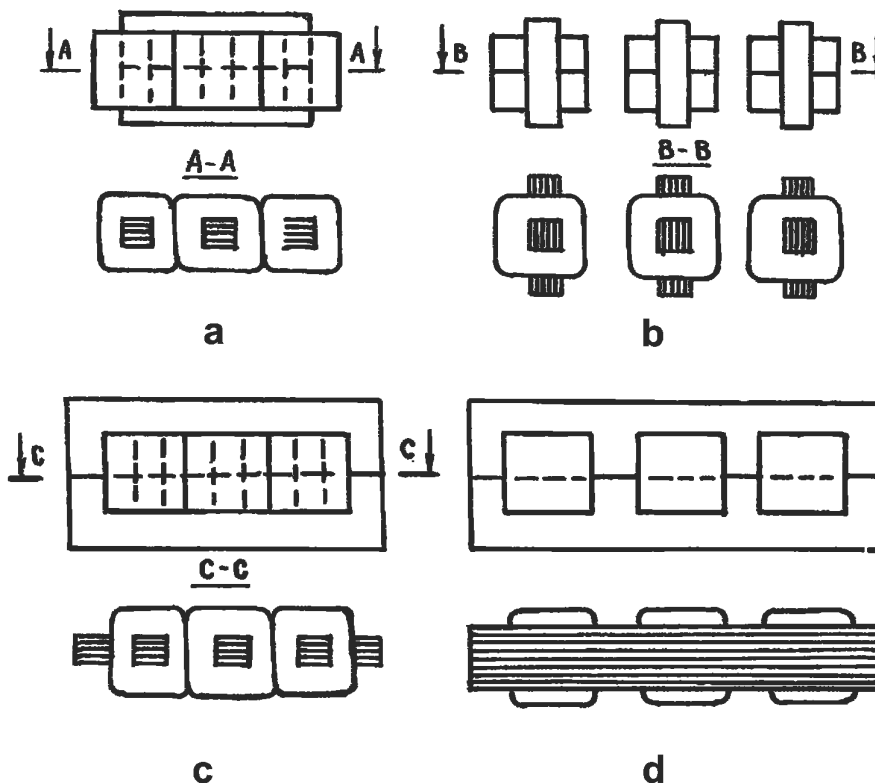


Fig. 4.8. Three-phase transformers core designs: a- three core-type; b- a group of three single-phase transformers; c- four core-type; d-five core-type

In three-phase three core-type transformers zero sequence magnetic fluxes has their paths through insulating ambient and the transformer tank. To conduct the magnetic flux on the path with so high magnetic reactance, it is necessary to have high magnetization current. That's why reactance  $x_{\mu 0}$  is small relatively to  $x_1$ , and in per unit is  $x_{*\mu 0} = 0,3...1,0$ . It is to be accounted when zero sequence equivalent circuits are formed. Exact value  $x_{\mu 0}$  is determined experimentally. For three-core type transformers, with some approximation  $x_{\mu 0} = (0,85...0,9)x_1$ , where  $x_1 = u_s \% / 100$ . As a rule in three-winding transformers one of windings is delta-connected. That's why  $x_{\mu 0} \equiv \infty$ . Main modifications of three-winding transformer windings connection and corresponding zero sequence equivalent circuits are shown in Fig. 4.7, f-h. There is no zero sequence current in winding *III* (Fig. 4.7,f). Therefore in this case

$$x_0 = x_I + x_{II} \quad (4.22)$$

It is supposed that in modification shown in Fig. 4.7,i, that path for zero sequence current circulation on the side of winding *III* is provided. In this case transformer has to be introduced in zero sequence circuit by its equivalent circuit shown in Fig g. 4. 7, i.

If windings are connected by  $Y_0 / \Delta / \Delta$  circuit (Fig. 4.6,j) zero sequence current of winding *I* is compensated by currents induced in windings *II* and *III*. In this case

$$x_0 = x_I + x_{II}x_{III} / (x_{II} + x_{III}) \quad (4.23)$$

Zero sequence reactance of three-phase two- winding transformer (which symmetrical splitting of its low voltage winding in two parts: with connection them in delta) can be estimated by formula (4.23) for which

$$x_I = 0,125x_{H-L}; \quad x_{II} = x_{III} = 1,75x_{H-L}, \quad \text{where } x_{H-L} = u_s \% / 100. \quad (4.24)$$

According to (4.23)

$$x_0 = 0,125x_{H-L} + 1,75x_{H-L} / 2 = x_{H-L} \quad (4.25)$$

Table 4.2 shows summary formulae for values of transformers zero sequence reactance.

There are magnetic and electrical coupling between windings of autotransformer. In this case conditions for zero sequence currents differ from those in transformers. It should be taken into account while making zero sequences equivalent circuits.

For dead ground of autotransformers neutral, their zero sequence equivalent circuits are the same as for transformers with corresponding connections of windings. Thus, if autotransformer hasn't third winding, and path for zero sequence current to be circulated in the secondary circuit is provided, its equivalent circuit (if magnetization current and reactance are neglected) is represented (Fig. 4.9,a) as total leakage inductive reactance of autotransformer. It is accordingly the three-winding transformer having circuit shown in Fig. 4.7,i, is similar to of autotransformer that have equivalent circuit of third delta-connected winding (Fig. 4.9,b) with which power autotransformers are usually provided.

Table 4.2  
Zero sequence reactance of transformers

Design of transformer	Connection of windings	Formula to determine $x_0$
Three-phase three core-type two winding transformer	$Y_0 / \Delta$ (Fig. 4.7,b)	$x_I + x_{II} x_{\mu 0} / (x_{II} + x_{\mu 0})$
	$Y_0 / Y$ (Fig.4.7,e)	$x_I + x_{\mu 0}$
	$Y_0 / Y_0$ (Fig.4.7,d)	$x_I + \frac{(x_{\text{ext}} + x_{II}) x_{\mu 0}}{x_{II} + x_{\mu 0} + x_{\text{ext}}}$
A group of three single-phase transformers, and four- or five core-type three-phase two-winding transformers	$Y_0 / \Delta$ (Fig.4.7,b)	$x_I + x_{II} = x_{I-II} = x_1$
	$Y_0 / Y$ (Fig.4.7,e)	$\infty$
	$Y_0 / Y_0$ (Fig.4.7,d)	$x_I + x_{II} + x_{\text{ext}} =$ $= x_{I-II} + x_{\text{ext}} = x_1 + x_{\text{ext}}$
Three-phase three-winding transformer	$Y_0 / \Delta / Y$ (Fig. 4.7,f)	$x_I + x_{II}$
	$Y_0 / \Delta / Y_0$ (Fig. 4.7j)	$x_I + \frac{(x_{\text{ext}} + x_{III}) x_{II}}{x_{II} + x_{III} + x_{\text{ext}}}$
	$Y_0 / \Delta / \Delta$ (Fig. 4.7,i)	$x_I + \frac{x_{III} x_{II}}{x_{II} + x_{III}}$

It is impossible to evaluate current in the neutral directly from zero sequence equivalent circuit of autotransformer. Under current directions shown in Fig. 4.9 unknown current in the circuit of neutral is equal to tripled difference of zero sequence currents of primary and secondary circuits,

that is  $\underline{I}_{0,N} = 3(\underline{I}_{0,I} - \underline{I}_{0,II})$ . And each of them has to be referred to its own voltage step but not to that the equivalent circuit was composed.

For autotransformer with neutral grounded through reactance  $x_N$  (Fig. 4.9,C) when zero sequence current is available the voltage drop equals to

$$\underline{U}_{0,N} = 3 j x_N \underline{I}_{0,N} = 3 j x_N (\underline{I}_{0,I} - \underline{I}_{0,II}) \quad (4.26)$$

If voltage on neutral is  $\underline{U}_{0,N}$  and voltage at clamps of phases  $I$  and  $II$  relative to the neutral are  $\underline{U}_{N,I}$  and  $\underline{U}_{N,II}$  accordingly, for resulting reactance of zero sequence between phases  $I$  and  $II$  clamps of autotransformer reduced to stage  $I$  it is possible to express

$$\begin{aligned} x'_{I-II} &= \left[ (\underline{U}_{N,I} + \underline{U}_{0,N}) - (\underline{U}_{N,II} + \underline{U}_{0,N}) \frac{\underline{U}_I}{\underline{U}_{II}} \right] / \underline{I}_{0,I} = \\ &= \left[ (\underline{U}_{N,I} - \underline{U}_{N,II}) + \underline{U}_{0,N} \left( 1 - \frac{\underline{U}_I}{\underline{U}_{II}} \right) \right] / \underline{I}_{0,I}. \end{aligned}$$



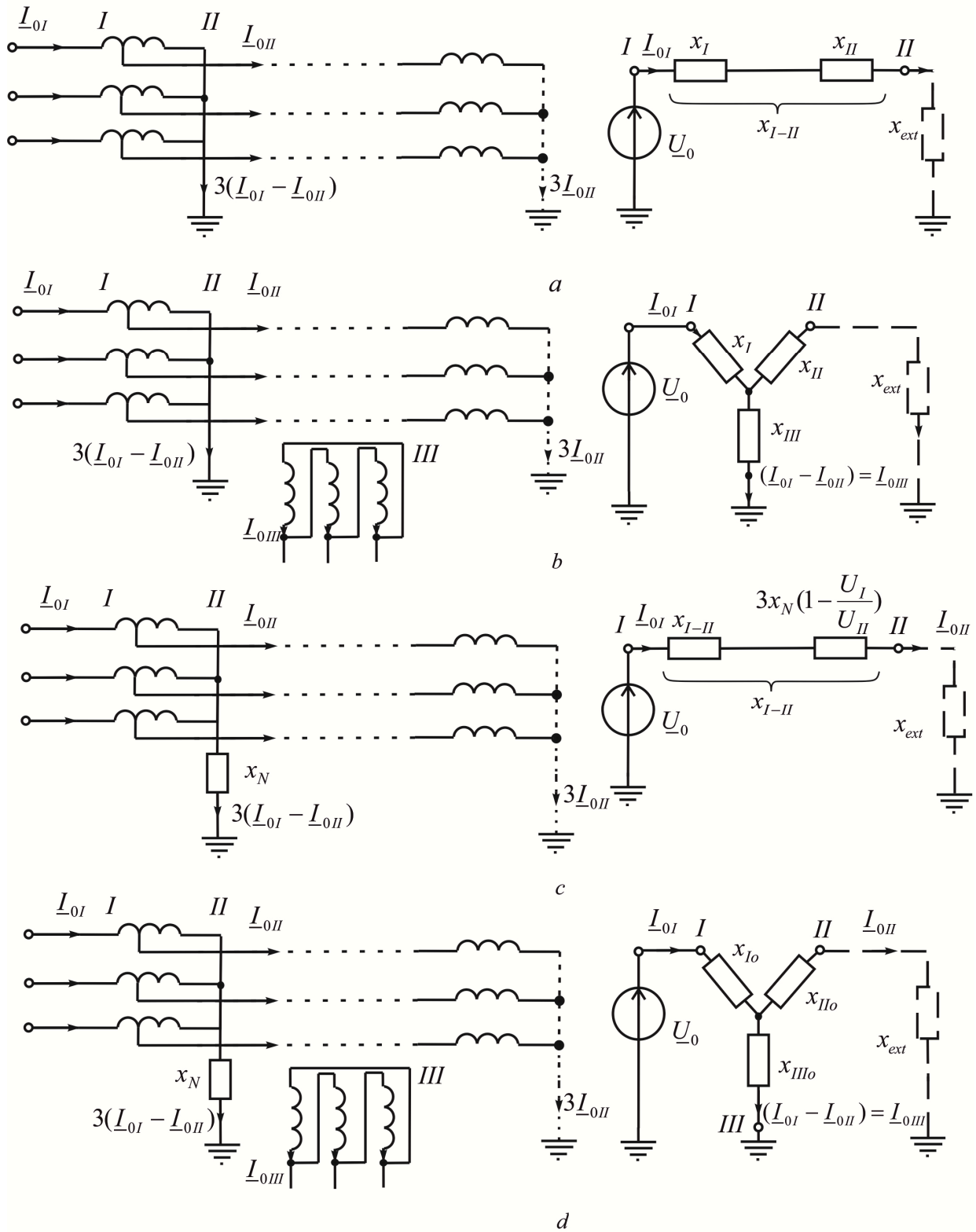


Fig. 4.9. Circuit of autotransformers windings and zero sequence equivalent circuits

As the first component  $(\underline{U}_{N,I} - \underline{U}_{N,II}) / I_{0,I}$  is the leakage reactance of autotransformer  $x_{I-II}$  referred to phase  $I$ , and  $\underline{U}_{0,N} / I_{0,I}$  on the basis of (4.26) is

$$\frac{\underline{U}_{0,N}}{I_{0,I}} = 3x_N \left( 1 - \frac{\underline{U}_I}{\underline{U}_{II}} \right),$$

then

$$x'_{I-II} = x_{I-II} + 3x_N \left( 1 - \frac{U_I}{U_{II}} \right)^2 \quad (4.27)$$

In the same way for three-winding autotransformer (Fig. 4.9,d), resulting reactances of zero sequence between other its windings pairs, also referred to phase  $I$  are equal to

$$x'_{I-III} = x_{I-III} + 3x_N, \quad (4.28)$$

$$x'_{II-III} = x_{II-III} + 3x_N \left( \frac{U_I}{U_{II}} \right)^2 \quad (4.29)$$

For the three-ray equivalent circuit of three-winding autotransformer resulting reactances of zero sequence for each the ray by formulae (4.27)-(4.29) are determined (Fig. 4.9,d):

$$\left. \begin{aligned} x_{0,I} &= x_I - 3x_N \left( \frac{U_I}{U_{II}} - 1 \right); \\ x_{0,II} &= x_{II} - 3x_N \left( \frac{U_I}{U_{II}} - 1 \right) \frac{U_I}{U_{II}}; \\ x_{0,III} &= x_{III} - 3x_N \frac{U_I}{U_{II}}. \end{aligned} \right\} \quad (4.30)$$

Two-winding autotransformer with ungrounded neutral in zero sequence equivalent circuit operates in the no-load mode (zero sequence current of autotransformer can be neglected, that is  $x_{\mu 0} \equiv \infty$ ). As for three-winding autotransformer, ungrounding the neutral does not stipulate no-load mode for zero sequence current because of present third delta- connected winding that creates conditions for this current to be circulated.

In overhead transmission lines zero sequence current circulates in current-carrying wires, and in grounded circuits (lightning protection ground wires, rail tracks along the line), and in ground-in which form a closed path. The main problem to compute values of zero sequences reactance is accounting current distributions in ground.

Consider single-wire line for which ground is return path. Inductive impedance ( $Ohm/km$ ) of such a line with sufficient for practice accuracy is determined as reactance of equivalent two-wire line "wire-ground" by the formula:

$$x_L = 0,145 \lg(D_e/R_e), \quad (4.31)$$

where  $R_e$  - is equivalent radius of wire, m;  $D_e = 66,4/\sqrt{f\lambda}$  - is depth of the return wire laying;  $D_e = 935$  m

For industrial frequency  $f = 50$  Hz and average value of ground conductivity  $\lambda = 10^{-4}$  ( $Ohm \cdot cm$ ), assumed as wet ground conductivity, the depth of return line laying is  $D_e = 935$  m (it is approximately assumed  $D_e = 1000$  m).

Equivalent radius  $R_e$  of split phase from  $n$  for similar wires is:

$$R_e = \sqrt[n]{kR_w d^{n-1}}, \quad (4.32)$$

where  $d_{av}$  is geometrical mean distance  $n$  between one phase wires which is determined by the number of possible different distances between two wires equal to the number of combinations from  $n$  components in twos, that is  $n(n-1)/2$ , then

$$d_{av} = \frac{n(n-1)}{2} \sqrt{d_{12}d_{13} \dots d_{1n}d_{23}d_{24} \dots d_{2n} \dots d_{(n-1)n}}. \quad (4.33)$$

If there is one wire in phase ( $n=1$ )

$$R_e = kR_w, \quad (4.34)$$

where  $R_w$  is an actual wire radius,  $m$ :  $k$  is a coefficient, which takes into account a part of magnetic flux, passing through the wire ( $k=0,779$  for solid round wires made of non-magnetic material, and  $k=0,724 \dots 0,771$  for wires made of steel and aluminum).

Taking into consideration such a fact that insignificant variations of coefficient  $k$  do not affect much a value of the lines inductive reactance (4.31) (that coefficient appears in expressions for impedances under the logarithm sign) it is possible to proceed from the average value  $k=0,9$ .

In split phase ( $n \geq 2$ ) and symmetrical location of wires one gets, according to (4.32) and (4.33):

$$\text{at } n=2 \quad R_e = \sqrt{kR_w d}; \quad (4.35)$$

$$\text{at } n=3 \quad R_e = \sqrt[3]{kR_w d^2}; \quad (4.36)$$

$$\text{at } n=4 \quad R_e = \sqrt[4]{\sqrt{2}kR_w d^3}. \quad (4.37)$$

Here  $d$  is a split pitch, i.e. distance between the closest wires in phase, m.

Resistance of line "wire-ground" is equal to the sum of wire resistance  $R_w$  that is determined by reference book data, and resistance  $r_g$  which takes into account power losses in ground as a result of current flowing in it. Resistance  $r_g$ , (Ohm/km), depends insignificantly on ground conductivity and is determined on the formula

$$r_g = \pi^2 f \cdot 10^{-4}. \quad (4.38)$$

If  $f=50$  Hz, the ground resistance is  $r_g=0,05$  (Ohm/km).

Total impedance of the path "wire-ground", Ohm/km, is

$$\underline{Z}_L = R_w + r_g + j0,1451 \lg(D_e/(kR_w)). \quad (4.39)$$

For lines with split wires it is necessary to assume  $R_w/n$  instead of  $R_w$ , and instead of  $kR_w$  resistance  $R_e$  should be taken.

If two lines “wire-ground” passing parallel to each other, mutual induction arises between them. Impedance of mutual inductance (Ohm/km) is

$$\underline{Z}_M = r_g + j0,1451g(D_e/D), \quad (4.40)$$

where  $D$  is distance between axes of line wires, m.

Three-phase overhead transmission line without ground wires can be shown as three one-phase lines “wire-ground”. Own resistance (Ohm/km) of three-phase line phase is determined by the formula (4.39). Mutual inductive reactance between phase wires is assumed equal to one another and is determined by (4.40) when instead of  $D$  geometric mean distance between phases A, B and C, m:

$$D = D_{av} = \sqrt[3]{D_{AB}D_{BC}D_{CA}}. \quad (4.41)$$

For the line with horizontal suspension of phase (under symmetric position of end phases relative to a central one)  $D_{av} = 1,26D_{ph}$  ( $D_{ph}$  – distance between adjacent phases, m).

It is worth noticing that values of impedances determined for averaged distances between phases are in accordance with the condition of wires transposition complete cycle.

In the terms of phase coordinates the parameters of three-phase line can be expressed as a matrix of impedances:

$$\underline{Z} = \begin{vmatrix} \underline{Z}_L & \underline{Z}_M & \underline{Z}_M \\ \underline{Z}_M & \underline{Z}_L & \underline{Z}_M \\ \underline{Z}_M & \underline{Z}_M & \underline{Z}_L \end{vmatrix}. \quad (4.42)$$

Express parameters of three-phase line may be expressed in terms of symmetrical components. For that the equation of voltage drop in phase impedance of the line section is to be written:

$$\Delta \underline{\vec{U}} = \underline{Z} \underline{\vec{I}}. \quad (4.43)$$

According to (4.6) and (4.7) voltage drops and currents in phase and symmetric coordinates are connected by relationships:

$$\Delta \underline{\vec{U}} = S \cdot \Delta \underline{\vec{U}}_S, \quad \underline{\vec{I}} = S \cdot \underline{\vec{I}}_S. \quad (4.44)$$

Having substituted in (4.43) those values and after multiplication of the obtained result by  $S^{-1}$  we receive

$$\Delta \underline{\vec{U}}_S = \underline{Z}_S \cdot \underline{\vec{I}}_S, \quad (4.45)$$

where  $\underline{Z}_S$  a matrix of total impedances in the terms of symmetrical coordinates

$$\begin{aligned} \underline{Z}_S = S^{-1} \underline{Z} \cdot S &= \frac{1}{3} \begin{vmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{vmatrix} \cdot \begin{vmatrix} \underline{Z}_L & \underline{Z}_M & \underline{Z}_M \\ \underline{Z}_M & \underline{Z}_L & \underline{Z}_M \\ \underline{Z}_M & \underline{Z}_M & \underline{Z}_L \end{vmatrix} \cdot \begin{vmatrix} a^2 & a & 1 \\ a & a^2 & 1 \\ 1 & 1 & 1 \end{vmatrix} = \\ &= \begin{vmatrix} \underline{Z}_L - \underline{Z}_M & 0 & 0 \\ 0 & \underline{Z}_L - \underline{Z}_M & 0 \\ 0 & 0 & \underline{Z}_L + 2\underline{Z}_M \end{vmatrix} = \begin{vmatrix} \underline{Z}_1 & 0 & 0 \\ 0 & \underline{Z}_2 & 0 \\ 0 & 0 & \underline{Z}_0 \end{vmatrix}. \end{aligned} \quad (4.46)$$

Having substituted in (4.46) values of  $\underline{Z}_L$  and  $\underline{Z}_M$  from (4.39) and (4.40), taking into account  $D_{av}$  and performed necessary transformations we have

$$\underline{Z}_1 = \underline{Z}_2 = R_w + j0,145 \lg(D_e / (kR_w)) , \quad (4.47)$$

$$\underline{Z}_0 = R_w + 0,15 + j0,435 \lg(D_e / (kR_{av})) , \quad (4.48)$$

where -  $R_{av}$  is an geometrical mean radius of three-wire system

$$R_{av} = \sqrt[3]{kR_w D_{av}^2} . \quad (4.49)$$

As the impedances of the lines  $\underline{Z}_L$  and  $\underline{Z}_M$  are comparable, impedances  $\underline{Z}_1$  and  $\underline{Z}_0$  differ greatly due to impedance of mutual induction  $\underline{Z}_M$ .

For multi-circuit lines without ground wires phase impedance and impedance of mutual induction between phases of each the line (per length unit) are determined using the same expressions (4.39) and (4.40) that for one-chain line with account of  $D_{av}$ . Besides, for zero sequence mutual inductance from co-parallel circuits is taken into consideration. Thus, for two parallel lines without ground wires impedance of mutual inductance (Ohm/km) between each of the wires of the first (*I*) and second (*II*) lines is

$$\underline{Z}_{I-II} = r_g + j0,145 \lg(D_e / D_{I-II}) , \quad (4.50)$$

where  $D_{I-II}$  is geometrical mean distance (m) between phases of circuits of lines *I* and *II*

$$D_{I-II} = \sqrt[9]{D_{AA'} D_{AB'} D_{AC'} D_{BA'} D_{BB'} D_{BC'} D_{CA'} D_{CB'} D_{CC'}} . \quad (4.51)$$

Impedance matrix of two parallel lines looks like

$$\underline{Z} = \begin{vmatrix} \underline{Z}_I & \underline{Z}_{I-II} \\ \underline{Z}_{II-I} & \underline{Z}_{II} \end{vmatrix} . \quad (4.52)$$

Submatrix  $\underline{Z}_I$  and  $\underline{Z}_{II}$  are analogous to matrix (4.42) in phase coordinates. Submatrix  $\underline{Z}_{I-II}$  and  $\underline{Z}_{II-I}$  are square matrix of the 3d order with elements having been determined on (4.50) wherein  $\underline{Z}_{I-II} = \underline{Z}_{II-I}^T$ .

Parameters of two parallel lines will be searched by analogy with abovementioned calculations ((4.43) - (4.46)) in the terms of symmetrical coordinates by analogy for one-chain three-phase line. To do that, write down the equation of voltage drops on phase impedances of the unit length section of two-chain three-phase line section:

$$\begin{vmatrix} \Delta \vec{U}_I \\ \Delta \vec{U}_{II} \end{vmatrix} = \begin{vmatrix} \underline{Z}_I & \underline{Z}_{I-II} \\ \underline{Z}_{II-I} & \underline{Z}_{II} \end{vmatrix} \cdot \begin{vmatrix} \vec{I}_I \\ \vec{I}_{II} \end{vmatrix} , \quad (4.53)$$

where  $\Delta \vec{U}_I = \begin{vmatrix} \Delta \vec{U}_{I,A} & \Delta \vec{U}_{I,B} & \Delta \vec{U}_{I,C} \end{vmatrix}$  is a vector column of the voltage drops within the *I* line section: the  $\Delta \vec{U}_{II} = \begin{vmatrix} \Delta \vec{U}_{II,A} & \Delta \vec{U}_{II,B} & \Delta \vec{U}_{II,C} \end{vmatrix}$  is a vector-column of voltage drops within the

$II$  line section:  $\vec{I}_I = \begin{vmatrix} \vec{I}_{I,A} & \vec{I}_{I,B} & \vec{I}_{I,C} \end{vmatrix}$  and  $\vec{I}_{II} = \begin{vmatrix} \vec{I}_{II,A} & \vec{I}_{II,B} & \vec{I}_{II,C} \end{vmatrix}$  are vectors-columns of the  $I$  and  $II$  lines currents.

From (4.53) we have two matrix equations:

$$\left. \begin{aligned} \Delta \vec{U}_I &= \underline{Z}_I \vec{I}_I + \underline{Z}_{I-II} \vec{I}_{II}; \\ \Delta \vec{U}_{II} &= \underline{Z}_{II-I} \vec{I}_I + \underline{Z}_{II} \vec{I}_{II}. \end{aligned} \right\} \quad (4.54)$$

Express voltages and currents in symmetrical coordinates and multiply the received equations by  $S^{-1}$  on the left:

$$\left. \begin{aligned} \Delta \vec{U}_{I,S} &= \underline{Z}_{I,S} \vec{I}_I + \underline{Z}_{I-II,S} \vec{I}_{II}; \\ \Delta \vec{U}_{II,S} &= \underline{Z}_{II-I,S} \vec{I}_I + \underline{Z}_{II,S} \vec{I}_{II}, \end{aligned} \right\} \quad (4.55)$$

where

$$\underline{Z}_{I,S} = S^{-1} \underline{Z}_I \cdot S = \begin{vmatrix} \underline{Z}_{I,1} & 0 & 0 \\ 0 & \underline{Z}_{I,2} & 0 \\ 0 & 0 & \underline{Z}_{I,0} \end{vmatrix}; \quad (4.56)$$

$$\underline{Z}_{II,S} = S^{-1} \underline{Z}_{II} \cdot S = \begin{vmatrix} \underline{Z}_{II,1} & 0 & 0 \\ 0 & \underline{Z}_{II,2} & 0 \\ 0 & 0 & \underline{Z}_{II,0} \end{vmatrix}; \quad (4.57)$$

$$\underline{Z}_{I-II,S} = S^{-1} \underline{Z}_{I-II} \cdot S = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3\underline{Z}_{I-II} \end{vmatrix}. \quad (6.58)$$

The elements matrix (4.56) and (4.57) determine on (4.47) and (4.48) and elements of matrix (4.58) by (4.50). The first equation (4.55) in detail:

$$\left. \begin{aligned} \Delta \vec{U}_{I,1} &= \underline{Z}_{I,1} \vec{I}_{I,1}; \\ \Delta \vec{U}_{I,2} &= \underline{Z}_{I,2} \vec{I}_{I,2}; \\ \Delta \vec{U}_{I,0} &= \underline{Z}_{I,0} \vec{I}_{I,0} + 3\underline{Z}_{I-II} \vec{I}_{II,0}. \end{aligned} \right\} \quad (6.59)$$

Analyzing these expressions, it is seen that parallel line affects only the parameters of zero sequence equivalent circuit. It can be explained by the fact that positive and negative sequences currents create balanced phasor system, and their resulting magnetic fields are equal to zero. Resulting magnetic field created by zero sequence current of three-phases of the line is not equal to zero, and has the same direction as magnetic field created by zero sequence current of parallel line. Resulting magnetic field increases. Due to it the parameters zero sequence equivalent circuits of both the lines vary.

The third equation of system (4.59) can be changed in such a way:

$$\Delta \vec{U}_{I,0} = (\underline{Z}_{I,0} - 3\underline{Z}_{I-II}) \vec{I}_{I,0} + 3\underline{Z}_{I-II} (\vec{I}_{II,0} + \vec{I}_{I,0}) \quad (4.60)$$

Symbolize

$$\begin{aligned} \underline{Z}_{I-II,0} &= r_{I-II,0} + jx_{I-II,0} = 3\underline{Z}_{I-II} = \\ &= 0,15 + j0,435 \lg(D_e/D_{I-II}). \end{aligned} \quad (4.61)$$

Mutual inductance especially reveals itself when both chains of the power electric line are located on one support, and value  $x_{I-II,0}$  is within 0,9...1.0 (Ohm/km). When distance between the lines increases mutual inductance values and  $x_{I-II,0}$  decrease and if distance  $D_{I-II} \geq 500m$  they can be neglected. Equivalent circuit in Fig. 4.10, a conforms to expression (4.60). Mutual inductance impedance is brought in the sum (difference) of two circuits currents. Fig. 4.10,b shows equivalent circuit of two parallel lines to calculate short-circuit within one of parallel line. Such a circuit can be obtained if two parallel lines are divided into two sections, and circuit in Fig. 4.10,a is applied for the both. Relative distance from short-circuit point to the nodes  $M$  and  $N$  in Fig. 4.10,b circuit is denoted as  $n$  and  $1-n$ .

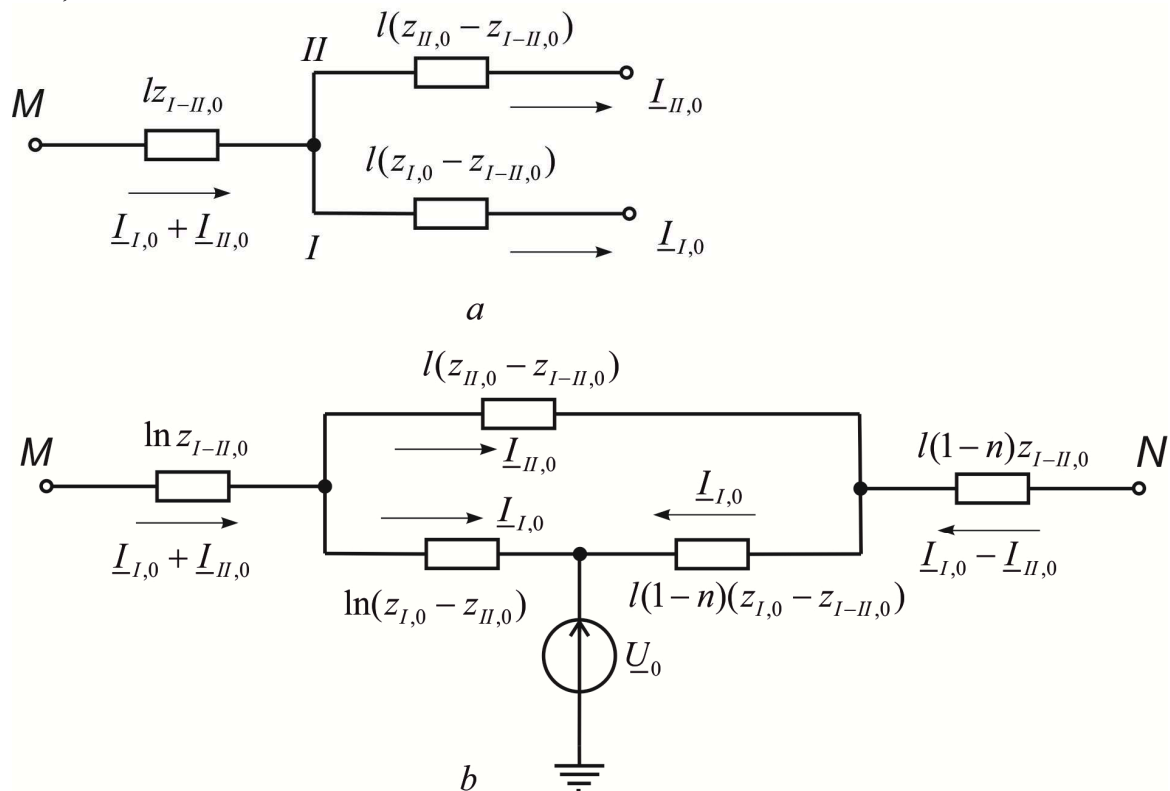


Fig. 4.10. Zero sequence equivalent circuits of a two-chain power electric line for the cases: a- circuits are connected at one end; b- circuitshave two common points of short-circuit within one of the lines

Often parallel lines have equal parameters and work for common buses, that is  $\underline{Z}_{II,0} = \underline{Z}_{I,0}$ ;  $\Delta \underline{U}_{I,0} = \Delta \underline{U}_{II,0}$ ;  $\underline{I}_{I,0} = \underline{I}_{II,0}$ . Resulting impedance of such lines is

$$\underline{Z}_0 = (\underline{Z}_{I,0} + \underline{Z}_{I-II,0})/2 \quad (4.62)$$

More detailed cases are shown in [35].

Estimate influence of the ground wires on the value of zero sequence impedance of overhead transmission line. The ground wires are used as the means of lightning protection, placing them in the upper point of support. Until recently the ground wires was grounded at each support. Presence of induced currents is possible in short-circuits arising. The induced current can be significant at flowing within the balanced system of phase currents due to which impedances of positive and negative sequences do not have much influence on zero sequences impedances of overhead transmission lines.

For the lines mainly of great length, another approach to the ground wires grounding is recently applied. The wire is suspended to insulators and cut for a number of sections. One end of each the section is grounded, and at the other end a disruptive distance between the wire and the ground is remained. That distance is broken down if overvoltage has value greater than specified. Such grounding construction, the ground wires does not practically affect the zero sequence impedance of overhead line.

Own impedances of cable grounded from both ends can be determined if it is considered as independent one-wire line "wire-ground". Analogously to (4.39) under  $f = 50$  Hz impedance of cable (Ohm/km)

$$\underline{Z}_{r,0} = r_r + 0,05 + j0,1451 \lg(D_{\text{layment}}/R_{r,e}), \quad (4.63)$$

where  $r_r$  is a cable resistance;  $R_{r,e}$  is an equivalent radius of the wire.

Impedance of mutual inductance (Ohm/km) between a wire and the grounding wire according to (4.40)

$$\underline{Z}_{r,w} = 0,05 + j0,1451 \lg(D_{\text{layment}}/D_{w,r}), \quad (4.64)$$

where  $D_{w,r}$  is an geometrical mean distance between line wires and cable

$$D_{w,r} = \sqrt[3]{D_{AT}D_{BT}D_{CT}}. \quad (4.65)$$

Matrix of impedances of line with one ground wire is

$$\underline{Z}^T = \begin{vmatrix} \underline{Z} & \underline{Z}_{r,w} \\ \underline{Z}_{r,w} & \underline{Z}_{r,0} \end{vmatrix}, \quad (4.66)$$

where  $\underline{Z}$  is a Submatrix of own and mutual impedances of wires to be analogous to matrix (4.42);  $\underline{Z}_{r,w}$  is a Submatrix of mutual impedances between wires and the ground wire which components are calculated on (4.64), and  $\underline{Z}_{r,w} = \underline{Z}_{r,w}^T$ ;  $\underline{Z}_{r,0}$  is a cable impedance calculated by (4.63).

Balance equation of voltage drops for the group of elements-three phases and a ground wire looks like

$$\begin{vmatrix} \Delta \underline{\vec{U}} \\ \Delta \underline{\vec{U}}_r \end{vmatrix} = \begin{vmatrix} \underline{Z} & \underline{Z}_{r,w} \\ \underline{Z}_{w,r} & \underline{Z}_{r,0} \end{vmatrix} \cdot \begin{vmatrix} \underline{\vec{I}} \\ \underline{\vec{I}}_r \end{vmatrix}, \quad (4.67)$$

where  $\Delta \underline{\vec{U}} = |\Delta \underline{U}_A \quad \Delta \underline{U}_B \quad \Delta \underline{U}_C|^T$  is a vector-column of voltage drops within the line section;  $\Delta \underline{\vec{U}}_r$  is one-dimensional vector of voltage drop within the wire;  $\underline{\vec{I}} = |\underline{\vec{I}}_A \quad \underline{\vec{I}}_B \quad \underline{\vec{I}}_C|^T$  is a vector-column of phase line currents; and  $\underline{\vec{I}}_r$  is one-dimensional current vector in cable.



Express equation (4.67) by two matrix equations:

$$\Delta \underline{\vec{U}} = \underline{Z} \underline{\vec{I}} + \underline{Z}_{r,w} \underline{\vec{I}}_r; \quad (4.68)$$

$$\Delta \underline{\vec{U}}_r = \underline{Z}_{w,r} \underline{\vec{I}} + \underline{Z}_{r,0} \underline{\vec{I}}_r. \quad (4.69)$$

As the ground wire is grounded at both sides,  $\Delta \underline{\vec{U}}_r = 0$ . From equation (4.69) find

$$\underline{\vec{I}}_r = -\underline{Z}_{r,0}^{-1} \underline{Z}_{w,r} \underline{\vec{I}}. \quad (4.70)$$

After substitution of the obtained current value in (4.68)

$$\Delta \underline{\vec{U}} = \underline{Z}_w^T \underline{\vec{I}} = (\underline{Z} - \underline{Z}_{r,w} \underline{Z}_{r,0}^{-1} \underline{Z}_{w,r}) \underline{\vec{I}}. \quad (4.71)$$

Matrix  $\underline{Z}_r^T$  of own and mutual impedances of wires without accounting the grounding wire can be obtained deleting the line and column with index "T" and recalculating other components of matrix (4.66) by formula

$$\underline{Z}_{ij}^T = \underline{Z}_{ij} - \underline{Z}_{j,r} \underline{Z}_{r,i} / \underline{Z}_{r,0}, \quad (4.72)$$

where  $i$  is line index; and  $j$  is column index.

New matrix looks like

$$\underline{Z}_r^T = \begin{vmatrix} \underline{Z}_L - \frac{\underline{Z}_{r,w}^2}{\underline{Z}_{r,0}} & \underline{Z}_M - \frac{\underline{Z}_{r,w}^2}{\underline{Z}_{r,0}} & \underline{Z}_M - \frac{\underline{Z}_{r,w}^2}{\underline{Z}_{r,0}} \\ \underline{Z}_M - \frac{\underline{Z}_{r,w}^2}{\underline{Z}_{r,0}} & \underline{Z}_L - \frac{\underline{Z}_{r,w}^2}{\underline{Z}_{r,0}} & \underline{Z}_M - \frac{\underline{Z}_{r,w}^2}{\underline{Z}_{r,0}} \\ \underline{Z}_M - \frac{\underline{Z}_{r,w}^2}{\underline{Z}_{r,0}} & \underline{Z}_M - \frac{\underline{Z}_{r,w}^2}{\underline{Z}_{r,0}} & \underline{Z}_L - \frac{\underline{Z}_{r,w}^2}{\underline{Z}_{r,0}} \end{vmatrix}. \quad (4.73)$$

Find symmetric components of impedances of a three-phase line with one ground wire. Analogously to (6.46) matrix of the sequences impedances can be written as

$$\underline{Z}_S^T = S^{-1} \underline{Z}_r^T \cdot S = \begin{vmatrix} \underline{Z}_1^T & 0 & 0 \\ 0 & \underline{Z}_2^T & 0 \\ 0 & 0 & \underline{Z}_0^T \end{vmatrix} =$$

$$= \begin{vmatrix} \underline{Z}_L - \underline{Z}_M & 0 & 0 \\ 0 & \underline{Z}_L - \underline{Z}_M & 0 \\ 0 & 0 & \underline{Z}_L + 2\underline{Z}_M - 3\underline{Z}_{r,w}^2 / \underline{Z}_{r,0} \end{vmatrix} =$$

$$= \begin{vmatrix} \underline{Z}_1 & 0 & 0 \\ 0 & \underline{Z}_2 & 0 \\ 0 & 0 & \underline{Z}_0 - 3\underline{Z}_{r,w}^2/\underline{Z}_{r,0} \end{vmatrix}. \quad (4.74)$$

A ground wire grounded from the both ends affects only the parameters of zero sequence. As current direction in cable is opposite to current in phases, resulting magnetic field decreases so inductive reactance of zero sequence decreases too. As for the ground wire influence on impedance of zero sequence of lines that influence depends on arguments of  $\underline{Z}_0$ ,  $\underline{Z}_{r,0}$ ,  $\underline{Z}_{r,w}$ .

Zero sequence impedance of line with two ground wires can be calculated if they are replaced by equivalent one which geometric radius is

$$R_{r,av} = \sqrt{R_{r,e}D_r}, \quad (4.75)$$

where  $D_r$  is distance (m) between the phase wires and the ground wires.

Own electric impedance (Ohm/km) of the equivalent ground wire is

$$\underline{Z}_{r,0,e} = r_r / 2 + 0,05 + j0,145 \lg(D_e/R_{r,e}). \quad (4.76)$$

Impedance of mutual inductance (Ohm/km) between the line wire and equivalent ground wire is

$$\underline{Z}_{r,w} = 0,05 + j0,145 \lg(D_e/D_{w,r,av}), \quad (4.77)$$

where  $D_{w,r,av}$  is a geometrical mean distance (m) between phase wires and the ground wires

$$D_{w,r,av} = \sqrt[6]{D_{A,T1}D_{B,T1}D_{C,T1}D_{A,T2}D_{B,T2}D_{C,T2}}. \quad (4.78)$$

Impedances of some sequences of lines where the ground wires are not taken into account are found on formulae (4.47) and (4.48). Then obtained values of impedances  $\underline{Z}_1$ ,  $\underline{Z}_2$ ,  $\underline{Z}_0$ ,  $\underline{Z}_{r,0,e}$ ,  $\underline{Z}_{r,w}$  are used to obtain elements of matrix (4.74) where the ground wires are taken into account. The ground wires do not influence parameters of equivalent circuits of negative and positive sequences.

In approximate practical calculation average values of ratios between inductive reactances  $x_0/x_1$  for overhead transmission lines can be assumed equal to:

<i>one-circuit, 110 ...220 kV without the ground wire or</i>	
<i>with steel the ground wire</i> .....	3.5
<i>the same with grounded well conducting the ground wire</i> .....	2.0
<i>two-circuit, 110...220 kV without the ground wires or</i>	
<i>with steel the ground wires</i> .....	5.5
<i>the same with grounded well conducting the ground wires</i> .....	3.0
<i>single, 330 kV</i> .....	4.1
<i>single, 500 kV</i> .....	4.2
<i>two parallel, 330 kV, following one line path</i> .....	5.9
<i>two parallel, 500 kV, following one line path</i> .....	6.2

Capacitive reactance of line is given in catalogues as line impedance (Ohm/km). Total capacitive reactance of line is calculated dividing given linear value on the length (km) of line.

For cable line, impedances of positive (negative) sequence can be calculated as it is done for overhead transmission line. Inductive reactance of cable lines is much less than that of overhead transmission lines due to shorter distances between phases. Capacitive conductivity of cable lines is more than that of overhead ones. It is stipulated by shorter distances and high value of insulation permittivity.

As a rule cable sheath is grounded on its ends and sleeves. It creates parallel to ground circuits for zero sequence currents. The affect of a cable sheath is similar to the ground wires of the overhead lines. Not only own impedance of shell influences greatly the current distribution between shell and ground but also its ground impedance, which values depend on character of cable laying (trench, blocks, etc.) and a number of other factors hampering to calculate value  $Z_0$  of cable for certain.

For approximate calculations for three-core cables with  $r_0 \approx 10r_1$ ,  $x_0 = (3,5...4,6)x_1$ , and when cables are single-core ones  $x_0 = (0,8...1,0)x_1$ . Capacitive impedance of cable is given in catalogues as resistivity (Ohm/km).

To obtain reliable data concerning values of cable impedance  $Z_0$  corresponding tests under real conditions should be made.

#### 4.6. Drawing up equivalent circuits for symmetrical components

To calculate any mode with asymmetrical fault by method of symmetrical components it is necessary first of all to draw up equivalent circuits for all three (positive, negative, and zero) sequences. Equivalent circuit of certain sequence includes equivalents of the network elements in which under given asymmetry, currents of correspondent sequences flow. Parameters of element equivalents of a sequence equivalent circuits components are expressed reducing them accordingly to a level of voltage being chosen as the main one or chosen base conditions in concrete or in per unit of measurement. When equations are solved analytically, the resulting impedances of some sequences are found relatively to that point where asymmetry has arisen. Resulting electromotive force is found using the equivalent circuit for positive sequence.

**Equivalent circuit for positive sequence** has to include equivalents of all elements of the electrical installation for made design circuit of short-circuit in which positive sequence current flow. Synchronous generators, synchronous compensators, synchronous and induction electric being subject to account, while calculating initial current value of asymmetric short circuit, are introduced in equivalent circuit of positive sequence by electromotive forces and subtransient reactances. All other elements are represented by their own equivalent circuits with their impedances.

**Equivalent circuit for negative sequence** is analogous its structure to equivalent circuit for positive sequence as positive and negative sequences currents circulate in the same paths. To obtain negative sequence equivalent circuit, electromotive forces of all generating sources should be taken equal to zero, and inductive reactance of synchronous motors and loads are to be replaced with inductive reactances of these elements negative sequence. In simplified practical calculations it is allowed to assume the equality of elements impedances in positive and negative sequences circuits. Zero potential point of all generating and loading branches is considered as the beginning of equivalent circuit for negative sequence. The point in which considered asymmetry has arisen is assumed as the end of equivalent circuit for negative sequence. Voltages of correspondent sequences are applied to the ends of their equivalent circuits.

**Equivalent circuit for zero sequence** differs from equivalent circuits for positive and negative sequences as zero sequence currents in paths different from those for symmetrical system

currents. That's why equivalent circuit for zero sequence depends much upon winding connection of transformers and autotransformers, and their neutrals conditions (Fig. 4.7 and 4.9).

Equivalent circuit for zero sequence drawing up starts from the point where asymmetry arose, supposing that zero sequence voltage is applied there. Depending upon the kind of asymmetry this voltage is applied either relative to ground (lateral asymmetry, Fig. 4.11,a) or in series in the phase conductors (longitudinal asymmetry, Fig. 4.11,b). Then possible paths of zero sequence currents are revealed within every electrically connected circuit.

Zero sequence voltage  $\underline{U}_{s0}$  is applied relative to ground and if capacitive susceptance to the ground is not available, for zero sequence currents circulation it is necessary to have at least one grounded neutral in the same electrically connected circuit. If there are several grounded neutrals, a few parallel loops for zero sequence current are available.

If asymmetry is of longitudinal type, that is zero sequence voltage is introduced in succession into phase wires, zero sequence current circulation is possible even if grounded neutrals are not available if there is a loop on bypass paths of the same electrically connected circuit. If those paths are not available zero sequence current circulation is possible in the same circuit only when there are grounded neutrals on both sides from the place where zero sequence voltage  $\Delta\underline{U}_{L0}$  has been applied.

Such facts must be taken into consideration while compiling equivalent circuit for zero sequence:

- if transformer winding on the side of asymmetrical fault (short circuit) is delta- connected or star-connected with ungrounded neutral, both transformer itself and elements following it (in the direction of the fault) shouldn't be introduced in equivalent circuit;
- if transformer windings are connected  $Y_0/\Delta$ , and star-connected winding with grounded neutral faces the point of asymmetric fault, only elements located between the fault point and transformer, and the transformer itself have to be introduced in equivalent circuit;
- if a few overhead transmission lines are laid in one path, it is necessary to take into consideration mutual inductance between lines using equivalent circuits from [17, 35];
- if transformer (generator, motor, load) neutral is grounded via resistor or any other component, it has to be introduced in equivalent circuit by tripled impedance value. It is explained by the fact, that equivalent circuit is drawn up for one phase but zero sequence currents sum of all three phases is circulated through the neutral circuit. To account voltage drop in circuit impedance it should be tripled.

Fig. 4.12 shows how equivalent circuit for zero sequence is drawn up when zero sequence voltage arises between wires and ground (lateral asymmetry). Arrows in Fig. 4.12,b show the paths of zero sequence currents circulation under the conditions to be considered. Windings of transformers, autotransformer and other circuit elements have ordinal numbers which are kept to mark elements of equivalent circuit for zero sequence (Fig. 4.12,c). As there is a path for zero sequence currents in the average voltage circuit of autotransformer it entered with its full equivalent circuit. Zero sequence current circulation in winding  $I2$  of transformer  $T3$  is provided by grounded neutral of load. This transformer is supposed to be three-core type that's why its impedance of zero sequence magnetizing has been taken into consideration. As for another transformer and autotransformer no data concerning their design are practically needed as they have delta-connected windings.

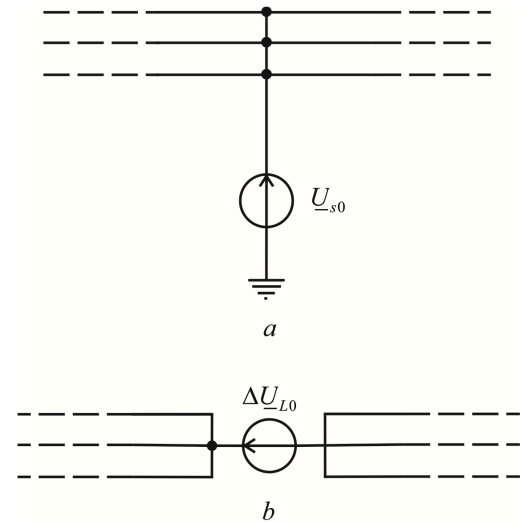


Fig. 4.11. Switching of zero sequence voltage source: a – under lateral asymmetry; b – under longitudinal asymmetry

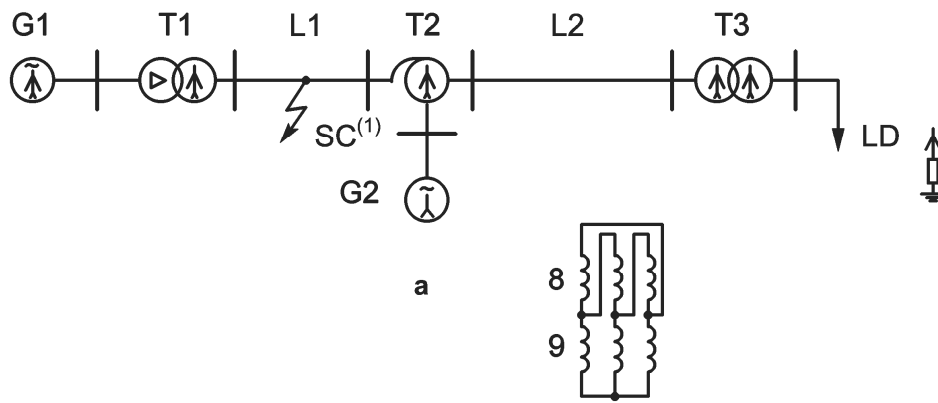


Fig. 4.12,a An example of equivalent circuit for zero sequence under lateral asymmetry: a - design circuit; b - equivalent circuit three-phase representation; c - equivalent circuit of zero sequence

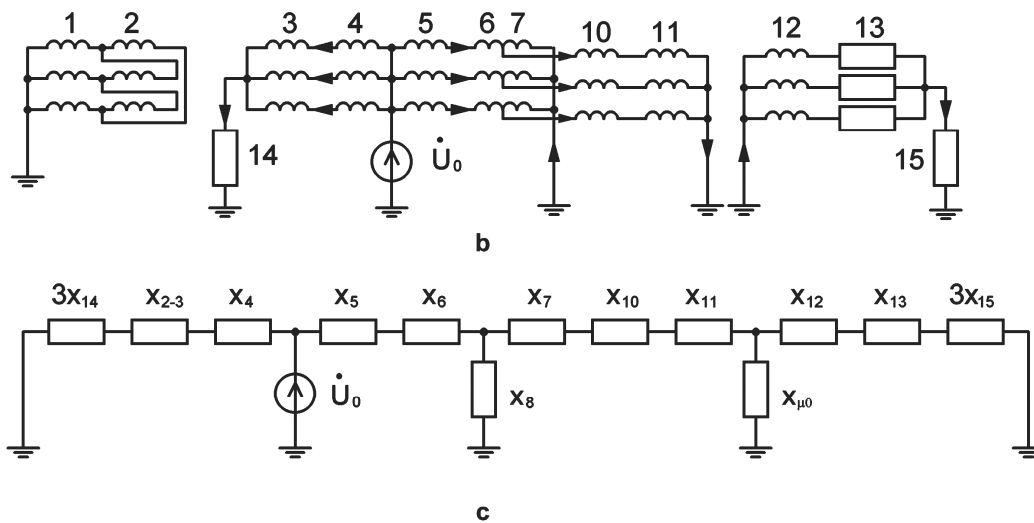


Fig. 4.12,b,c

If it is supposed that zero sequence voltage is applied in the same point in line wire branch, the equivalent circuit for zero sequence will stay to be the same but its resulting impedance will be absolutely different one.

As mutual inductance of zero sequence between parallel circuits of overhead line can influence considerably, it has to be accounted while drawing up equivalent circuit for zero sequence by introduction of correspondent equivalent circuits.

#### 4.7. Resulting electromotive forces and impedances of equivalent circuits for separate sequences

While calculating modes with asymmetric faults the resulting impedances of equivalent circuits for the sequences relative the points in which asymmetry has arisen are determined. Transformations in equivalent circuits necessary to do that are performed according to abovementioned recommendations. Besides, it is necessary to mean fundamental difference for equivalent circuits conversation under lateral asymmetry and longitudinal one. That difference can be shown in the specific circuit (Fig. 4.13,a), which elements are numbered in brackets and their numbers are used to mark correspondent elements in equivalent circuits for separate sequences.

Under lateral asymmetry in the point *M* equivalent circuit for positive sequence looks like that shown in Fig. 4.13,b. Elements *1* and *2*, and *5* and *6* series connected are marked with numbers *8* and *9* correspondingly. To determine resulting electromotive forces and impedances relatively to point *M* it is enough to replace the branch *9* from  $\dot{E} = 0$ , and the branch obtained by taking the sum

of element 8, and elements 3, 4 connected in parallel, and having electromotive force  $\dot{E}_1$  with one equivalent (Fig. 4.13,c). Equivalent circuit for negative sequence and its transformation are analogous excluding absence of electromotive forces of sources in it. Equivalent circuit for zero sequence (Fig. 4.13,d) is also easily transformed into equivalent one by series and parallel addition of branch impedances.

Suppose longitudinal asymmetry takes place in the point  $M$ . In this case positive sequence voltage should be introduced at the circuit fault of element 4 (Fig. 4.13, e). To determine resulting electromotive forces and impedances of circuit relatively point  $M$  it is necessary, in this case to add series element 8 and 9. Then the obtained branch 10 with electromotive force  $\dot{E}_1$  and branch 3 (Fig. 4.13, f) are to be replaced with equivalent one to get unknown resulting electromotive force relatively to point  $M$ , and to obtain resulting impedance relatively to the same point it is enough to add impedance of element 4 to impedance of obtained equivalent branch. Equivalent circuit for negative sequence is analogous to that one shown in Fig. 4.13,e. It hasn't only the electromotive force of the source. It's resulting impedance is found similarly.

Two-chain line is introduced in the equivalent circuit for zero sequence (Fig. 4.13,h) by its three-ray equivalent circuit with elements 11-13 to take into account mutual inductance between circuits being under different conditions now. To determine here resulting impedance of the element  $II$  it is necessary to add in parallel way its impedance to a sum of impedances of elements 2, 13, 5 and 7 (the latter have tripled values of impedance), and element 12 impedance add to the sum obtained.

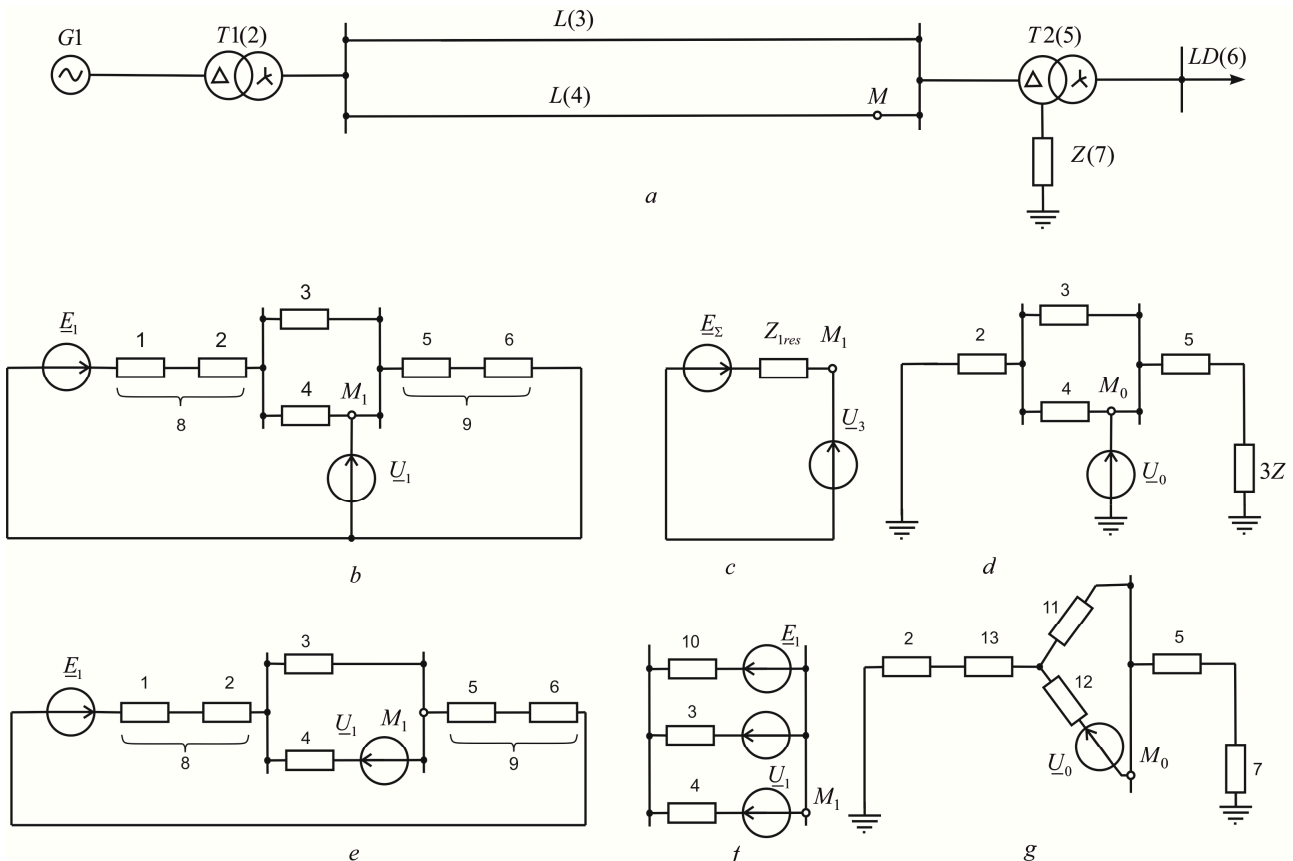


Fig. 4.13. An example of equivalent circuits of separate sequences and determinations of resulting electromotive forces, and impedances: a – design circuit; b-d- lateral asymmetry in the point  $M$ ; (e-g)- longitudinal asymmetry in the point  $M$

Relations between values of resulting impedances of the same sequence under lateral- and longitudinal asymmetries in one and the same point can be rather different depending on type, point of asymmetry and other factors.

**Test questions**

1. What are the main advantages of symmetrical components method? Why is modes calculation with asymmetric faults performed on one special phase?
2. What is the sense of key items of symmetric components method?
3. What is calculation of modes with asymmetrical faults by the method of symmetrical components?
4. In what way is the system of asymmetrical phasor divided into three symmetrical phasor systems?
5. How is it possible to obtain asymmetrical phasor system on the basis of symmetrical phasor systems (of positive, negative, and zero sequence)?
6. What are impedances of positive, negative, and zero sequences of circuit different elements at the short-circuit operation?
7. Why does the negative sequence impedance of rotating machines differ from positive sequence impedance?
8. Why are impedance values of positive  $\underline{Z}_1$ , and zero  $\underline{Z}_2$  sequences different in general case for one and the same element of electric circuit?
9. In what way are impedances of zero sequence for two-and three-winding transformers and autotransformers determined?
10. What is the explanation for the fact that impedance per phase for zero sequence of three-bar transformer is not equal to impedance per phase for positive sequence  $\underline{Z}_1$ , but  $\underline{Z}_1 = \underline{Z}_2$  ( $\underline{Z}_2$  is impedance per phase for negative sequence)?
11. What is influence of the ground wire on the overhead transmission line zero sequence impedance?
12. In what way are equivalent circuits for different sequences are drawn up under asymmetric short circuits?
13. What are the peculiarities of equivalent circuit for zero sequence?
14. In what way are resulting impedances of equivalent circuits for different sequences determined?

**Topics for essay**

1. Application of symmetrical components method for analysis and calculation of asymmetrical short circuits.
2. Positive, negative, and zero sequences impedances of different circuit components at short circuit operation.
3. Equivalent circuits drawing up for zero sequence and their peculiarities.
4. Equivalent circuit for zero sequence for parallel power transmission lines.

## **CHAPTER 5: LATERAL ASYMMETRY**

- 5.1. Starting statements
- 5.2. Single-phase short circuit
- 5.3. Two-phase short circuit
- 5.4. Two-phase short circuit to ground
- 5.5. Account of contact resistance at the point of short circuit
- 5.6. Equivalence rule for the current of positive sequence
- 5.7. Integrated equivalent circuits
- 5.8. Comparison of currents at different short circuits
- 5.9. Symmetrical current and voltage components transformation
- 5.10. Methods of asymmetrical short circuit calculation.

Test questions

Topics for essay



### 5.1. Starting statements

Calculating a single lateral asymmetry by method of symmetrical components of currents and voltages of a separate sequence for the special phase  $A$  according to the equation (4.9) can be expressed by the equation set

$$\left. \begin{aligned} \underline{U}_{s,A1} &= \underline{E}_{A\Sigma} - \underline{Z}_{1,\text{res}} \underline{I}_{s,A1}; \\ \underline{U}_{s,A2} &= 0 - \underline{Z}_{2,\text{res}} \underline{I}_{s,A2}; \\ \underline{U}_{s,A0} &= 0 - \underline{Z}_{0,\text{res}} \underline{I}_{s,A0}. \end{aligned} \right\} \quad (5.1)$$

The equation set has six unknown variables, and to determine them 3 additional equations have to be made up on a basis of boundary conditions for specific kind of asymmetry. ( $\underline{I}_{s,A1}$ ,  $\underline{I}_{s,A2}$ ,  $\underline{I}_{s,A0}$ ,  $\underline{U}_{s,A1}$ ,  $\underline{U}_{s,A2}$ ,  $\underline{U}_{s,A0}$ ).

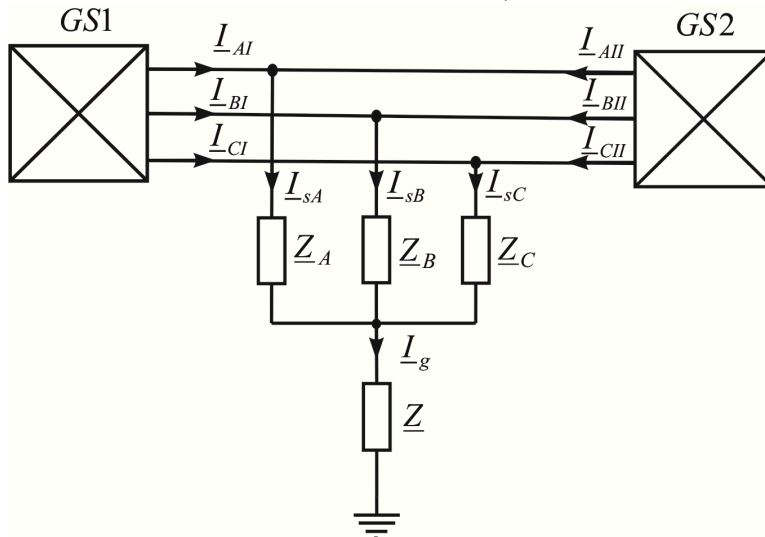


Fig. 5.1. Design circuit for single lateral asymmetry of the three-phase voltage system

The design circuit for a single lateral asymmetry can be generally expressed at any point of a three-phase system as connection of unequal impedances (Fig. 5.1). When phases  $A$ ,  $B$  and  $C$  are closed through impedances  $\underline{Z}_A$ ,  $\underline{Z}_B$ ,  $\underline{Z}_C$  and connected to ground through common impedance  $\underline{Z}$  we can get asymmetrical short circuit of any kind considering a part of impedances to be equal to zero or infinity. Short circuit occurs in the branch with zero impedance. Currents and voltages of the branches are ones at the points of a

short circuit interrelated by matrix equation describing boundary conditions

$$\underline{\vec{U}}_s = \underline{Z} \cdot \underline{\vec{I}}_s \quad (5.2)$$

where  $\underline{\vec{U}}_s = |\underline{U}_A \quad \underline{U}_B \quad \underline{U}_C|^T$ ;  $\underline{\vec{I}}_s = |\underline{I}_A \quad \underline{I}_B \quad \underline{I}_C|^T$  is columns vectors of voltages and currents at the points of a short circuit;  $\underline{Z}$  - is matrix of impedances

$$\underline{Z} = \begin{vmatrix} \underline{Z}_A + \underline{Z} & \underline{Z} & \underline{Z} \\ \underline{Z} & \underline{Z}_B + \underline{Z} & \underline{Z} \\ \underline{Z} & \underline{Z} & \underline{Z}_C + \underline{Z} \end{vmatrix} \quad (5.3)$$

Boundary conditions (5.2) on the basis of (4.6) and (4.7) can be expressed by symmetrical components

$$S \cdot \underline{\vec{U}}_{s,S} = S \cdot \underline{Z} \cdot \underline{\vec{I}}_{s,S} \quad (5.4)$$

taking into account that  $\underline{\vec{U}}_{s,S} = S^{-1} \cdot \underline{Z} \cdot S \cdot \underline{\vec{I}}_{s,S} = \underline{Z}_S \cdot \underline{\vec{I}}_{s,S}$

where  $\underline{Z}_S = S^{-1} \cdot \underline{Z} \cdot S$ .

Solving equations (5.1) simultaneously with the obtained equation set (5.4), we can determine currents and voltages of separate sequences at the point of a short circuit. This method enables obtaining general solution that is a source for all particular solutions. But general solution results in bulky expressions. Therefore it is more single and obvious to make solution for each kind of lateral asymmetry using boundary conditions only for this kind of asymmetry.

Let us consider principal kinds of asymmetrical short circuits: one-phase, two-phase and two-phase to ground. Besides the assumptions given in 2.4, we additionally consider.

- “metallic” short circuit takes place in every case (with consideration of contact resistances of electric arc, etc.) causing in extra complications as impedance  $\underline{Z}$  (Fig. 5.1) is equal to zero;
- equivalent circuits for separate sequences are equivalent relating to the point of a short circuit (resultant  $\dot{E}_{A\Sigma}$  and resultant impedances  $\underline{Z}_{1,\text{res}}$ ,  $\underline{Z}_{2,\text{res}}$ ,  $\underline{Z}_{0,\text{res}}$  have been found);
- phase  $A$  is the special phase when determining boundary conditions;
- direction to the point of a short circuit is taken as a positive direction (for phase currents and their symmetrical components as well);
- to simplify expressions the index of the short circuit kind is kept only in boundary conditions and final results.

Phase voltages and currents according to (4.4) are determined by the following expressions through symmetrical components of the special phase as

$$\left. \begin{aligned} \underline{U}_{sA} &= \underline{U}_{sA1} + \underline{U}_{sA2} + \underline{U}_{sA0}; \\ \underline{U}_{sB} &= a^2 \underline{U}_{sA1} + a \underline{U}_{sA2} + \underline{U}_{sA0}; \\ \underline{U}_{sC} &= a \underline{U}_{sA1} + a^2 \underline{U}_{sA2} + \underline{U}_{sA0}; \end{aligned} \right\} \quad (5.5)$$

$$\left. \begin{aligned} \underline{I}_{sA} &= \underline{I}_{sA1} + \underline{I}_{sA2} + \underline{I}_{sA0}; \\ \underline{I}_{sB} &= a^2 \underline{I}_{sA1} + a \underline{I}_{sA2} + \underline{I}_{sA0}; \\ \underline{I}_{sC} &= a \underline{I}_{sA1} + a^2 \underline{I}_{sA2} + \underline{I}_{sA0}; \end{aligned} \right\} \quad (5.6)$$

where symmetrical components of currents and voltages for phase  $B$  and  $C$  are determined using turning operator  $a$ .

## 5.2. Single-phase short circuit

Boundary condition of a single-phase short circuit can be calculated using design circuit (Fig. 5.2, a). It is deduced from Fig. 5. 1 considering impedance values  $\underline{Z}_A$  is equal to zero and  $\underline{Z}_B$  and  $\underline{Z}_C$  - are equal to infinity (Fig. 5.1). Let's write down boundary conditions for phase voltages and currents as

$$\left. \begin{aligned} \underline{U}_{sA}^{(1)} &= 0; \\ \underline{I}_{sB}^{(1)} &= 0; \\ \underline{I}_{sC}^{(1)} &= 0. \end{aligned} \right\} \quad (5.7)$$

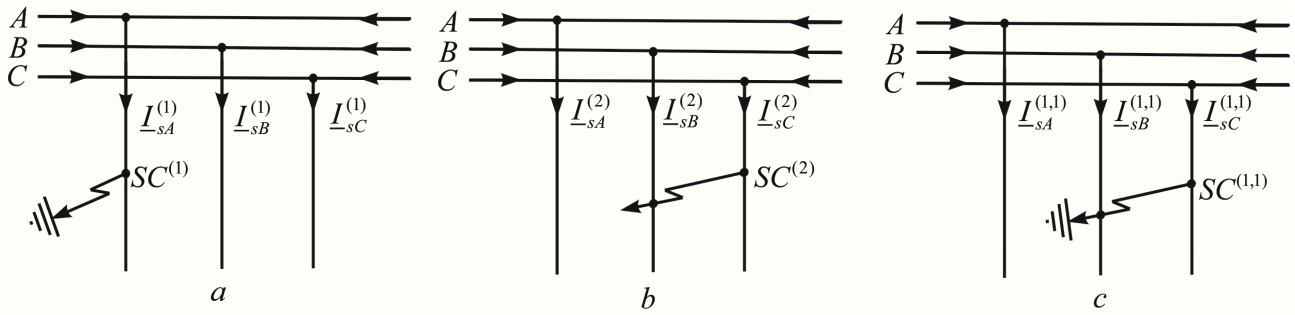


Fig. 5.2. Design circuits of asymmetrical short circuit: a – single-phase; b – two-phase; c – two-phase to ground

Transform (5.7) using symmetrical components of the special phase  $A$  according to equations (5.5) and (5.6)

$$\left. \begin{aligned} \underline{U}_{sA} &= \underline{U}_{sA1} + \underline{U}_{sA2} + \underline{U}_{sA0} = 0; \\ \underline{I}_{sB} &= a^2 \underline{I}_{sA1} + a \underline{I}_{sA2} + \underline{I}_{sA0} = 0; \\ \underline{I}_{sC} &= a \underline{I}_{sA1} + a^2 \underline{I}_{sA2} + \underline{I}_{sA0} = 0. \end{aligned} \right\} \quad (5.8)$$

Solving equations (5.1) simultaneously with the obtained equations (5.8) we determine currents and voltages of the sequences. Substituting  $\underline{U}_{sA1}$ ,  $\underline{U}_{sA2}$ ,  $\underline{U}_{sA0}$  from (5.1) into equations (5.8) for  $\underline{U}_{sA}$ . As a result a system of three equations of currents of separate sequences is obtained

$$\left. \begin{aligned} \underline{Z}_{1\text{res}} \underline{I}_{sA1} + \underline{Z}_{2\text{res}} \underline{I}_{sA2} + \underline{Z}_{0\text{res}} \underline{I}_{sA0} &= \underline{E}_{A\Sigma}; \\ a^2 \underline{I}_{sA1} + a \underline{I}_{sA2} + \underline{I}_{sA0} &= 0; \\ a \underline{I}_{sA1} + a^2 \underline{I}_{sA2} + \underline{I}_{sA0} &= 0. \end{aligned} \right\} \quad (5.9)$$

According to Cramer's formula sequence currents are equal to:

$$\underline{I}_{sA1} = \Delta_1 / \Delta; \quad \underline{I}_{sA2} = \Delta_2 / \Delta; \quad \underline{I}_{sA0} = \Delta_0 / \Delta.$$

Here  $\Delta$  - is a determinant of the equation set (5.9)

$$\Delta = \begin{vmatrix} \underline{Z}_{1\text{res}} & \underline{Z}_{2\text{res}} & \underline{Z}_{0\text{res}} \\ a^2 & a & 1 \\ a & a^2 & 1 \end{vmatrix} = (a - a^2)(\underline{Z}_{1\text{res}} + \underline{Z}_{2\text{res}} + \underline{Z}_{0\text{res}});$$

$\Delta_1, \Delta_2, \Delta_0$  - are complementary determinants obtained by substitution of  $i$ -column of a determinant by the column of absolute terms of a system?

$$\Delta_1 = \Delta_2 = \Delta_0 = \underline{E}_{A\Sigma}(a - a^2).$$

Then

$$\underline{I}_{sA1}^{(1)} = \underline{I}_{sA2}^{(1)} = \underline{I}_{sA0}^{(1)} = \underline{E}_{A\Sigma} / (\underline{Z}_{1\text{res}} + \underline{Z}_{2\text{res}} + \underline{Z}_{0\text{res}}). \quad (5.10)$$

We can find symmetrical voltage components of the special phase  $A$  at the point of a short circuit by equations (5.1)

$$\left. \begin{aligned} \underline{U}_{sA1}^{(1)} &= \dot{E}_{A\Sigma} - \underline{Z}_{1\text{res}} \underline{I}_{sA1}^{(1)} = (\underline{Z}_{2\text{res}} + \underline{Z}_{0\text{res}}) \underline{I}_{sA1}^{(1)}; \\ \underline{U}_{sA2}^{(1)} &= -\underline{Z}_{2\text{res}} \underline{I}_{sA2}^{(1)}; \\ \underline{U}_{sA0}^{(1)} &= -\underline{Z}_{0\text{res}} \underline{I}_{sA0}^{(1)}. \end{aligned} \right\} \quad (5.11)$$

Currents and voltages of phases at the point of a short circuit can be calculated analytically using equations (5.6) and (5.5):

$$\left. \begin{aligned} \underline{I}_{sA}^{(1)} &= 3 \underline{I}_{sA1}^{(1)}; \\ \underline{I}_{sB}^{(1)} &= 0; \\ \underline{I}_{sC}^{(1)} &= 0; \end{aligned} \right\} \quad (5.12)$$

$$\left. \begin{aligned} \underline{U}_{sA}^{(1)} &= 0; \\ \underline{U}_{sB}^{(1)} &= \left[ (a^2 - a) \underline{Z}_{2\text{res}} + (a^2 - 1) \underline{Z}_{0\text{res}} \right] \cdot \underline{I}_{sA1}^{(1)}; \\ \underline{U}_{sC}^{(1)} &= \left[ (a - a^2) \underline{Z}_{2\text{res}} + (1 - a^2) \underline{Z}_{0\text{res}} \right] \cdot \underline{I}_{sA1}^{(1)}. \end{aligned} \right\} \quad (5.13)$$

Coefficient of proportionality between the phase current and the positive sequence current for the special phase  $A$

$$m^{(1)} = \underline{I}_{sA}^{(1)} / \underline{I}_{sA1}^{(1)} = 3. \quad (5.14)$$

Phase currents and voltages after calculation of the symmetrical components of the special phase  $A$  can be found graphically by plotting on a scale corresponding phasor diagrams (index “k” should be omitted at all phasors when plotting) Let’s plot a phasor diagram of current

$$\underline{U}_{A\Sigma} = jU_{A\Sigma}, \quad \underline{Z}_{1\text{res}} = jx_{1\text{res}}, \quad \underline{Z}_{2\text{res}} = jx_{2\text{res}}, \quad \underline{Z}_{0\text{res}} = jx_{0\text{res}}.$$

Then  $\underline{I}_{sA1} = I_{sA1}$ ,  $\underline{U}_{sA1} = jU_{sA1}$ . On the real axis of the complex plane three equal parallel phasors  $\underline{I}_{sA1}$ ,  $\underline{I}_{sA2}$ ,  $\underline{I}_{sA0}$  are plotted. Summing up phase phasors of separate sequences according to (4.3), the phasor diagram of the phase currents of voltages is obtained (Fig 5.3,b). The angle  $\theta_U$  between voltages of undamaged phase depends on  $x_{2\text{res}}$  and  $x_{0\text{res}}$  and can vary within limits  $\pi/3 \leq \theta_U \leq \pi$ . If  $x_{0\text{res}} \rightarrow 0$ , then  $\theta_U \rightarrow 180^\circ$ , when  $x_{0\text{res}} = \infty$ , the angle  $\theta_U = 60^\circ$ .

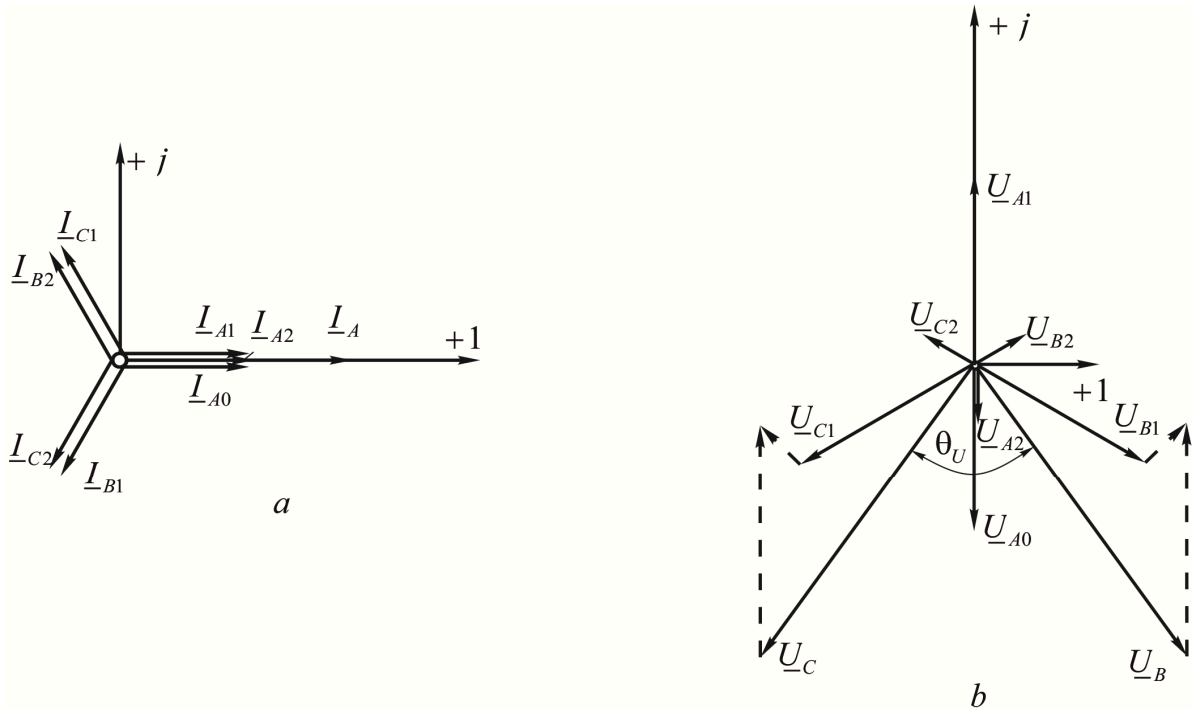


Fig. 5.3. The phasors diagrams of phase currents and voltages and their symmetrical components at the point of a single phase short circuit

Under single phase ground short difference between phase voltage and phase voltage to zero point of phasor system should be distinguished. These two voltages differ by the component of a zero sequence phase voltage. If there is no zero sequence voltage, the phase voltage to zero point of phasor system is the phase voltage to ground.

### 5.3. Two-phase short circuit

We can get two-phase short circuit design circuit SC<sup>(2)</sup> (Fig. 5.2,b) from Fig. 5.1 when impedance  $\underline{Z}_A$  is equal to infinity and  $\underline{Z}_B$  and  $\underline{Z}_C$  are equal to zero. Boundary conditions in phase quantities are:

$$\left. \begin{aligned} \underline{I}_{sA}^{(2)} &= 0; \\ \underline{I}_{sB}^{(2)} &= -\underline{I}_{sC}^{(2)}; \\ \underline{U}_{sB}^{(2)} &= \underline{U}_{sC}^{(2)}. \end{aligned} \right\} \quad (5.15)$$

Boundary conditions being transformed with use of symmetrical components of the special phase  $A$  according to equations (5.5) and (5.6) lead to:

$$\left. \begin{aligned} \underline{I}_{sA} &= \underline{I}_{sA1} + \underline{I}_{sA2} + \underline{I}_{sA0} = 0; \\ \underline{I}_{sB} + \underline{I}_{sC} &= (a^2 + a)\underline{I}_{sA1} + (a + a^2)\underline{I}_{sA2} + 2\underline{I}_{sA0} = 0; \\ \underline{U}_{sB} - \underline{U}_{sC} &= (a^2 - a)\underline{U}_{sA1} + (a - a^2)\underline{U}_{sA2} = 0. \end{aligned} \right\} \quad (5.16)$$

Solving together sets of equations (5.1) and (5.6), we find currents and voltages of the sequences for the special phase  $A$  at the point of short circuit. For that we substitute expressions for  $\underline{U}_{sA1}$  and  $\underline{U}_{sA2}$  for equations (5.1) into equation (5.16)

$$\left. \begin{aligned} \underline{I}_{sA1} + \underline{I}_{sA2} + \underline{I}_{sA0} &= 0; \\ (a^2 + a)\underline{I}_{sA1} + (a + a^2)\underline{I}_{sA2} + 2\underline{I}_{sA0} &= 0; \\ (a^2 - a)\underline{Z}_{1res}\underline{I}_{sA1} + (a - a^2)\underline{Z}_{2res}\underline{I}_{sA2} &= (a^2 - a)\underline{E}_{A\Sigma}. \end{aligned} \right\} \quad (5.17)$$

The solution of the set of equations (5.17) for the currents

$$\underline{I}_{sA1} = \Delta_1 / \Delta; \quad \underline{I}_{sA2} = \Delta_2 / \Delta; \quad \underline{I}_{sA0} = \Delta_0 / \Delta,$$

where

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ a^2 + a & a + a^2 & 2 \\ (a^2 - a)\underline{Z}_{1res} & (a - a^2)\underline{Z}_{2res} & 0 \end{vmatrix} = 3(a^2 - a)(\underline{Z}_{1res} + \underline{Z}_{2res});$$

$$\Delta_1 = 3(a^2 - a)\underline{E}_{A\Sigma}; \quad \Delta_2 = -\Delta_1; \quad \Delta_0 = 0.$$

Then

$$\left. \begin{aligned} \underline{I}_{sA1}^{(2)} &= \underline{E}_{A\Sigma} / (\underline{Z}_{1res} + \underline{Z}_{2res}); \\ \underline{I}_{sA2}^{(2)} &= -\underline{I}_{sA1}^{(2)} = -\underline{E}_{A\Sigma} / (\underline{Z}_{1res} + \underline{Z}_{2res}); \\ \underline{I}_{sA0}^{(2)} &= 0. \end{aligned} \right\} \quad (5.18)$$

Voltages of separate sequences for the special phase  $A$  at the point of a short circuit are determined by (5.1)

$$\left. \begin{aligned} \underline{U}_{sA1}^{(2)} &= \underline{Z}_{1res}\underline{I}_{sA1}^{(2)}; \\ \underline{U}_{sA2}^{(2)} &= -\underline{Z}_{2res}\underline{I}_{sA2}^{(2)} = \underline{Z}_{2res}\underline{I}_{sA1}^{(2)} = \underline{U}_{sA1}^{(2)}; \\ \underline{U}_{sA0}^{(2)} &= 0. \end{aligned} \right\} \quad (5.19)$$

Determining voltage  $\underline{U}_{sA0}^{(2)}$  it should be considered that voltage  $\underline{U}_{sA0}^{(2)}$  at  $\underline{I}_{sA0}^{(2)} = 0$ , on basis of (5.1), is equal to zero in the electric supply systems with grounded neutral ( $x_{0res}$  has a finite value) and voltage  $\underline{U}_{sA0}^{(2)} = -\infty$ . In the networks with insulated neutral  $\underline{Z}_{0res} = \infty$ . One of voltage equations is omitted.

Currents and voltages of phases considering (5.5), (5.6) and (5.15):

$$\left. \begin{aligned} \underline{I}_{sA}^{(2)} &= 0; \\ \underline{I}_{sB}^{(2)} &= a^2 \underline{I}_{sA1}^{(2)} + a \underline{I}_{sA2}^{(2)} = (a^2 - a) \underline{I}_{sA1}^{(2)} = -j\sqrt{3} \underline{I}_{sA1}^{(2)}; \\ \underline{I}_{sC}^{(2)} &= a \underline{I}_{sA1}^{(2)} + a^2 \underline{I}_{sA2}^{(2)} = (a - a^2) \underline{I}_{sA1}^{(2)} = j\sqrt{3} \underline{I}_{sA1}^{(2)} = -\underline{I}_{sB}^{(2)}; \end{aligned} \right\} \quad (5.20)$$

$$\left. \begin{aligned} \underline{U}_{sA}^{(2)} &= \underline{U}_{sA1}^{(2)} + \underline{U}_{sA2}^{(2)} = 2\underline{U}_{sA1}^{(2)} = 2\underline{Z}_{2\text{res}} \underline{I}_{sA1}^{(2)}; \\ \underline{U}_{sB}^{(2)} &= a^2 \underline{U}_{sA1}^{(2)} + a \underline{U}_{sA2}^{(2)} = -\underline{U}_{sA1}^{(2)}; \\ \underline{U}_{sC}^{(2)} &= a \underline{U}_{sA1}^{(2)} + a^2 \underline{U}_{sA2}^{(2)} = -\underline{U}_{sA1}^{(2)}. \end{aligned} \right\} \quad (5.21)$$

Coefficient of proportionality of the damaged phase current and positive sequence current of the special phase  $A$  at the point of a short circuit is equal to

$$m^{(2)} = \left| \underline{I}_{sB}^{(2)} / \underline{I}_{sA1}^{(2)} \right| = \sqrt{3}. \quad (5.22)$$

Phasor diagrams of voltages and currents at the point of two-phase short circuit are given in Fig. 5.4,b. They are plotted for the conditions when on electric circuit is made equivalent by inductive reactances and electromotive force  $\underline{E}_{A\Sigma}$  is directed along the imaginary axis.

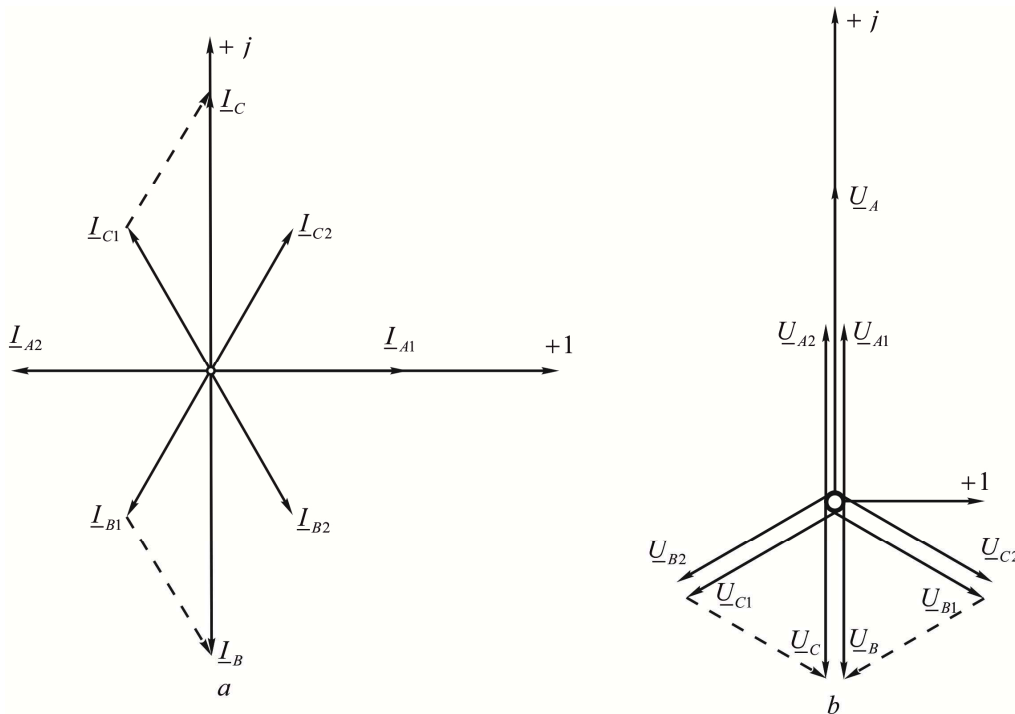


Fig. 5.4. Phasor diagrams of voltages (a) and currents (b) and their symmetrical components at the point of two-phase short circuit

#### 5.4. Two-phase short circuit to ground

Design circuit of a two-phase ground short circuit (Fig.5.2) can be received when impedance value  $\underline{Z}_A$  is equal to infinity and  $\underline{Z}_B$  and  $\underline{Z}_C$  are equal to zero (Fig. 5.1). The following boundary conditions in phase values correspond to.

$$\left. \begin{aligned} \underline{I}_{sA}^{(1,1)} &= 0; \\ \underline{U}_{sB}^{(1,1)} &= \underline{U}_{sC}^{(1,1)} = 0. \end{aligned} \right\} \quad (5.23)$$

Express them by symmetrical components of the special phase  $A$ :

$$\left. \begin{aligned} \underline{I}_{sA} &= \underline{I}_{sA1} + \underline{I}_{sA2} + \underline{I}_{sA0} = 0; \\ \underline{U}_{sB} &= a^2 \underline{U}_{sA1} + a \underline{U}_{sA2} + \underline{U}_{sA0} = 0; \\ \underline{U}_{sC} &= a \underline{U}_{sA1} + a^2 \underline{U}_{sA2} + \underline{U}_{sA0} = 0. \end{aligned} \right\} \quad (5.24)$$

From equations (5.1) we determine  $\underline{I}_{sA1}, \underline{I}_{sA2}, \underline{I}_{sA0}$  and substitute them into (5.24)

$$\left. \begin{aligned} \underline{U}_{sA1}/\underline{Z}_{1res} + \underline{U}_{sA2}/\underline{Z}_{2res} + \underline{U}_{sA0}/\underline{Z}_{0res} &= \underline{E}_{A\Sigma}/\underline{Z}_{1res}; \\ a^2 \underline{U}_{sA1} + a \underline{U}_{sA2} + \underline{U}_{sA0} &= 0; \\ a \underline{U}_{sA1} + a^2 \underline{U}_{sA2} + \underline{U}_{sA0} &= 0. \end{aligned} \right\} \quad (5.25)$$

Solution the set (5.25) for the voltages of positive, negative and zero sequence at the point of short circuit gives

$$\underline{U}_{sA1} = \Delta_1/\Delta; \quad \underline{U}_{sA2} = \Delta_2/\Delta; \quad \underline{U}_{sA0} = \Delta_0/\Delta,$$

where

$$\begin{aligned} \Delta &= \begin{vmatrix} 1/\underline{Z}_{1res} & 1/\underline{Z}_{2res} & 1/\underline{Z}_{0res} \\ a^2 & a & 1 \\ a & a^2 & 1 \end{vmatrix} = \\ &= (a - a^2) \left( \frac{1}{\underline{Z}_{1res}} + \frac{1}{\underline{Z}_{2res}} + \frac{1}{\underline{Z}_{0res}} \right); \\ \Delta_1 &= \Delta_2 = \Delta_0 = (a - a^2) \underline{E}_{A\Sigma}/\underline{Z}_{1res}. \end{aligned}$$

Then

$$\underline{U}_{sA1}^{(1,1)} = \underline{U}_{sA2}^{(1,1)} = \underline{U}_{sA0}^{(1,1)} = \underline{E}_{A\Sigma} \frac{1/\underline{Z}_{1res}}{1/\underline{Z}_{1res} + 1/\underline{Z}_{2res} + 1/\underline{Z}_{0res}}. \quad (5.26)$$



By substitution (5.26) into (5.1),

$$\left. \begin{aligned} \underline{I}_{sA1}^{(1,1)} &= \underline{E}_{A\Sigma} / \left[ \underline{Z}_{1res} + \underline{Z}_{2res} \underline{Z}_{0res} / (\underline{Z}_{2res} + \underline{Z}_{0res}) \right]; \\ \underline{I}_{sA2}^{(1,1)} &= -\underline{I}_{sA1}^{(1,1)} \underline{Z}_{0res} / (\underline{Z}_{2res} + \underline{Z}_{0res}); \\ \underline{I}_{sA0}^{(1,1)} &= -\underline{I}_{sA1}^{(1,1)} \underline{Z}_{2res} / (\underline{Z}_{2res} + \underline{Z}_{0res}). \end{aligned} \right\} \quad (5.27)$$

using equation (5.6) and (5.5), determination of phase currents and voltages is made

$$\left. \begin{aligned} \underline{I}_{sA}^{(1,1)} &= 0; \\ \underline{I}_{sB}^{(1,1)} &= \underline{I}_{sA1}^{(1,1)} \left[ a^2 - (\underline{Z}_{2res} + a\underline{Z}_{0res}) / (\underline{Z}_{2res} + \underline{Z}_{0res}) \right]; \\ \underline{I}_{sC}^{(1,1)} &= \underline{I}_{sA1}^{(1,1)} \left[ a - (\underline{Z}_{2res} + a^2 \underline{Z}_{0res}) / (\underline{Z}_{2res} + \underline{Z}_{0res}) \right]; \end{aligned} \right\} \quad (5.28)$$

$$\left. \begin{aligned} \underline{U}_{sA}^{(1,1)} &= 3\underline{I}_{sA1} \underline{Z}_{2res} \underline{Z}_{0res} / (\underline{Z}_{2res} + \underline{Z}_{0res}); \\ \underline{U}_{sB}^{(1,1)} &= 0; \\ \underline{U}_{sC}^{(1,1)} &= 0. \end{aligned} \right\} \quad (5.29)$$

The ground current

$$\underline{I}_g = \underline{I}_{sB} + \underline{I}_{sC} = 3\underline{I}_{sA0} \quad (5.30)$$

Coefficient of proportionality of the damaged phase current and the positive sequence current at the point of short circuit is evaluated by expressions

$$m^{(1,1)} = \left| a^2 - (\underline{Z}_{2res} + a\underline{Z}_{0res}) / \underline{Z}_{2res} + \underline{Z}_{0res} \right|$$

For the condition that resistances are equal to zero, the value

$$m^{(1,1)} = \sqrt{3} \sqrt{1 - x_{2res} x_{0res} / (x_{2res} + x_{0res})^2} \quad (5.31)$$

The value of  $m^{(1,1)}$  is within the limits  $1,5 \leq m^{(1,1)} \leq \sqrt{3}$  depending on  $x_{2res}$  and  $x_{0res}$  values relation.

In Fig. 5.5 phasor diagrams of currents and voltages at the point of two-pole short circuit to the ground are shown. They have been plotted for the same conditions as phasor diagrams in Fig. 5.3 and 5.4. The phase shift between damaged phases current  $\theta_I$  depends on ratio of impedance values  $\underline{Z}_{1res}$  and  $\underline{Z}_{0res}$  and can vary within limits  $60^\circ < \theta_I < 180^\circ$ . The upper boundary corresponds to the value  $\underline{Z}_{0res} = \infty$ , the lower - to  $\underline{Z}_0 \rightarrow 0$ , and for the condition  $\underline{Z}_{2res} = \underline{Z}_{0res}$  the angle  $\theta_I = 120^\circ$ . As well as at single-phase short circuit, zero point of the phasor system is shifted in regard to ground at the voltage of zero sequence.

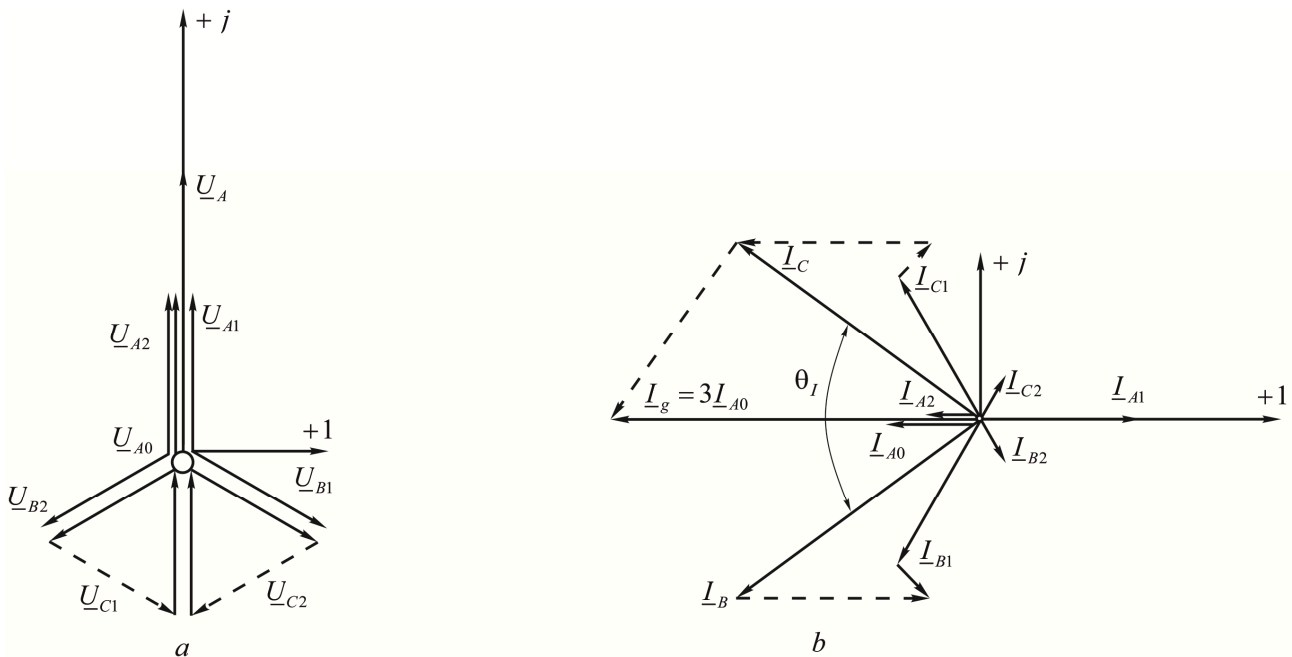


Fig. 5.5. Phasors diagrams of phase voltages (a) and currents (b) and their symmetrical components at the point of short circuit to ground

Therefore we should distinguish phase voltage related to ground and phase voltage related to zero point.

### 5.5. Account of contact resistance at the point of short circuit.

In distribution networks of enterprises account of contact resistance at the point of short circuit is of a great importance. Contact resistance is formed at this point. It consists of the resistance of occurred arc and the resistances of elements of current path from one phase to another or from phases to ground. Arc occurs either from the very beginning of the fault as at insulation break-through or arc over, or in some time interval when the element caused the short circuit is broken down. At short circuit between phases contact resistance is defined mainly by the resistance of electric arc.

In some cases, contact resistance is so small that it can be neglected. It is obvious that the current at such short circuit is bigger than at short circuit when contact resistance is available. So when possible maximum values are to be found the worst case conditions are considered assuming the contact resistance at the point of short circuit equal to zero.

Let's analyze consideration of contact resistance at asymmetrical short circuits of different types. The contact resistance is assumed to be mainly determined by the resistance in a first approximation ( $r_{\text{arc}}$ ).

Let two-phase short circuit between phases  $B$  and  $C$  occurred through the resistance of arc  $r_{\text{arc}}$ . It can be considered as a metallic two-phase short circuit of the branch where phases have equal resistances  $r_{\text{arc}}/2$  (Fig. 5.6,a). By this procedure asymmetrical subcircuit has been reduced into symmetrical one to use a symmetrical components method. Introduction of resistance  $r_{\text{arc}}/2$  into the phase  $A$  doesn't change boundary conditions of examined short circuit as there is no branch current in this phase.

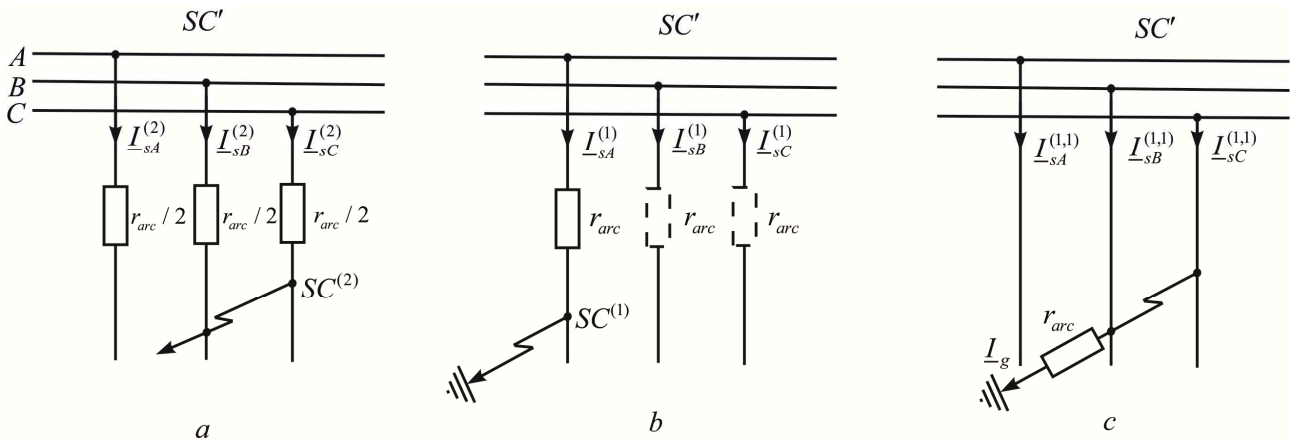


Fig. 5.6. Design circuit of asymmetrical short circuit with account of the contact resistance of arc: a-two-phase; b-single-phase; c-two-phase to ground

According to (5.18) and (5.19) for the point

$$\left. \begin{aligned} \underline{I}_{sA1}^{(2)} &= \underline{E}_{A\Sigma} / (\underline{Z}_{1res} + \underline{Z}_{2res} + r_{arc}); \\ \underline{I}_{sA2}^{(2)} &= -\underline{I}_{sA1}^{(2)}; \end{aligned} \right\} \quad (5.32)$$

$$\underline{U}_{sA1}^{(2)} = \underline{U}_{sA2}^{(2)} = \underline{I}_{sA1}^{(2)} (\underline{Z}_{2res} + r_{arc}). \quad (5.33)$$

By expressions (5.32) separate sequence currents and for the point  $SC^{(2)}$  can be determined. The voltages of separate sequences at the point  $SC^{(2)}$  are.

$$\left. \begin{aligned} \underline{U}_{s'A1}^{(2)} &= \underline{U}_{sA1}^{(2)} + \underline{I}_{sA1}^{(2)} \cdot r_{arc} / 2 = \underline{I}_{sA1}^{(2)} (\underline{Z}_{2res} + r_{arc}); \\ \underline{U}_{s'A2}^{(2)} &= \underline{U}_{sA2}^{(2)} + \underline{I}_{sA2}^{(2)} \cdot r_{arc} / 2 = \underline{I}_{sA1}^{(2)} \underline{Z}_{2res}. \end{aligned} \right\} \quad (5.34)$$

Consider the phase  $A$  closure to ground through the resistance of arc  $r_{arc}$  (Fig. 5.6, b). In order to keep the symmetry of a given circuit section. It is assumed that there are equal resistances in both other phases. It is true as according to boundary conditions for the fault currents  $\underline{I}_{sB}^{(1)} = 0$ ;  $\underline{I}_{sC}^{(1)} = 0$ .

The resulting impedance of every sequence has increased by  $r_{arc}$ . Therefore similarly to (5.10) currents of positive, negative and zero sequences at the point of a short circuit are:

$$\underline{I}_{sA1}^{(2)} = \underline{I}_{sA2}^{(2)} = \underline{I}_{sA0}^{(2)} = \underline{E}_{A\Sigma} / (\underline{Z}_{1res} + \underline{Z}_{2res} + \underline{Z}_{0res} + 3r_{arc}). \quad (5.35)$$

The subcircuit voltages of separate sequences at the point  $SC^{(1)}$  are

$$\left. \begin{aligned} \underline{U}_{sA1}^{(1)} &= \underline{I}_{sA1}^{(1)} (\underline{Z}_{2res} + \underline{Z}_{0res} + 2r_{arc}); \\ \underline{U}_{sA2}^{(1)} &= -\underline{I}_{sA2}^{(1)} (\underline{Z}_{2res} + r_{arc}); \\ \underline{U}_{sA0}^{(1)} &= -\underline{I}_{sA0}^{(1)} (\underline{Z}_{0res} + r_{arc}). \end{aligned} \right\} \quad (5.36)$$

The currents at the actual point of short circuit  $SC^{(1)}$  are determined by (5.35). The voltages of separate sequences at the point  $SC^{(1)}$  are determined by the second Kirchhoff's Law

$$\left. \begin{aligned} \underline{U}_{s'A_1}^{(1)} &= \underline{U}_{sA_1}^{(1)} + \underline{I}_{sA_1}^{(1)} r_{arc} = \underline{I}_{sA_1}^{(1)} (\underline{Z}_{2res} + \underline{Z}_{0res} + 3r_{arc}); \\ \underline{U}_{s'A_2}^{(1)} &= \underline{U}_{sA_2}^{(1)} + \underline{I}_{sA_2}^{(1)} r_{arc} = -\underline{I}_{sA_2}^{(1)} \underline{Z}_{2res}; \\ \underline{U}_{s'A_0}^{(1)} &= \underline{U}_{sA_0}^{(1)} + \underline{I}_{sA_0}^{(1)} r_{arc} = -\underline{I}_{sA_0}^{(1)} \underline{Z}_{0res}. \end{aligned} \right\} \quad (5.37)$$

Under the short circuit between phases  $B$  and  $C$  with simultaneous short circuit to ground through the resistance of arc  $r_{arc}$  (Fig. 5.6,b) the latter one will be present only in the circuit of zero sequence as its tripled value. So the currents of separate sequences at the place by analogy with (5.27) are:

$$\left. \begin{aligned} \underline{I}_{sA_1}^{(1,1)} &= \underline{E}_{A\Sigma}^{(1)} / \left[ \frac{\underline{Z}_{1res} + \underline{Z}_{2res} (\underline{Z}_{0res} + 3r_{arc})}{/(\underline{Z}_{2res} + \underline{Z}_{0res} + 3r_{arc})} \right]; \\ \underline{I}_{sA_2}^{(1,1)} &= -\underline{I}_{sA_1}^{(1,1)} \left[ (\underline{Z}_{0res} + 3r_{arc}) / (\underline{Z}_{2res} + \underline{Z}_{0res} + 3r_{arc}) \right]; \\ \underline{I}_{sA_0}^{(1,1)} &= -\underline{I}_{sA_1}^{(1,1)} \left[ \underline{Z}_{2res} / (\underline{Z}_{2res} + \underline{Z}_{0res} + 3r_{arc}) \right]. \end{aligned} \right\} \quad (5.38)$$

Symmetrical components of the voltages of positive, negative and zero sequences according to the solution of equation set (5.25), where  $r_{arc}$  is included only in the circuit of zero sequence, at the point  $SC^{(1,1)}$  are calculated by the expressions:

$$\left. \begin{aligned} \underline{U}_{sA_1}^{(1,1)} &= \underline{I}_{sA_1}^{(1,1)} \underline{Z}_{2res} (\underline{Z}_{0res} + 3r_{arc}) / \\ &\quad / (\underline{Z}_{2res} + \underline{Z}_{0res} + 3r_{arc}); \\ \underline{U}_{sA_2}^{(1,1)} &= -\underline{I}_{sA_2}^{(1,1)} \underline{Z}_{2res}; \\ \underline{U}_{sA_0}^{(1,1)} &= -\underline{I}_{sA_0}^{(1,1)} \underline{Z}_{0res}. \end{aligned} \right\} \quad (5.39)$$

The currents at the actual point of short circuit  $SC'^{(1,1)}$  are equal to the currents in the branches at the point  $SC^{(1,1)}$ . The voltages of positive and negative sequences are the same, as the resistance  $r_{arc}$  is not included in the equivalent circuits of positive and negative sequences. The voltage of zero sequence at the point  $SC'^{(1,1)}$  gets value

$$\underline{U}_{s'A_0}^{(1,1)} = \underline{U}_{sA_0}^{(1,1)} + \underline{I}_{sA_0}^{(1,1)} 3r_{arc}. \quad (5.40)$$

### 5.6. Equivalence rule for the current of positive sequence

Here the obtained expressions of symmetrical components of currents and voltages at the point of asymmetrical short circuit, tabulated in the Table 5.1 are analyzed. It is seen that the currents of negative and zero sequences  $\underline{I}_{sA_2}$  and  $\underline{I}_{sA_0}$ , total currents in the faulted phases  $\underline{I}_{sA}$ ,  $\underline{I}_{sB}$ ,  $\underline{I}_{sC}$  voltage of all sequences  $\underline{U}_{sA_1}$ ,  $\underline{U}_{sA_2}$ ,  $\underline{U}_{sA_0}$  and total voltages in phases are proportional to the positive sequence current at the point of short circuit.

Therefore the calculation of asymmetrical short circuit of any kind first of all comprises determination of positive sequence current at the point of short circuit.

The structure of expressions (5.10), (5.18) and (5.27) allows to write down in general from the positive sequence current of the phase  $A$  at asymmetrical short circuit of any kind using parameters of the equivalent circuit

$$\underline{I}_{sA1}^{(n)} = \underline{E}_{A\Sigma} / \left( \underline{Z}_{ires} + \underline{Z}_{\Delta}^{(n)} \right), \tag{5.41}$$

where index  $(n)$  labels a type of short circuit;  $\underline{Z}_{\Delta}^{(n)}$  - labels additional impedance depending on a short circuit types (Table 5.1).

Phase currents at the point of a short circuit are proportional to the positive sequence current of a particular phase  $A$  (5.12), (5.20), (5.28). Taking it into account, the magnitude of the special phase current at the point of asymmetrical short circuit in general form is determined by expression

$$\underline{I}_{F,A}^{(n)} = m^{(n)} \underline{I}_{sA1}^{(n)}, \tag{5.42}$$

where  $m^{(n)}$  - is a proportionality factor that can be found by data from the Table 5.1.

The generalized statement (5.42) allows to formulate *equivalence rule for the positive sequence current*: positive sequence current of asymmetrical short circuit of any time can be defined as the current under three-phase short circuit at the point, remoted from the actual point of short circuit for additional impedance  $\underline{Z}_{\Delta}^{(n)}$  for every kind of short circuit is determined by resulting impedances of negative and positive sequences relative to the point of short circuit

For voltage  $\underline{U}_{sA1}$  (see Table 5.1) is true

$$\underline{U}_{sA1}^{(n)} = \underline{Z}_{\Delta}^{(n)} \underline{I}_{sA1}^{(n)} \tag{5.43}$$

- that is consequence of the rule formulated above.

Current of three-phase short circuit can be calculated by the equivalence rule of positive sequence at  $m^{(3)} = 1$  and  $\underline{Z}_{\Delta}^{(3)} = 0$ .

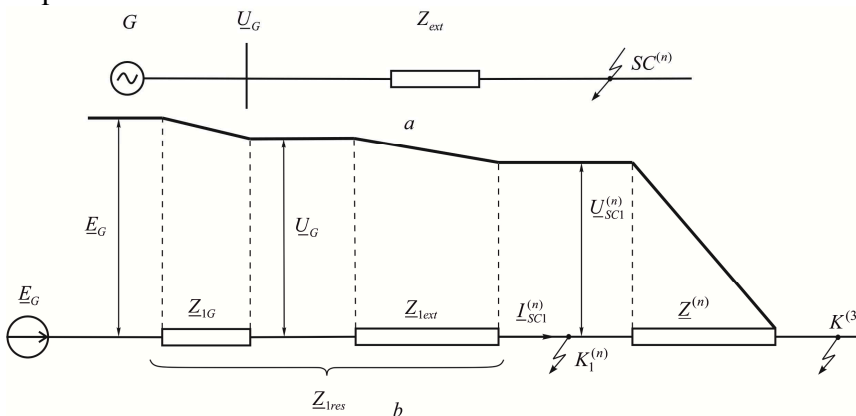


Fig . 5.7. To determination of asymmetrical fault current (a) through the current of conditional three-phase short circuit (b)

Fig. 5.7 illustrates the equivalence rule for positive sequence current: at equivalent three-phase short circuit at the point  $SC^{(3)}$  the current and voltage of positive sequence can be found at the point of a given asymmetrical short circuit.

The positive sequence current at the point of short circuit as well as currents of other sequences depends on element impedances of all

sequences of the analyzed circuit including the resistance of arc. Consider a transformer with either single-phase or double-phase short circuit to ground on its terminals. If the neutral of this transformer is grounded through a resistance it will influence currents of all sequences though currents of positive and negative sequences do not flow through this resistance. The equivalence rule for positive sequence current has the following meaning: the above expressions for determination the current of three-phase short circuit along with the methods of its calculation can be extended to asymmetrical types of short circuit.

So variation in time of the periodic component of a generator of the positive sequence current when damper winding is absent; with the account of automatic excitation control device (when thyristor system of excitation is used), so, at any value of  $(n)$ , asymmetrical short circuit behind the external inductive reactance  $x_{\text{ext}}$ , can be presented by expression:

$$I_{FIt}^{(n)} = I_{\text{ps1}}^{(n)} + \left( I_{1(0)}^{\prime(n)} - I_{\text{ps1}}^{(n)} \right) e^{-\frac{t}{T_d^{\prime(n)}}} \leq \frac{U_{\text{rated}}}{x_{1\text{ext}} + x_{\Delta}^{(n)}}. \quad (5.44)$$

here  $I_{\text{ps1}}^{(n)}$  - is the steady-state current of the positive sequence under maximum field current

$$I_{\text{ps1}}^{(n)} = E_{q,ps} / \left( x_d + x_{1\text{ext}} + x_{\Delta}^{(n)} \right); \quad (5.45)$$

$I_{1(0)}^{\prime(n)}$  - is the initial transient current of a positive sequence

$$I_{1(0)}^{\prime(n)} = E'_{q(0)} / \left( x'_d + x_{1\text{ext}} + x_{\Delta}^{(n)} \right); \quad (5.46)$$

$T_d^{\prime(n)}$  - is the damping time constant of free transient current of a positive sequence

$$T_d^{\prime(n)} = T_{f0} \left( x'_d + x_{1\text{ext}} + x_{\Delta}^{(n)} \right) / \left( x_d + x_{1\text{ext}} + x_{\Delta}^{(n)} \right). \quad (5.47)$$

The restriction in expression (5.44) is caused by the fact that under excitation control the generator voltage of a positive sequence cannot exceed the rated value. Periodic component will vary according to the same law as follows from the expression (5.42). To multiply both summands (5.44) by  $m^{(n)}$ , is sufficient to get its value at any instant.

Expressions describing the transient in a generator with damper winding circuit for any kind of short circuit are similar to expressions for three-phase short circuit. The value of a positive sequence current at the point of a short circuit determined by (5.44), as well as values of currents of other sequences tied to it depend on all elements impedances of a circuit examined.

Properties of transient at asymmetrical short circuit when compared with three-phase short circuit appear in the character of stator current transient component variation. Let's examine these features on example when asymmetrical short circuit occurs behind purely inductive reactance in the circuit of the stator of the generator without damper winding with thyristor system of excitation. Variation in time of the r.m.s. value of periodic component of positive sequence current is presented by expression (5.44).

Considering short circuits of different kinds occur consequently at the same point of a system under the same conditions we can write down the following inequalities using data of Table 5.1 for additional reactance. If only the pure inductive circuit is considered, then

$$x_{\Delta}^{(1)} > x_{\Delta}^{(2)} > x_{\Delta}^{(1,1)} > x_{\Delta}^{(3)} = 0. \quad (5.48)$$

The following inequalities are true for the current of positive sequence at the point of short circuit (5.49)

$$I_{sA1}^{(1)} < I_{sA1}^{(2)} < I_{sA1}^{(1,1)} < I_{sA1}^{(3)}. \quad (5.49)$$

The following relations are true for the voltage of positive sequence according to (5.41) and (5.43)

$$U_{sA1}^{(1)} > U_{sA1}^{(2)} > U_{sA1}^{(1,1)} > U_{sA}^{(3)} = 0. \quad (5.50)$$

According to (5. 47) damping time constants of transient current component for short circuit of different kinds are correlated by inequalities (5, 51)

$$T_d^{(1)} > T_d^{(2)} > T_d^{(1,1)} > T_d^{(3)}. \quad (5.51)$$

From these relations we can see that the more the additional reactance  $x_{\Delta}^{(n)}$ , characterizing a type of asymmetrical short circuit is, the slower is transient in a generator stator circuit. The slowest damping of current takes place at single-phase short circuit to ground.

The operation of a device for automatic excitation control that can react the changes of positive sequence voltage also depends on  $x_{\Delta}^{(n)}$ . When the value of  $x_{\Delta}^{(n)}$  grows, the process of excitation forcing slows down. But as value of  $x_{\Delta}^{(n)}$  increases, it causes less positive sequence voltage of generator reduction, its recovery (when it is possible) takes shorter time, and steady-state comes faster.

According to equivalence rule for positive sequence current we can write general expression for equivalent damping time constant of a stator current aperiodic component at short circuit of any kind

$$T_{a,res}^{(n)} = \frac{x_{2res} + x_{\Delta}^{(n)}}{\omega(r_{1res} + r_{\Delta}^{(n)})}, \quad (5.52)$$

where  $r_{\Delta}^{(n)}$  is the additional resistance for the given kind of short circuit. For the condition that inductive components of impedance in the circuit are equal to zero,  $r_{\Delta}^{(n)}$  is defined similarly to  $x_{\Delta}^{(n)}$ .

## 5.7. Integrated equivalent circuits

Relations between symmetrical components of currents and voltages at the point of short circuit allow designing so called integrated equivalent circuits for asymmetrical short circuit of any kind. They are compiled for special phase  $A$  in single-phase representation using resulting emf and impedances of separate sequences.

Fig. 5.8 presents some circuits. The first one is the design circuit for short circuit in electric doubly-fed supply system at the point of short circuit (the point  $SC^{(n)}$  of power line) where lateral asymmetry occurred. Other circuits correspond equivalent circuits for special phase  $A$  in single-phase of positive, negative and zero sequence. When integrated equivalent circuits is compiled the direction the beginning B1 (B2, B0) of equivalent circuit to its end E1 (E2, E0) – is taken as positive direction of current that is the point of short circuit. To obtain integrated equivalent circuit, equivalent circuits of positive, negative and zero sequences should be connected by terminals B1 (B2, B0) and E1 (E2, E0) according to the relations obtained for currents and voltages of the special phase for the given short circuit (pp. 5.2-5.4)

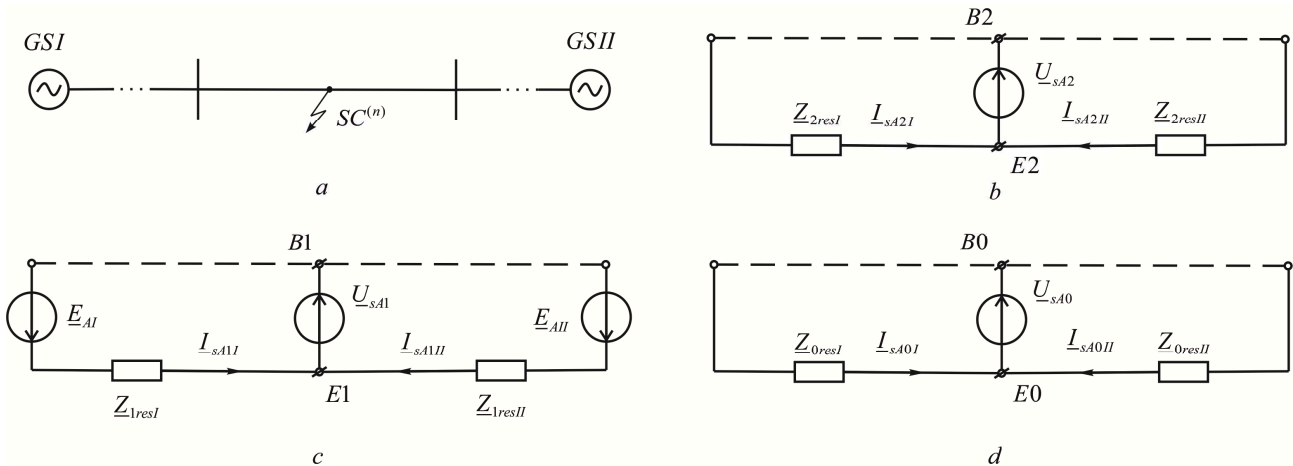


Fig. 5.8. Electric doubly-fed supply network under asymmetrical short circuit: a- design circuit; b, c, d- equivalent circuits of positive, negative and zero sequences for special phase A

Fig. 5.9 presents integrated equivalent circuit under asymmetrical short circuit to ground. It is compiled of equivalent circuit for positive sequence and equivalent circuits of negative and zero sequences (Fig. 5.8) according to (5.8)

$$\underline{U}_{sA1}^{(1)} + \underline{U}_{sA2}^{(1)} + \underline{U}_{sA0}^{(1)} = 0$$

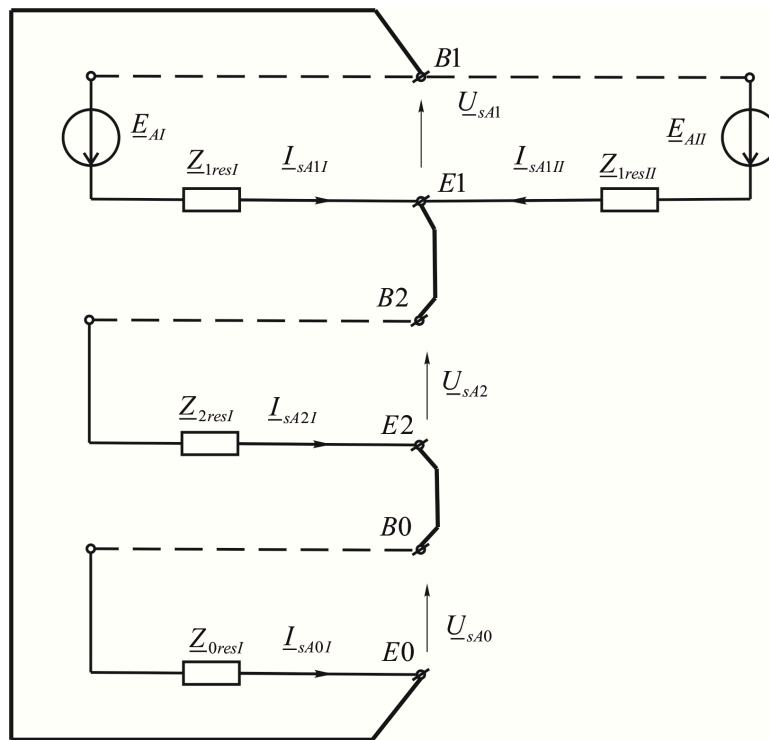


Fig. 5.9. Integrated equivalent circuit for the special phase A under single-phase short circuit

In the integrated equivalent circuit the current relation (5.10) is fulfilled

$$\underline{I}_{sA1}^{(1)} = \underline{I}_{sA2}^{(1)} = \underline{I}_{sA0}^{(1)}$$



Integrated equivalent circuit for two-phase short circuit is given in Fig. 5.10. It is made up on the ration of sequence voltages (5.19)

$$\underline{U}_{sA1}^{(2)} = \underline{U}_{sA2}^{(2)}; \underline{U}_{sA0}^{(2)} = 0$$

Currents in its branches correspond the conditions (5.18)

$$\underline{I}_{sA1}^{(2)} = -\underline{I}_{sA2}^{(2)}; \underline{I}_{sA0}^{(2)} = 0.$$

The integrated equivalent circuit (Fig. 5.11), made up using ratio of sequence voltages (5.26) corresponds two-phase short circuit to ground.

$$\underline{U}_{sA1}^{(1,1)} = \underline{U}_{sA2}^{(1,1)} = \underline{U}_{sA0}^{(1,1)}.$$

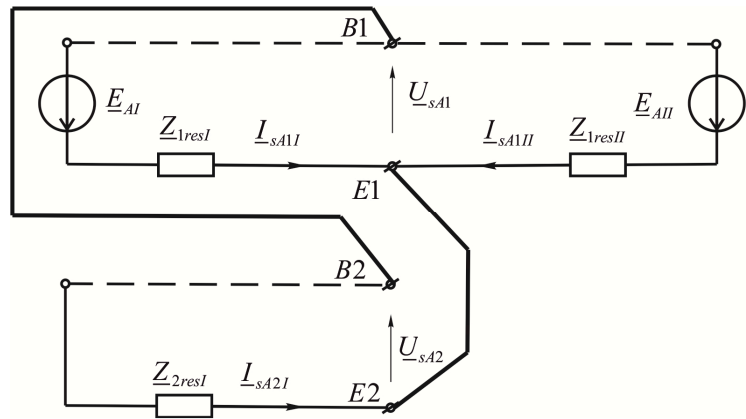


Fig. 5.10. Integrated equivalent circuit of special phase A under two-phase short circuit

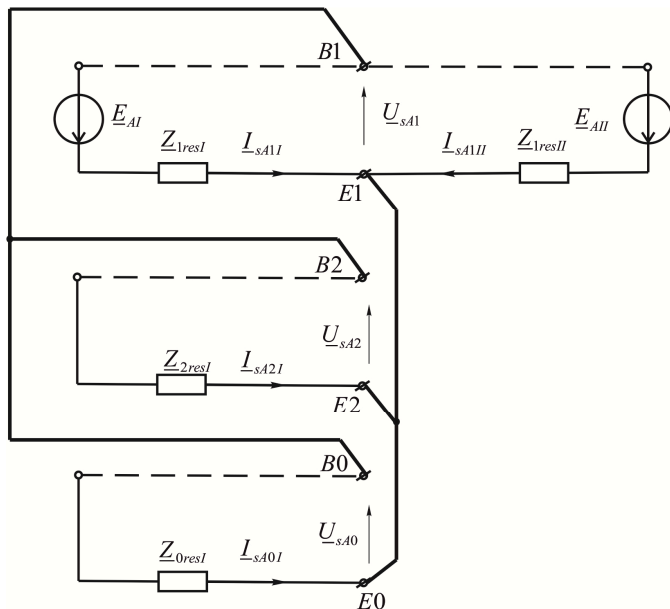


Fig. 5.11. The integrated equivalent circuit for special phase A at two-phase short circuit to ground

The identity (5.24) is fulfilled in branches of equivalent circuit for currents of sequences.

$$\underline{I}_{sA1}^{(1,1)} = -\left(\underline{I}_{sA2}^{(1,1)} + \underline{I}_{sA0}^{(1,1)}\right)$$

Making calculations with the use of integrated equivalent circuits, equivalent circuits of separate sequences are made up and connected according to Fig. 5.9-5.11 for the short circuit of a given kind. In order to find currents of positive, negative and zero sequences in an element, as a first step impedances modeling this element in the circuits of positive, negative and zero sequences should be determined. Then the currents in the branches that include this impedance should be determined. The voltage of a separate sequence for a given node of the integrated equivalent circuit is

defined as the node voltage relating to the reference point of equivalent circuit of corresponding sequence.

Integrated equivalent circuits are easy to use for calculations of asymmetrical short circuits using static models. Values of separate sequence currents and voltages are measured.

### 5.8. Comparison of currents at different short circuits

Comparison of currents at different short circuits is of a practical interest. It permits using the known value of three-phase short circuit for the given point of the power supply system to assess possible current values for asymmetrical short circuit as a first approximation. Calculation of the three-phase short circuit current value is simpler than for other kinds. Further determined boundary relations are true only for currents at the given point of short circuit when  $t = 0$ .

Current at the place of short circuit of any kind can be determined according to the expression (when  $r \rightarrow 0$ )

$$I_F^{(n)} = m^{(n)} I_{sA1}^{(1)} = m^{(n)} E_{A\Sigma} / \left( x_{1res} + x_{\Delta}^{(n)} \right) \quad (5.53)$$

Ratios of the current periodic components of any asymmetrical short circuit to the current periodic component for three-phase short circuit can be presented as

$$SC^{(n;3)} = \frac{I_F^{(n)}}{I_F^{(3)}} = \frac{m^{(n)} E_{A\Sigma}^{(n)} / \left( x_{1res} + x_{\Delta}^{(n)} \right)}{E_{A\Sigma}^{(3)} / x_{1res}} = \frac{m^{(n)} E_{A\Sigma}^{(n)} / E_{A\Sigma}^{(3)}}{1 + x_{\Delta}^{(n)} / x_{1res}} \quad (5.54)$$

Expression (5.54) is valid for any instant of time and accounts the equivalent circuit of a generator for positive sequence. Corresponding emf should be taken for it. For approximate estimation of variation limits for  $SC^{(n;3)}$ , the difference between  $E_{A\Sigma}^{(n)}$  and  $E_{A\Sigma}^{(3)}$  can be neglected and equality  $E_{A\Sigma}^{(n)} = E_{A\Sigma}^{(3)}$  is true for the instant  $t = 0$ . In this case the expression (5.54) becomes simpler

$$SC^{(n;3)} = m^{(n)} / \left( 1 + x_{\Delta}^{(n)} / x_{1res} \right) \quad (5.55)$$

Under this assumption the maximum error takes place when relation between the of steady-state currents or currents the nearest to steady-state ones is defined. It is explained by the fact that generator is introduced to the equivalent circuit at the three-phase short circuit by emf  $E_{q,ps}$  and reactance  $x_d$ , and at the single-phase short circuit – by emf equal to zero. However, this case has no practical importance.

Now consider, what limiting values the ratio  $SC^{(n;3)}$  can take at different kinds of short circuit.

**Two-phase short circuit** According to the data of Table 5.1  $m^{(2)} = \sqrt{3}$ ,  $x_{\Delta}^{(2)} \approx x_{2res}$ . Then according to the expression (5.55)

$$SC^{(2;3)} = \sqrt{3} / \left( 1 + x_{2res} / x_{1res} \right) \quad (5.56)$$

Let's analyze  $SC^{(n;3)}$  values under short circuit close to terminals of a generator and at short circuit at remote point.

When short circuit occurs near terminals of generator  $x_{ext} = 0$ , and at the initial instant of short circuit ( $t = 0$ )  $x_{1G} = x_d''$ ,  $x_{2G} \approx x_d''$ .

Therefore

$$x_{1res} \approx x_{2res} \text{ and } SC^{(2;3)} = \sqrt{3} / 2 \quad (5.57)$$

In steady state ( $t \rightarrow \infty$ ), when  $x_{1G} = x_d$ ,  $x_{2G} \ll x_d$  and  $x_{1res} \gg x_{2res}$ , we can assume that  $x_{2res} \approx 0$ . Then  $SC^{(2;3)} \rightarrow \sqrt{3}$ . Hence, ratio  $SC^{(2;3)}$  is in the limits:

$$\sqrt{3} / 2 \leq SC^{(2;3)} < \sqrt{3} \quad (5.58)$$

When short circuit occurs at the point remote from the terminals of a generator value  $x_{ext}$  is high It can occur in electric supply system of industrial enterprise of average capacity, that has no own thermal power station. In this case

$$x_{2\text{res}}/x_{1\text{res}} = (x_{2\text{G}} + x_{\text{ext}})/(x_{1\text{G}} + x_{\text{ext}}) \approx 1 \quad (5.59)$$

independently from the instant of short circuit, and  $SC^{(2;3)} = \sqrt{3}/2$ . It means, there is

approximately constant ratio  $I_{\text{F}}^{(2)} \approx (\sqrt{3}/2)I_{\text{F}}^{(3)}$  between current of two or three-phase short circuit during the whole transient..

### **Single-phase short circuit**

From Table 5.1  $m^{(1)} = 3$ ;  $x_{\Delta}^{(1)} = x_{2\text{res}} + x_{0\text{res}}$ . Then according to (5.55) multiplicity factor is equal to

$$\begin{aligned} SC^{(1;3)} &= 3/\left[1 + (x_{2\text{res}} + x_{0\text{res}})/x_{1\text{res}}\right] = \\ &= 3/\left(1 + x_{2\text{res}}/x_{1\text{res}} + x_{0\text{res}}/x_{1\text{res}}\right), \end{aligned} \quad (5.60)$$

that depends greatly on the value of zero sequence impedance.

Value of inductive reactance  $x_{0\text{res}}$  varies over a wide range (practically from 0 to  $\infty$ ). It can be varied by changing a number of neutrals of transformers in networks with mains voltage 110 kV and over.

Let's consider interval of values  $SC^{(1,3)}$  at short circuit near terminals of generator when  $x_{\text{ext}} \rightarrow 0$ , and  $x_{1\text{res}} \gg x_{2\text{res}}$ . It can be assumed that  $x_{2\text{res}}/x_{1\text{res}} \approx 0$ . Then when  $x_{0\text{res}} \approx 0$  limiting value  $SC^{(1,3)} \rightarrow 3$ , when  $x_{0\text{res}} \rightarrow \infty$  is  $SC^{(1,3)} = 0$ .

Hence values of ratio  $SC^{(1,3)}$  is in limits

$$0 \leq SC^{(1,3)} < 3. \quad (5.61)$$

At short circuit in the remote point of an electric supply system  $x_{1\text{res}} \approx x_{2\text{res}}$ , and limiting values corresponding to  $x_{0\text{res}} = 0$  and  $x_{0\text{res}} \rightarrow \infty$  are

$$0 \leq SC^{(1,3)} < 1,5. \quad (5.62)$$

Variation of values of coefficient  $SC^{(1,3)}$  according to the ratio  $x_{0\text{res}}/x_{1\text{res}}$  in the short circuit at remote point is shown in Fig. 5.12.

### **Two-phase-to-ground short circuit**

From Table 5.1 the values  $m^{(1,1)}$  and  $x_{\Delta}^{(1,1)}$  can be taken. Then according to the expression (5.55)

$$\begin{aligned} SC^{(1,1;3)} &= \sqrt{3} \sqrt{1 - x_{2\text{res}}x_{0\text{res}}/(x_{2\text{res}} + x_{0\text{res}})^2} / \\ & / \left[1 + (x_{2\text{res}}x_{0\text{res}}/(x_{2\text{res}} + x_{0\text{res}})/x_{1\text{res}})\right]. \end{aligned} \quad (5.63)$$

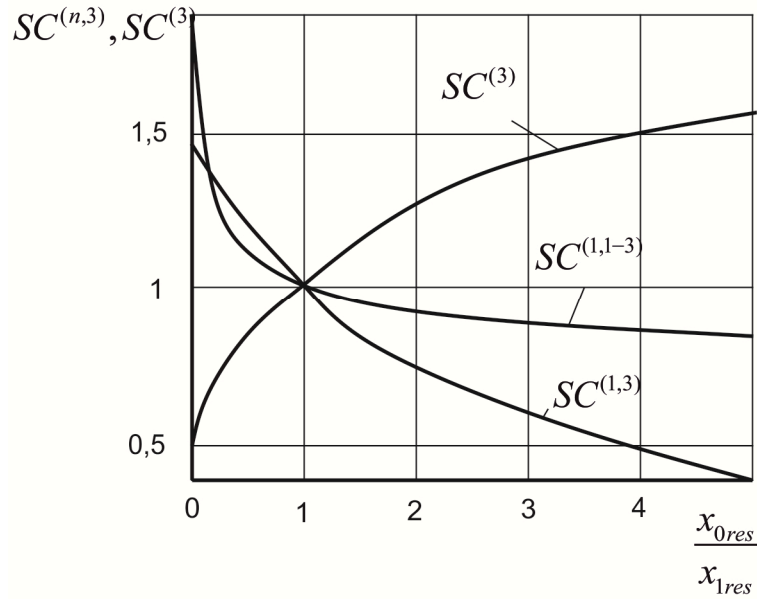


Fig. 5.12. Ratio of currents of different short circuits according to the ratio of reactances  $x_{0res}/x_{1res}$

As it was indicated above (see Fig. 5.4) depending on the ratio  $x_{2res}/x_{0res}$ , the value  $m^{(1,1)}$  is in the limits  $1,5 \leq m^{(1,1)} \leq \sqrt{3}$ , the upper found corresponds to  $x_{2res}/x_{0res}$  that is equal to zero or infinity, the lower one  $x_{2res}/x_{0res}$ .

If  $x_{0res} = 0$ , then  $SC^{(1,1;3)} = \sqrt{3}$ . When  $x_{0res} = \infty$ , integrated equivalent circuit at  $K^{(1,1)}$  (Fig. 5.11) transforms into integrated equivalent circuit at  $SC^{(2)}$  (Fig. 5.10), and as a result

$$SC^{(1,1;3)} = \sqrt{3} \left( 1 + x_{2res}/x_{1res} \right). \quad (5.64)$$

At short circuit at remote point where  $x_{2res} \approx x_{1res}$

$$SC^{(1,1;3)} = \sqrt{3}/2. \quad (5.65)$$

Therefore values of coefficient  $SC^{(1,1;3)}$  are varied in limits

$$\sqrt{3}/2 \leq SC^{(1,1;3)} \leq \sqrt{3}, \quad (5.66)$$

as at two-phase short circuit

Fig. 5.12 presents variation of coefficient  $SC^{(1,1;3)}$  values according to ratio  $x_{0res}/x_{1res}$  at short circuit at remote point. From here it follows that when  $x_{0res}/x_{1res} = 0, 2 \dots 1$  the current of single-phase short the circuit is slightly higher than current of two-phase short circuit. At the same time the inverse ratio takes place at any other value of  $x_{0res}/x_{1res}$ .

Comparison of values of ground current at single-phase and two phase short circuit is of a practical interest too. Using (5.10), (5.12), (5.27) and (5.30), the ratio  $SC^{(g)} = I_g^{(1)}/I_g^{(1,1)} = I_{s0}^{(1)}/I_{s0}^{(1,1)}$  can be presented as

$$SC^{(3)} = \frac{x_{1res}x_{2res} + x_{0res}x_{1res} + x_{2res}x_{0res}}{x_{1res}x_{2res} + x_{2res}^2 + x_{2res}x_{0res}}, \quad (5.67)$$

from where it follows that depending on relation between  $x_{2res}$  and  $x_{1res}x_{0res}$  according to ratio  $SC^{(3)} > 1$  or  $SC^{(3)} < 1$ . When  $x_{2res} = x_{1res}$  the expression changes to

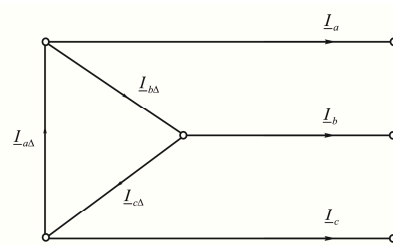
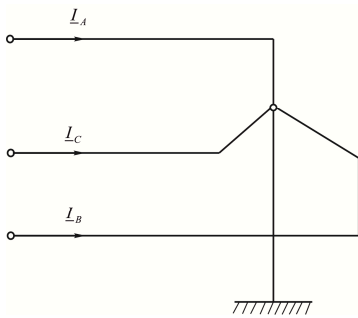
$$SC^{(3)} = (1 + 2x_{0res}/x_{1res}) / (2 + x_{0res}/x_{1res}). \quad (5.68)$$

Dependence of the ratio  $SC^{(3)}$  on  $x_{0res}/x_{1res}$  is illustrated by the curve (Fig. 5.12). Ground currents for short circuits compared are equal only when  $x_{0res} = x_{1res}$  when  $x_{0res} > x_{1res}$  ground current is higher at single-phase short circuit. If  $x_{0res} < x_{1res}$ , the two-phase short circuit current has higher value. Ratios (5.67) and (5.68) are valid for zero sequence of any circuit branch current as it is proportional to the place of short circuit.

### 5.9. Symmetrical current and voltage components transformation

Phase branch currents and node voltages, calculated with use of symmetrical components are valid only for the design circuit sections that connected electrically with the point of short circuit. It is caused by the fact that determining parameters of equivalent circuit elements we use line values (magnitudes) of transformation ratio, that is transformer groups are ignored.

In the process of currents and voltages transformation, their vectors magnitude and angles are changed. Phase of complex transformation ratio can be accounted by displacement the obtained branch currents and voltages by the angles that are defined by transformers winding connection groups.



Groups  $Y/Y-0$  and  $Y/\Delta-11$  are taken as standard. Fig. 5.13, presents transformer with winding connection  $Y_0/\Delta-11$ . If phase currents  $\underline{I}_A, \underline{I}_B, \underline{I}_C$  are preset according to positive directions indicated on Fig. 5.13, currents of phase wires beyond delta connected winding are

Fig. 5.13. Current distribution for transformer with winding connection  $Y_0/\Delta-11$

$$\left. \begin{aligned} \underline{I}_a &= \underline{I}_{a\Delta} - \underline{I}_{b\Delta} = (\underline{I}_A - \underline{I}_B)w_Y/w_\Delta = (\underline{I}_A - \underline{I}_B)K/\sqrt{3}; \\ \underline{I}_b &= \underline{I}_{b\Delta} - \underline{I}_{c\Delta} = (\underline{I}_B - \underline{I}_C)w_Y/w_\Delta = (\underline{I}_B - \underline{I}_C)K/\sqrt{3}; \\ \underline{I}_c &= \underline{I}_{c\Delta} - \underline{I}_{a\Delta} = (\underline{I}_C - \underline{I}_A)w_Y/w_\Delta = (\underline{I}_C - \underline{I}_A)K/\sqrt{3} \end{aligned} \right\} \quad (5.69)$$

or in matrix form

$$\begin{pmatrix} \underline{I}_a \\ \underline{I}_b \\ \underline{I}_c \end{pmatrix} = \frac{K}{\sqrt{3}} \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \underline{I}_A \\ \underline{I}_B \\ \underline{I}_C \end{pmatrix} = \frac{K}{\sqrt{3}} \mathbf{A}^{(11)} \begin{pmatrix} \underline{I}_A \\ \underline{I}_B \\ \underline{I}_C \end{pmatrix}, \quad (5.70)$$

where  $K$  – is line transformation ratio equal to the rated transformer line voltages ratio. If numbers of winding turns of high and low transformer voltage are denoted as  $w_Y$  and  $w_\Delta$ , and high voltage  $U_{HV}$ , is taken as main stage, line transformation ratio for transformers with an odd vector group is

$$K = \sqrt{3} w_Y / w_\Delta \approx U_{HV} / U_{LV}. \quad (5.71)$$

For transformers with an even vector group

$$K = w_Y / w_\Delta. \quad (5.72)$$

On the other hand when currents  $\underline{I}_a, \underline{I}_b, \underline{I}_c$  on the stage of delta connected winding, currents of star connected transformer winding can be expressed as

$$\left. \begin{aligned} \underline{I}_A - \underline{I}_0 &= (\underline{I}_a - \underline{I}_c) / (K\sqrt{3}); \\ \underline{I}_B - \underline{I}_0 &= (\underline{I}_b - \underline{I}_a) / (K\sqrt{3}); \\ \underline{I}_C - \underline{I}_0 &= (\underline{I}_c - \underline{I}_b) / (K\sqrt{3}) \end{aligned} \right\} \quad (5.73)$$

or in matrix form

$$\begin{pmatrix} \underline{I}_A - \underline{I}_0 \\ \underline{I}_B - \underline{I}_0 \\ \underline{I}_C - \underline{I}_0 \end{pmatrix} = \frac{1}{K\sqrt{3}} \begin{pmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} \underline{I}_a \\ \underline{I}_b \\ \underline{I}_c \end{pmatrix} = \frac{1}{K\sqrt{3}} \mathbf{B}^{(11)} \begin{pmatrix} \underline{I}_a \\ \underline{I}_b \\ \underline{I}_c \end{pmatrix}. \quad (5.74)$$

Expressions (5.69) and (5.73) can be transformed expressing currents using symmetrical sequence components for the special phase  $A$ :

$$\left. \begin{aligned} \underline{I}_a &= (\underline{I}_{A1} + \underline{I}_{A2} + \underline{I}_{A0} - a^2 \underline{I}_{A1} - a \underline{I}_{A2} - \underline{I}_{A0}) K / \sqrt{3} = \\ &= (\underline{I}_{A1} e^{j30^\circ} + \underline{I}_{A2} e^{-j30^\circ}) K; \\ \underline{I}_b &= (a^2 \underline{I}_{A1} + a \underline{I}_{A2} + \underline{I}_{A0} - a \underline{I}_{A1} - a^2 \underline{I}_{A2} - \underline{I}_{A0}) K / \sqrt{3} = \\ &= (\underline{I}_{A1} e^{j90^\circ} + \underline{I}_{A2} e^{-j90^\circ}) K; \\ \underline{I}_c &= (a \underline{I}_{A1} + a^2 \underline{I}_{A2} + \underline{I}_{A0} - \underline{I}_{A1} - \underline{I}_{A2} - \underline{I}_{A0}) K / \sqrt{3} = \\ &= (\underline{I}_{A1} e^{j150^\circ} + \underline{I}_{A2} e^{-j150^\circ}) K \end{aligned} \right\} \quad (5.75)$$

and

$$\left. \begin{aligned}
 \underline{I}_A - \underline{I}_0 &= \left( \underline{I}_{a1} + \underline{I}_{a2} + \underline{I}_{a0} - a\underline{I}_{a1} - a^2\underline{I}_{a2} - \underline{I}_{a0} \right) / (K\sqrt{3}) = \\
 &= \left( \underline{I}_{a1}e^{-j30^\circ} + \underline{I}_{a2}e^{j30^\circ} \right) / K; \\
 \underline{I}_B - \underline{I}_0 &= \left( a^2\underline{I}_{a1} + a\underline{I}_{a2} + \underline{I}_{a0} - \underline{I}_{a1} - \underline{I}_{a2} - \underline{I}_{a0} \right) / (K\sqrt{3}) = \\
 &= \left( \underline{I}_{a1}e^{-j150^\circ} + \underline{I}_{a2}e^{j150^\circ} \right) / K; \\
 \underline{I}_C - \underline{I}_0 &= \left( a\underline{I}_{a1} + a^2\underline{I}_{a2} + \underline{I}_{a0} - a^2\underline{I}_{a1} - a\underline{I}_{a2} - \underline{I}_{a0} \right) / (K\sqrt{3}) = \\
 &= \left( \underline{I}_{a1}e^{j90^\circ} + \underline{I}_{a2}e^{-j90^\circ} \right) / K.
 \end{aligned} \right\} \quad (5.76)$$

Similar transformation can be done with voltages. Voltages on both sides of transformer stated by symmetrical components for the special phase  $A$ :

$$\left. \begin{aligned}
 \underline{U}_a &= \left( \underline{U}_{A1}e^{-j30^\circ} + \underline{U}_{A2}e^{j30^\circ} \right) K; \\
 \underline{U}_b &= \left( \underline{U}_{A1}e^{-j90^\circ} + \underline{U}_{A2}e^{j90^\circ} \right) K; \\
 \underline{U}_c &= \left( \underline{U}_{A1}e^{j150^\circ} + \underline{U}_{A2}e^{-j150^\circ} \right) K
 \end{aligned} \right\} \quad (5.77)$$

and

$$\left. \begin{aligned}
 \underline{U}_A &= \left( \underline{U}_{a1}e^{-j30^\circ} + \underline{U}_{a2}e^{j30^\circ} \right) K; \\
 \underline{U}_B &= \left( \underline{U}_{a1}e^{-j150^\circ} + \underline{U}_{a2}e^{j150^\circ} \right) K; \\
 \underline{U}_C &= \left( \underline{U}_{a1}e^{j90^\circ} + \underline{U}_{a2}e^{-j90^\circ} \right) K.
 \end{aligned} \right\} \quad (5.78)$$

Phase voltages relative to zero point of phasor system are defined by expressions (5.77) and (5.78) In operation mode without zero sequence components phase to-ground voltages are defined by expressions (5.77) and (5.78)

Structure of expressions (5.75) and (5.77) indicates that at transition from star connection side to delta connection side in the transformer with vector group  $Y_0/\Delta-11$ , positive sequence vectors are turned for  $30^\circ$  in the direction of vector rotation and negative sequence vectors are turned for  $30^\circ$  in the opposite direction (Fig. 5.14). At transition of through a transformer in the reverse direction there occur such angular displacements that vectors of symmetrical sequence components change sign. It is proved by expressions (5.76) and (5.78).

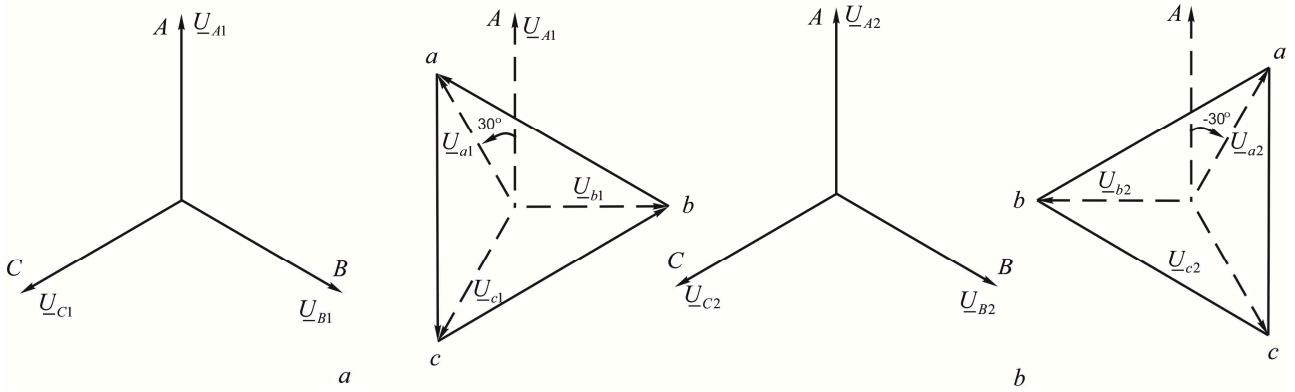


Fig. 5.14. Vectors of symmetrical components for a transformer with vector group  $Y_0/\Delta-11$ :  
a – positive sequence; b– negative sequence

Similar transformation can be done for other vector groups as well. Let's consider transformer with  $N$ -vector group. Similarly to expressions (5.70) and (5.74) can be found ratio of currents of different transformer side winding

$$\begin{pmatrix} \underline{I}_a \\ \underline{I}_b \\ \underline{I}_c \end{pmatrix} = \frac{K}{\sqrt{3}} \mathbf{A}^{(N)} \begin{pmatrix} \underline{I}_A \\ \underline{I}_B \\ \underline{I}_C \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} \underline{I}_A - \underline{I}_0 \\ \underline{I}_B - \underline{I}_0 \\ \underline{I}_C - \underline{I}_0 \end{pmatrix} = \frac{1}{K\sqrt{3}} \mathbf{B}^{(N)} \begin{pmatrix} \underline{I}_a \\ \underline{I}_b \\ \underline{I}_c \end{pmatrix}. \quad (5.79)$$

Matrix value  $\mathbf{A}^{(N)}$  and  $\mathbf{B}^{(N)}$  for different vector groups are given in Table 5.2. Analyzing symmetrical sequence components using ratio (4.6)-(4.7), we have for the special phase  $A$ :

$$\left. \begin{aligned} \begin{pmatrix} \underline{I}_{a1} \\ \underline{I}_{a2} \\ \underline{I}_{a0} \end{pmatrix} &= S^{-1} \begin{pmatrix} \underline{I}_a \\ \underline{I}_b \\ \underline{I}_c \end{pmatrix} = S^{-1} \frac{K}{\sqrt{3}} \mathbf{A}^{(N)} \begin{pmatrix} \underline{I}_A \\ \underline{I}_B \\ \underline{I}_C \end{pmatrix} = S^{-1} \frac{K}{\sqrt{3}} \mathbf{A}^{(N)} S \begin{pmatrix} \underline{I}_{A1} \\ \underline{I}_{A2} \\ \underline{I}_{A0} \end{pmatrix} ; \\ \begin{pmatrix} \underline{I}_{A1} \\ \underline{I}_{A2} \\ \underline{I}_{A0} \end{pmatrix} &= S^{-1} \begin{pmatrix} \underline{I}_A - \underline{I}_0 \\ \underline{I}_B - \underline{I}_0 \\ \underline{I}_C - \underline{I}_0 \end{pmatrix} = S^{-1} \frac{1}{K\sqrt{3}} \mathbf{B}^{(N)} \begin{pmatrix} \underline{I}_a \\ \underline{I}_b \\ \underline{I}_c \end{pmatrix} = S^{-1} \frac{1}{K\sqrt{3}} \mathbf{B}^{(N)} S \begin{pmatrix} \underline{I}_{a1} \\ \underline{I}_{a2} \\ \underline{I}_{a0} \end{pmatrix} \end{aligned} \right\} \quad (5.80)$$

$$\text{or} \quad \vec{\underline{I}}_{as} = \mathbf{M}_S^{I(N)} \cdot \vec{\underline{I}}_{AS} \quad \text{and} \quad \vec{\underline{I}}_{AS} = \mathbf{N}_S^{I(N)} \cdot \vec{\underline{I}}_{as}, \quad \text{where} \quad \mathbf{M}_S^{I(N)} = S^{-1} \frac{K}{\sqrt{3}} \mathbf{A}^{(N)} \cdot S,$$

$$\mathbf{N}_S^{I(N)} = S^{-1} \frac{1}{K\sqrt{3}} \mathbf{B}^{(N)} \cdot S,$$



that in expended form is presented as

$$\mathbf{M}^{(I)N} = \begin{vmatrix} Ke^{-j30^\circ N} & 0 & 0 \\ 0 & Ke^{j30^\circ N} & 0 \\ 0 & 0 & 0 \end{vmatrix},$$

$$\mathbf{N}^{(I)N} = \begin{vmatrix} K^{-1}e^{j30^\circ N} & 0 & 0 \\ 0 & K^{-1}e^{-j30^\circ N} & 0 \\ 0 & 0 & 0 \end{vmatrix}. \quad (5.81)$$

Similarly, ratio of symmetrical voltage components for a special phase  $A$  can be received using expressions (5.77), (5.78), (5.80) and (5.81), at transition from star to delta connection

$$\begin{vmatrix} \underline{U}_{a1} \\ \underline{U}_{a2} \\ \underline{U}_{a0} \end{vmatrix} = \mathbf{M}^{U(N)} \begin{vmatrix} \underline{U}_{A1} \\ \underline{U}_{A2} \\ \underline{U}_{A0} \end{vmatrix} = \begin{vmatrix} K^{-1}e^{-j30^\circ N} & 0 & 0 \\ 0 & K^{-1}e^{j30^\circ N} & 0 \\ 0 & 0 & 0 \end{vmatrix} \cdot \begin{vmatrix} \underline{U}_{A1} \\ \underline{U}_{A2} \\ \underline{U}_{A0} \end{vmatrix}; \quad (5.82)$$

at transition from delta to star

$$\begin{vmatrix} \underline{U}_{A1} \\ \underline{U}_{A2} \\ \underline{U}_{A0} \end{vmatrix} = \mathbf{N}^{U(N)} \begin{vmatrix} \underline{U}_{a1} \\ \underline{U}_{a2} \\ \underline{U}_{a0} \end{vmatrix} = \begin{vmatrix} Ke^{j30^\circ N} & 0 & 0 \\ 0 & Ke^{-j30^\circ N} & 0 \\ 0 & 0 & 0 \end{vmatrix} \cdot \begin{vmatrix} \underline{U}_{a1} \\ \underline{U}_{a2} \\ \underline{U}_{a0} \end{vmatrix}.$$

At transition through transformer with vector group 12 vectors of symmetrical current and voltage components don't change in phase. According to expressions (5.80) - (5.82) rule of transformation of separate sequence symmetrical components can be stated. In transformer with  $N$ -vector group at transition from Star-side to delta side positive sequence current and voltage vectors are shifted at the angle  $-30^\circ N$ , negative sequence current and voltage vectors are shifted at the angle  $+30^\circ N$ , and at transition from delta side to Y-side displacement angle of corresponding vectors change at  $+30^\circ N$  and  $-30^\circ N$ .

At odd vector group when it is not necessary to know true relative orientation of vectors diagrams on the both sides of transformer winding its winding can be considered as connected according to group 3 (or 9) (Table. 5.2). In such case vectors of positive and negative sequences are

turned at  $90^\circ$  to opposite sides (Fig. 5.15). Obviously, positive sequence vectors can be remained unshifted but negative sequence vectors should be shifted at  $180^\circ$ . It results in the following rule: at transition through the transformer with windings connection  $Y/\Delta$  or  $\Delta/Y$  it is sufficient to reverse the sign of negative sequence vectors only.

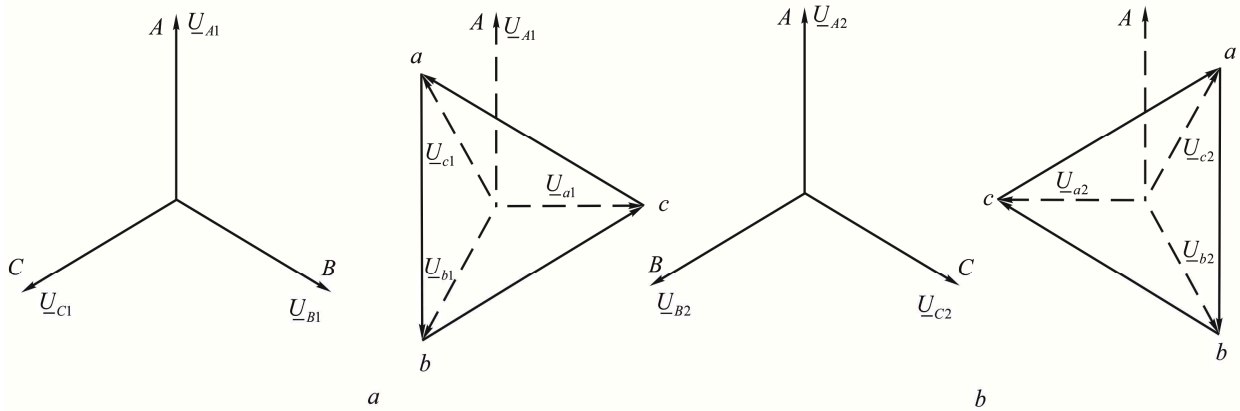


Fig. 5.15. Vectors of symmetrical components for transformer with vector group  $Y/\Delta-3$ :  
 a – positive sequence; b – negative sequence

It should be considered that ignoring actual group results in mismatch of denomination of wires beyond the transformer with marking of corresponding real vector group.

If current and voltages vectors are expressed in per unit, only angular displacement specified by correspondent vector group should be considered at transformation. Vector current and voltage diagrams at a random point of analyzed circuit can be built.

Using vector diagrams for current and voltages at the point of short circuit, it is possible to receive current and voltage vector diagrams for any point of the examined circuit.

In Fig. 5.16 vector current and voltage diagrams at the short circuit of different types at points  $\alpha, \beta$  of the circuit (Fig. 5.16,a), located on different transformer windings referred to point  $SC^{(n)}$  are given, as an example. Let's take that a short circuit consists of inductive reactances only and transformer windings are connected  $Y_0/\Delta-11$ . To make possible comparison of diagrams on the both sides of a transformer currents and voltages are considered to be expressed in per unit or reduced to the same voltage level.

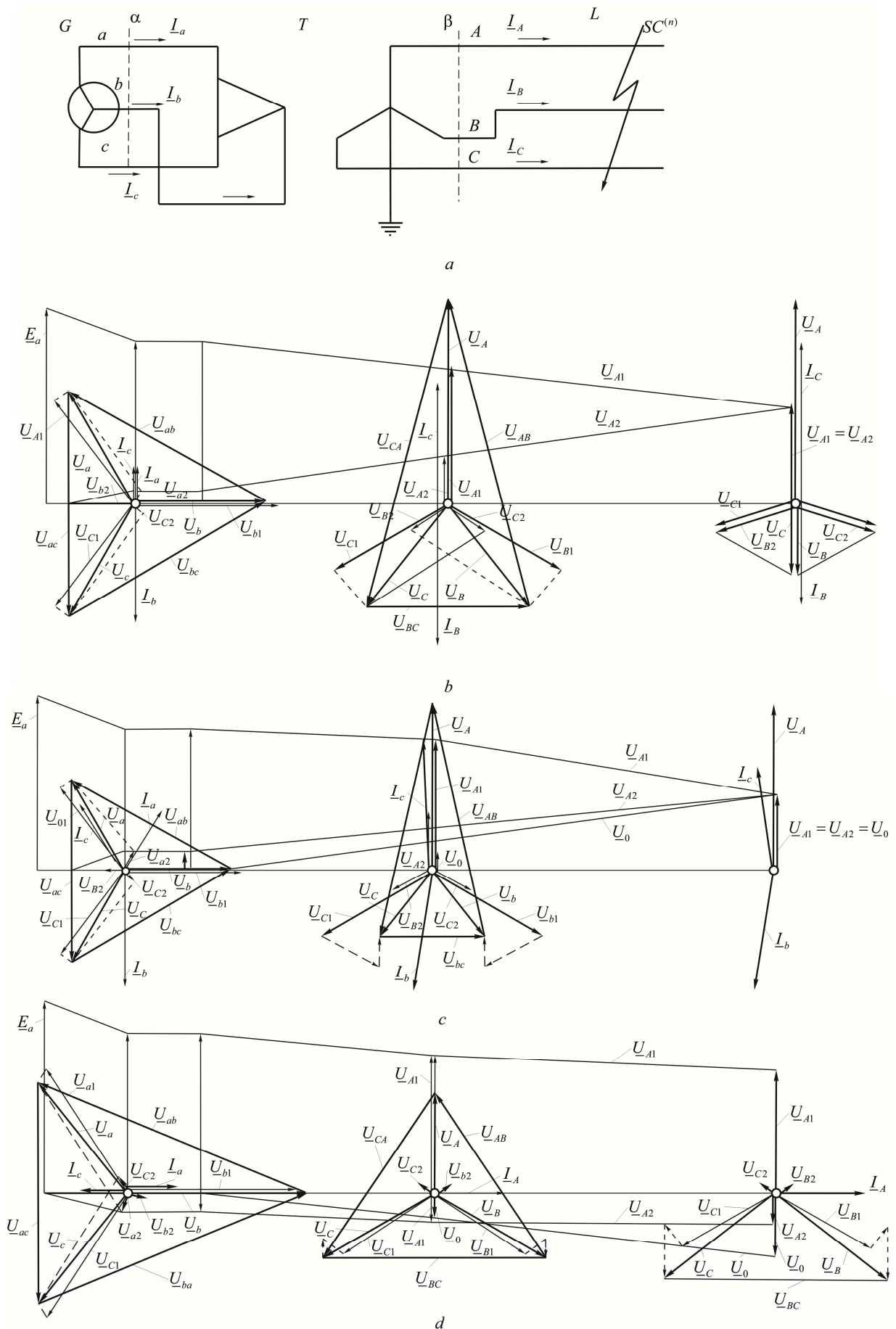


Fig. 5.16. Vector diagrams for different points of electric power supply network. (a) – design circuit for calculation of phase voltages and currents and their components at asymmetrical short circuit; (b) at two-phase short circuit (c) at two-phase to ground short circuit; d single-phase short circuit)

Separate sequence voltage vectors at points  $\alpha$ ,  $\beta$ ,  $SC^{(n)}$  of analyzed circuit for each asymmetrical short circuit are given in Fig. 5.16, b-d. Approaching the generator, positive sequence voltage rises but negative and zero sequence voltages reduce regarding absolute value. Zero sequence voltage doesn't occur beyond transformer at transformer winding connection given in Fig. 5.16,a. As follows from ratio  $\underline{U}_{A2}$  and  $\underline{U}_{A0}$  at the point of short circuit  $x_{0res} > x_{2res}$ .

Maximum distortion of vector voltage diagram always occurs at the point of short circuit. For points located nearer to the generator this distortion becomes progressively less. System of electromotive force vectors remains completely symmetrical. As transformer has odd vector group the single-phase short circuit on the Y-connected side is taken by generator as two-phase short circuit.

Building voltage vector diagrams for other network sections the following should be considered:

1) positive sequence voltage has minimum value at the point of short circuit and it rises approaching the source but negative and zero sequence voltages have maximum magnitudes at the point of short circuit and reduce approaching the source;

2) at transition through transformer positive and negative sequence vector systems are turned respectively clockwise and anticlockwise referred to their position at the point of short circuit, where rotation angle depends on a vector group;

3) connected transformer windings limit the area of zero sequence current flow in the network. In integrated equivalent circuits they are the starting point for zero sequence equivalent circuit.

### 5.10. Methods of asymmetrical short circuit calculation

Methods of calculation of transient at three-phase short circuit according to the equivalence rule for positive sequence current can be used for calculation of transient at any asymmetrical short circuit. Periodic component of positive sequence current of asymmetrical short circuit is determined similarly to the periodic component of current of three-phase short circuit but only at the point

remote from the actual point of short circuit by an additional resistor  $\underline{Z}_{\Delta}^{(n)}$ . Integrated equivalent circuit can be used calculating the current. Negative sequence current and zero sequence current as well as voltages of separate sequences at the point of short circuit can be determined by positive sequence current (Table 5.1).

Let's consider mode parameters performances calculation at asymmetrical short circuit and give correspondent calculation algorithms.

#### *Calculation of initial values of mode parameters periodic components*

In general case equivalent circuits for all sequences positive, negative and zero sequences are compiled. Equivalent circuits parameters for positive sequence are determined for the instant  $t = 0$ . In this case all generators, compensators and loads are given only by their subtransient impedances and electromotive forces. Positive sequence voltage at any point of power supply system at any asymmetrical short circuit is always higher than at three-phase short circuit at the same point, powering from separate motors at asymmetrical short circuit occurs more rarely than at three-phase short circuit. Loads and separate motors except for powerful motors directly connected to the point of short circuit are often neglected at approximate evaluation of asymmetrical short circuit surge current.

The negative sequence equivalent circuit in approximate calculation is obtained from the positive sequence equivalent circuit excluding emf and taking  $\underline{Z}_{2res} \equiv \underline{Z}_{1res}$ .

Succession of calculation of initial stages of mode parameters periodic components is as follows:

1) equivalent circuit for positive, negative and zero sequences is designed determining equivalent element (impedances) parameters and sources electromotive forces

2) resulting impedances  $\underline{Z}_{1res}$ ,  $\underline{Z}_{2res}$ ,  $\underline{Z}_{0res}$  are determined transforming equivalent circuits for separate sequences referred to the point of short circuit

3) positive sequence current at the point of short circuit is determined by formulae (5.41)

4) positive and negative sequence currents and voltages of separate sequences at the point of short circuit are determined depending on a kind of short circuit according to formulae of table 5.1

5) separate sequence current distribution is calculated by correspondent initial design circuits considering when necessary positive and negative sequence current vectors shifting at a correspondent angles at their transformation

6) phase currents and voltages for the indicated types of asymmetrical short circuit are found according to expressions of Table 5.1.

**Calculation of mode parameters of asymmetrical short circuit for the time  $t > 0$  using standard curves for generators**

Standard curves can be used for determining of the positive sequence current periodic component and hence of total current at unspecified instant of transient at asymmetrical short circuit.

The calculation algorithm is similar to the algorithm for three-phase short circuit (chapter 3) when calculated circuit has only one equivalent source (synchronous generator, compensator or a group of generators with equal parameters and operating in equal conditions referred to the point of short circuit):

1) positive sequence current  $I_G^{(n)}$  is determined ;

2) electrical remoteness of the point of equivalent three-phase short circuit from generator is defined

$$I_{*1,G}^{(n)} = I_{1,G}^{(n)} / I_{G,rated} \quad (5.83)$$

where  $I_{G,rated}$  - is rated generator current reduced to the value level of mains voltage where short current occurred ;

3) by the value found by (5.83), a proper standard curve is chosen and the ratio (5.83) is determined for the required instant

$$\gamma_t^{(n)} = I_{1,G,t}^{(n)} / I_{1,G}^{(n)} \quad (5.84)$$

4) according to ratio  $\gamma_t^{(n)}$  the required value of asymmetrical short circuit current periodic component at the time  $t$  is determined:

$$I_{G,t}^{(n)} = m^{(n)} \gamma_t^{(n)} I_{1,G}^{(n)} \quad (5.85)$$

If there are several sources in the network, they should be divided into 2 groups. One of them comprises all sources electrically located near the point of short circuit (sources of finite power), another one all sources electrically remote of the point of short circuit (sources of infinite power). In

common case the point of short circuit can be beyond reactance  $x_s$ , that is general for these sources groups. Even if  $x_{s,1} = 0$ , in the equivalent circuit for positive sequence, in the integrated equivalent circuit the reactance  $x_{s,1}$  is for  $x_A^{(n)}$ .

The following algorithm is used for determining asymmetrical short circuit mode parameters in this case:

1) positive sequence current at the point of short circuit  $I_{sA1\Sigma}^{(n)}$  is determined;

2) positive sequence currents for each source at the instant  $t = 0$  are determined;

3) electrical remoteness of sources from the point of short circuit is estimated by ratio

$$I_{Gi}^{(n)} / I_{G,rated,i}^{(n)}$$

4) individually introduced into the calculated circuit sources are subdivided into two groups by electrical remoteness  $I_{Gi}''/I_{G,rated,i}^2 \leq 2$  and  $I_{Gj}''/I_{G,rated,j}^2 > 2$ ;

5) integrated equivalent circuit is reduced to the form of three-ray star where the reactance  $x_{\Delta}^{(n)}$  is introduced into the branch with the point  $SC_1^{(n)}$ ;

6) positive sequence total current  $I_{Gj\Sigma}''^{(n)}$  of branches  $I_{Gj}''/I_{G,rated,j}^2 > 2$  is determined;

7) the ratio  $I_{Gj\Sigma}''^{(n)}/I_{sA1\Sigma}''^{(n)}$  is found;

8) the ratio  $I_{Gt}/I_G''$  for calculated instant and electrical remoteness  $I_{Gj\Sigma}''^{(n)}/\sum_{j=1}^2 I_{G,rated,j}''$  is

found, and then using additional standard curve  $(I_{F,t}/I'') = f_2(I_{Gt}/I_G'')$ , correspondent to the ratio found in p.7, the ratio

$$\gamma_t^{(n)} = I_{G,t}/I'' \quad ; \text{ is determined} \quad (5.86)$$

9) the required periodic rms current component at the point of short circuit for the time instant  $t$  is determined:

$$I_{F,t}^{(n)} = m^{(n)} \gamma_t^{(n)} I_{sA1\Sigma}''^{(n)} \quad (5.87)$$

#### **Calculation of current at asymmetrical short circuit for $t > 0$ using calculated curves for generators**

The succession of calculation with the use of design curves is as follows:

1) positive, negative and zero sequence equivalent circuits are made up and their parameters are defined;

2) these equivalent circuits are transformed to equivalent circuits of separate sequences with  $x_{1res}$ ,  $x_{2res}$ ,  $x_{0res}$  referred to the point of short circuit: resulting reactance of integrated equivalent circuit for asymmetrical short circuit of a given kind is determined by

$$x_{res}^{(n)} = x_{1res} + x_{\Delta}^{(n)} \quad (5.88)$$

3) rated impedance of powering rays is determined (the number of rays can be  $j = 1, \dots, N$ ): in general case for generator  $j$ -th ray

$$x_{*(rated)des,j}^{(n)} = x_{res}^{(n)} S_{rated,j} / (C_j U_b^2) \quad (5.89)$$

or

$$x_{*(rated)des,j}^{(n)} = x_{*(b)res}^{(n)} S_{rated,j} / (C_j S_b), \quad (5.90)$$

where  $S_{rated,j}$  - is rated power of the selected  $j$ -th generator;  $C_j$  - is coefficient of current distribution for the  $j$ -th ray, determined in positive sequence circuit (calculating in general case  $C = I$ , and total rated power of all generators in the circuit is taken as  $S_{rated,j}$ ); the reactance for a ray with infinite unrestricted power source is determined by expression

$$x_{*GS} = (x_{*1res} + x_{*\Delta}^{(n)}) / C_{GS} \quad (5.91)$$

where  $C_{GS}$  - is coefficient of current distribution for a ray trough which connection to the source of infinite power is made (if such a source is connected to the point of short circuit by several rays the sum of corresponding current distribution coefficients is taken as  $C_{GS}$ );

4) positive sequence current in per unit  $I_{sA1t,j}^{(n)}$  is determined for the source of limited power (with  $x_{*des} < 3$ ) according to design curves by the design impedance value of j-th for a given time  $t$ ; positive sequence current is determined in concrete units

$$I_{sA1t,j}^{(n)} = I_{*(rated)A1t,j} \cdot I_{rated,j}; \quad (5.92)$$

5) when the point of short circuit is electrically remotod from the sources,  $x_{*(des)j}^{(n)} \geq 3$  and it can be assumed

$$I_{sA1t,j}^{(n)} = I_{rated,j} / x_{*(rated)des,j}^{(n)} \quad (5.93)$$

or positive sequence current generated by such a source for all time instants is determined according to the formula

$$I_{sA1t,GS} = I_b / x_{*(b)GS} \quad (5.94)$$

if source reactance  $x_{*(b)GS}$  is given in per unit or according to the formula

$$I_{sA1t,GS} = U_b / (\sqrt{3}x_{GS}), \quad (5.95)$$

is reactance  $x_{GS}$  is given in named measurements units

6) periodic current component r.m.s. at the point of asymmetrical short circuit for instant  $t$  is

$$I_{F,t}^{(n)} = I_{s,t}^{(n)} = m^{(n)} \left( \sum_{j=1}^N I_{sA1t,j}^{(n)} + I_{sA1t,GS} \right). \quad (5.96)$$

**Current calculation asymmetrical short circuit by method of rectified characteristics** starts with making up equivalent circuits for positive, negative and zero sequences. Synchronous generators with excitation control device are introduced by inductive reactance  $x_t$  and emf  $E_t$  (or voltage on generator terminals), with values calculated for mode of a generator with automatic excitation control device, feeding an independent load, is determined comparing external relative to the generator terminals impedance of circuit with the correspondent critical impedance, calculated for the instant  $t$  (see chapter 3). For power supply system with several power sources operation mode is chosen approximately with the following re-estimation: . The estimation is accomplished by either positive sequence generator current with its critical current or positive sequence residual voltage on generator terminals with its rated voltage. For excitation rise mode the condition  $I_{1,G,t} \geq I_{gr,t}$  or  $U_{1,G,t} < U_{rated}$ , should be provided and for normal voltage mode  $I_{1,G,t} \leq I_{cr,t}$  or  $U_{1,G,t} = U_{rated}$ .

The network as the source of infinite power is introduced into equivalent circuit of positive sequence by corresponding constant emf and impedance equal to zero. Loads should be introduced into the equivalent circuit at points of their actual connection with  $x_{*l,ld} = 1, 2$  and  $E_{ld} = 0$ .

Component impedances in equivalent circuits for negative and zero sequences do not depend on time and are the same as when  $t = 0$ .

The algorithm of asymmetrical short circuit current calculation by method of rectified characteristic is the following

1) operation modes for synchronous generators with automatic excitation control device is chosen, and their corresponding parameters are defined;

2) equivalent circuits for positive, negative and zero sequences are designed calculating parameters of these circuits;

3) equivalent circuits of separate sequences are made equalized relative to the point of short circuit determining resulting sequence impedances and emfs

4) positive sequence current at the point of short circuit is calculated according to the formula (5.41);

5) currents in the branches of synchronous generators with automatic excitation control are determined; the currents are compared with corresponding critical currents;

6) the choice of proper modes is checked up. If mode of generators with automatic excitation control is chosen right, next stage of calculation is accomplished. If the mode for one generator at least is chosen wrong calculation is accomplished again taking another mode for the generator that is the first stage of calculation is repeated. For a given generator emf is taken equal to voltage and impedance is taken equal to zero. In this case only positive sequence equivalent circuit is changed. Negative and zero sequence equivalent circuits and corresponding resulting sequence impedances remain unchanged.

7) the following stages of calculation coincide with the calculation of initial value of asymmetrical short circuit periodic current component beginning with stage 4.

### Test questions

1. What types of asymmetrical faults can occur in electric power supply network and what are their peculiarities?
2. How can the stage of lateral asymmetry in power supply network be presented in general case?
3. What are boundary conditions for short circuits of all kinds?
4. What is the difference between equivalent circuits for positive, negative, and zero sequences?
5. How are currents and voltages of single-phase short circuit determined?
6. What are vector diagrams of currents and voltages of the special phase short circuit?
7. How are currents and voltages of two-phase short circuit determined?
8. What are vector diagrams of currents and voltages of two-phase short circuit?
9. How are currents and voltages of two-phase to-ground short circuit?
10. What are vector diagrams of currents and voltages of two-phase to-ground short circuit?
11. What is proportionally coefficient for currents of short circuit of different kinds?
12. What is equivalence rule for positive sequence current?
13. How is additional impedance for different kinds of asymmetrical short circuit determined?
14. How can actual value of a periodic current component at the initial instant of asymmetrical short circuit be found?
15. What are integrated equivalent circuits of asymmetrical short circuits?
16. What are dependences between additional impedances (currents, voltages) at different kinds of short circuits?
17. What are practical cases when maximum (minimum) values of asymmetrical short circuits currents ratios occur?
18. How are vector diagrams of currents and voltages of different kinds of short circuit changed with remoteness from the point of short circuit and at passing the transformer with vector group  $Y/Y-12$  and  $\Delta/Y-11$ ?
19. How is initial value of asymmetrical short circuit periodic current component calculated?
20. What are characteristics of asymmetrical short circuit current calculation by method of typical curves; by method of design curves; by method rectified characteristics?

### Topics for essay

1. Calculation of asymmetrical short circuit (single-phase, two-phase and two-phase to-ground) by method of symmetrical components.
2. General calculation ratios of asymmetrical short circuits at one point and their current and voltage vector diagrams
3. Integrated equivalent circuits for asymmetrical short circuit. Examples of equivalent circuits designing.



## **CHAPTER 6: LONGITUDINAL ASYMMETRY AND COMPLEX FAULTS**

- 6.1. General statements
- 6.2. Open-phase fault in three-phase network
- 6.3. Two-phase open fault
- 6.4. Connection of unequal resistors in network phases
- 6.5. Double ground connection
- 6.6. Single-phase short circuit with simultaneous open-phase fault

Test questions

Topics for essay

### 6.1. General statements

The final aim of analysis of faults, such as longitudinal asymmetry and complex failure is *calculation* of currents and voltages in circuits, branches, nodes, given points of electric supply network and point of fault. This calculation is necessary for substantiation of basic data for selection of electric power supply network equipment, protection of its elements and for adjusting and analysis of system automatic devices operating modes.

For analysis formalization the longitudinal asymmetry is represented as connection of unequal impedances in each phase of three-phase system. Only harmonic of the main mode is considered and the following assumptions are made:

- connection of impedances in networks phase at constant electromotive force of a source is equivalent to shunting the same impedances in other phases;
- shunting of an impedance in network phase is equivalent to connection the same impedance with opposite sign;
- open-phase fault is equivalent to connection of a voltage source at the fault point, which magnitude equal to the voltage drop at the part of the broken phase.

Application of symmetrical components method is effective for calculation of longitudinal asymmetry ( $L$ ) as well as for lateral asymmetry. According to it calculated values can be evaluated using phase  $A$  symmetrical components, taken as a **special phase**,

$$\left. \begin{aligned} \Delta \underline{U}_{LA} &= \Delta \underline{U}_{LA1} + \Delta \underline{U}_{LA2} + \Delta \underline{U}_{LA0}; \\ \Delta \underline{U}_{LB} &= a^2 \Delta \underline{U}_{LA1} + a \Delta \underline{U}_{LA2} + \Delta \underline{U}_{LA0}; \\ \Delta \underline{U}_{LC} &= a \Delta \underline{U}_{LA1} + a^2 \Delta \underline{U}_{LA2} + \Delta \underline{U}_{LA0} \end{aligned} \right\} \quad (6.1)$$

and

$$\left. \begin{aligned} \underline{I}_{LA} &= \underline{I}_{LA1} + \underline{I}_{LA2} + \underline{I}_{LA0}; \\ \underline{I}_{LB} &= a^2 \underline{I}_{LA1} + a \underline{I}_{LA2} + \underline{I}_{LA0}; \\ \underline{I}_{LC} &= a \underline{I}_{LA1} + a^2 \underline{I}_{LA2} + \underline{I}_{LA0}, \end{aligned} \right\} \quad (6.2)$$

where  $\underline{I}_{LA}, \underline{I}_{LB}, \underline{I}_{LC}$  and  $\Delta \underline{U}_{LA}, \Delta \underline{U}_{LB}, \Delta \underline{U}_{LC}$  - are currents and voltages of asymmetrical phase system  $A, B, C$ ;  $\underline{I}_{LA1}, \underline{I}_{LA2}, \underline{I}_{LA0}$  and  $\Delta \underline{U}_{LA1}, \Delta \underline{U}_{LA2}, \Delta \underline{U}_{LA0}$  - currents and voltages symmetrical components of positive, negative and zero sequences.

Currents of separate sequences cause voltage drops of corresponding sequences. Their ratio is described by the system of independent equations:

$$\left. \begin{aligned} \Delta \underline{U}_{LA1} &= \underline{E}_{A\Sigma} - \underline{z}_{1\text{res}} \underline{I}_{LA1}; \\ \Delta \underline{U}_{LA2} &= -\underline{z}_{2\text{res}} \underline{I}_{LA2}; \\ \Delta \underline{U}_{LA0} &= -\underline{z}_{0\text{res}} \underline{I}_{LA0}, \end{aligned} \right\} \quad (6.3)$$

where  $\underline{E}_{A\Sigma}$  - is the total power sources electromotive force, acting only in positive sequence circuit;  $\underline{z}_{1\text{res}}, \underline{z}_{2\text{res}}, \underline{z}_{0\text{res}}$  - are total impedances of separate sequences relative to longitudinal symmetry fault point.

Thus, calculated ratios can be obtained by principles same to those for lateral asymmetry, based on solution of equation system (6.1) - (6.3) with consideration of boundary conditions

peculiar to asymmetry. The task of solution is to estimate correlation equations of asymmetrical phase values with their symmetrical components.

Real electric circuit scheme for single longitudinal asymmetry (one-phase and two-phases open fault, switching of unequal impedances) can be reduced to equivalent circuits without open fault. This is done by introduction of voltage source at the point of a fault. Its value must be equal to voltage drop at the point of longitudinal asymmetry. The equivalent circuit obtained can be analyzed by analytical methods of electric circuit theory.

For the **special phase**, as in the case of lateral asymmetry, equivalent circuits for separate sequences are made-up. Design ratios for synthesis of integrated equivalent circuit of specific kind of longitudinal asymmetry can be estimated using them. Using the integrated equivalent circuit of the **special phase** currents and voltages at arbitrary point of electric network are estimated.

Single longitudinal asymmetry in three-phase network can be caused by non-simultaneous phase switching, open fault, unequal phase load, etc. The longitudinal asymmetry occurs at non-simultaneous disconnection of switching unit contacts (an arc caused by switching off a current occurs non-simultaneously between contacts in different phases), at blowing of fuses in one or two phases, non-synchronous starting of synchronous motors, emergency switching of electric network's phases.

The longitudinal and lateral asymmetries can occur simultaneously in electric networks in different combinations, these modes are **complex failures**. They can be caused by superimposition of emergencies, or emergency with its protective cutout (for example non-simultaneous asymmetrical short circuit cutout in doubly-fed supply network, simultaneous arise of asymmetrical short circuits in several points of network, ground short circuit with simultaneous open-phase fault, etc).

When analyzing complex failures, calculation algorithm is to be implemented for each point where longitudinal symmetry was violated. Method of symmetrical components allows presenting such points of **special phase** via three symmetrical currents and voltages. So, double asymmetry implies calculation of 12 unknown symmetrical components. To determine them, 12 equations must be made up. Boundary conditions for each point of symmetry breakdown stipulate three equations, describing relationships between symmetrical components of current and voltage.

### 6.2. Open-phase fault in three-phase network

Open-phase fault in three-phase system (Fig. 6.1) causes asymmetrical mode, characterized at the point of fault by the following boundary conditions:

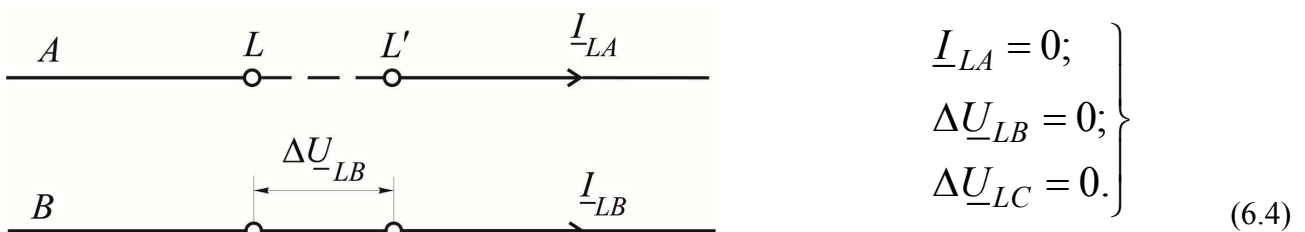


Fig. 6.1. Three-phase subcircuit with A-phase fault at the point  $L - L'$

An additional voltage source  $\Delta \dot{U}_{LA}$  (Fig. 6.2, a) is introduced at the phase fault point for this emergency analysis. Then, equivalent circuits for separate sequences are to be made-up for separate sequences (Fig. 6.2, b-c).

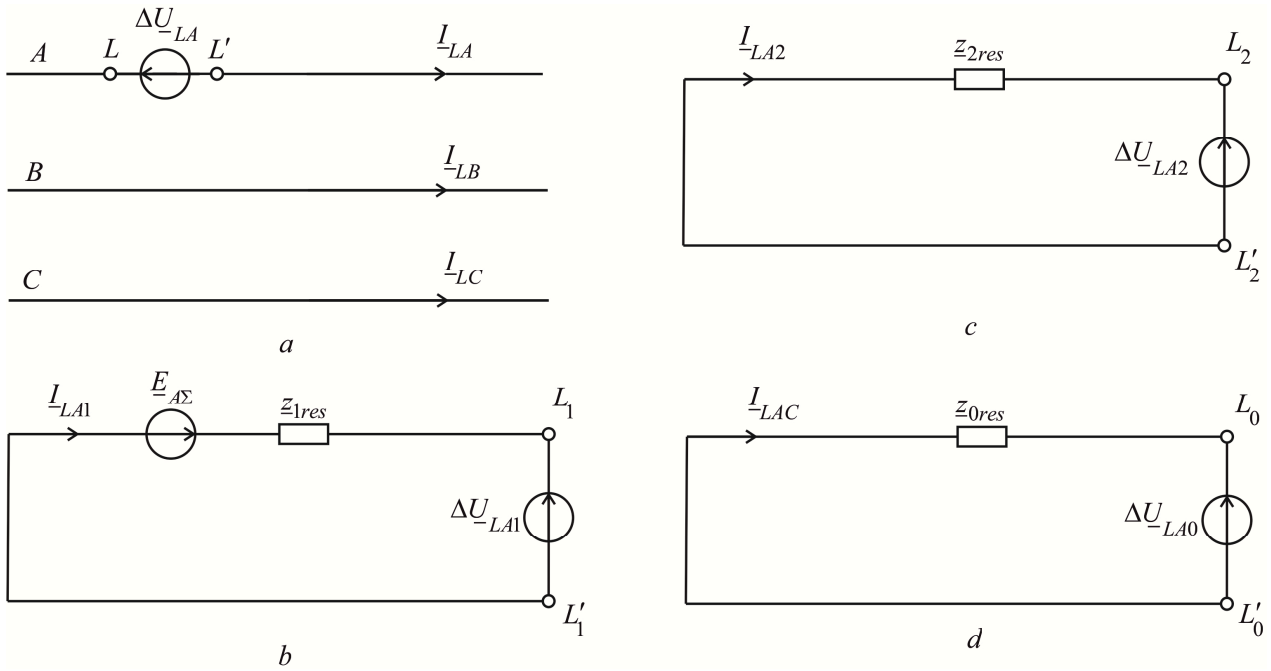


Fig. 6.2. Circuits for analysis of longitudinal asymmetry under  $A$ -phase open fault:  $a$  – design circuit;  $b$  – equivalent circuit for positive sequence;  $c$  – equivalent circuit for negative sequence;  $d$  – equivalent circuit for zero sequence

Comparison of voltage drops on unfaulted phases, expressed by its symmetrical components of  $A$ -phase results in:

$$a^2 \Delta \underline{U}_{-LA1} + a \Delta \underline{U}_{-LA2} + \Delta \underline{U}_{-LA0} = a \Delta \underline{U}_{-LA1} + a^2 \Delta \underline{U}_{-LA2} + \Delta \underline{U}_{-LA0} \Rightarrow \\ \Rightarrow \Delta \underline{U}_{-LA1} = \Delta \underline{U}_{-LA2};$$

$$\Delta \underline{U}_{-LB} = (a^2 + a) \Delta \underline{U}_{-LA1} + \Delta \underline{U}_{-LA0} = -\Delta \underline{U}_{-LA1} + \Delta \underline{U}_{-LA0} = 0 \Rightarrow \\ \Rightarrow \Delta \underline{U}_{-LA1} = \Delta \underline{U}_{-LA0}.$$

Thus, boundary conditions can be presented by symmetrical components (6.4) as

$$\Delta \underline{U}_{-LA1} = \Delta \underline{U}_{-LA2} = \Delta \underline{U}_{-LA0} = \Delta \underline{U}_{-LA} / 3; \quad (6.5)$$

$$\underline{I}_{-LA1} = -(\underline{I}_{-LA2} + \underline{I}_{-LA0}). \quad (6.6)$$

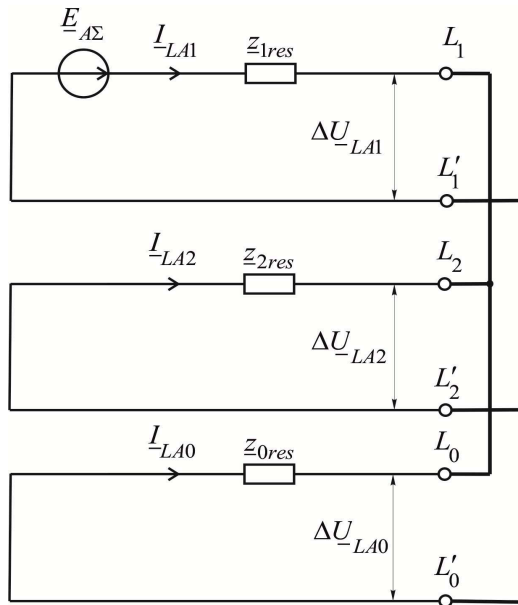


Fig. 6.3. Integrated equivalent circuit at A-phase fault

These equations allow compiling integrated equivalent circuit for longitudinal asymmetry being considered (Fig. 6.3).

According to it calculating equations for positive sequence current determination should be generated:

$$\underline{I}_{-LA1} = \underline{E}_{A\Sigma} / (\underline{z}_{1res} + \underline{z}_{2res}), \quad (6.7)$$

and for determination of positive sequence voltage drop at the point of fault

$$\Delta \underline{U}_{-LA1} = \underline{I}_{-LA1} \underline{z}_{LL1}, \quad (6.8)$$

where  $\underline{z}_{LL1} = \underline{z}_{2res} \underline{z}_{0res} / (\underline{z}_{2res} + \underline{z}_{0res})$  - additional impedances in positive sequence equivalent circuit relative to terminals  $L_1 - L'_1$  by arms of equivalent circuits of negative and zero sequences (Fig. 6.3).

Currents of negative and zero sequences flowing in other arms of equivalent circuit can be determined taking into account (6.5), second and third equation of (6.3) by

$$\underline{I}_{-LA2} = -\underline{I}_{-LA1} \underline{z}_{0res} / (\underline{z}_{2res} + \underline{z}_{0res}) = -\underline{I}_{-LA1} \underline{z}_{LL1} / \underline{z}_{2res}; \quad (6.9)$$

$$\underline{I}_{-LA0} = -\underline{I}_{-LA1} \underline{z}_{2res} / (\underline{z}_{2res} + \underline{z}_{0res}) = -\underline{I}_{-LA1} \underline{z}_{LL1} / \underline{z}_{0res}. \quad (6.10)$$

Currents of negative and zero sequences can be expressed using impedances and EMFs of integrated equivalent circuits by

$$\underline{I}_{-LA2} = -\underline{E}_{A\Sigma} \underline{z}_{LL1} / [\underline{z}_{2res} (\underline{z}_{1res} + \underline{z}_{LL1})]; \quad (6.11)$$

$$\underline{I}_{-LA0} = -\underline{E}_{A\Sigma} \underline{z}_{LL1} / [\underline{z}_{0res} (\underline{z}_{1res} + \underline{z}_{LL1})]. \quad (6.12)$$

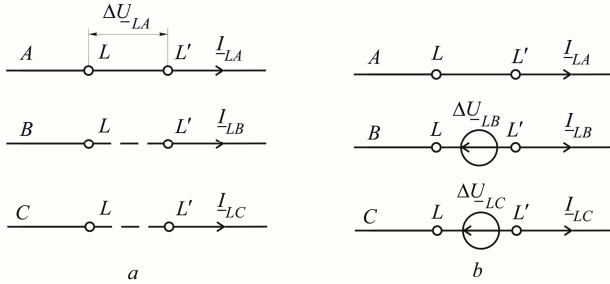
Expression for voltage source, introduced at the point of a fault, is determined by (6.5) and (6.3)

$$\Delta \underline{U}_{-LA} = 3 \underline{E}_{A\Sigma} \underline{z}_{LL1} / (\underline{z}_{1res} + \underline{z}_{LL1}). \quad (6.13)$$

These design formulae (6.5-6.13) are equations describing ratios between symmetrical mode components for special phase. Currents and voltages in other phases are expressed via turning operator using (6.1) and (6.2). Voltages at arbitrary points of network are determined by calculated currents using (6.7), (6.11) and (6.12) according to transformed integrated equivalent circuit (Fig. 6.3) relative to certain point of network for estimation of impedance between this point and power source.

### 6.3. Two-phase open fault

Two-phases open fault (Fig. 6.4,a) is described at the point of a fault by the following boundary conditions:



$$\left. \begin{aligned} \underline{I}_{LB} &= 0; \\ \underline{I}_{LC} &= 0; \\ \Delta \underline{U}_{LA} &= 0. \end{aligned} \right\} \quad (6.14)$$

Fig. 6.4. Three-phase network subcircuit under phases B and C open fault and its design circuit (b)

Design circuit allows generating integrated equivalent circuit (Fig. 6.5). Assuming phase A as healthy boundary conditions (6.14) can be expressed via symmetrical components of current in special phase as:

$$\underline{I}_{LB} = a^2 \underline{I}_{LA1} + a \underline{I}_{LA2} + \underline{I}_{LA0}; \quad (6.15)$$

$$\underline{I}_{LC} = a \underline{I}_{LA1} + a^2 \underline{I}_{LA2} + \underline{I}_{LA0}. \quad (6.16)$$

Difference of obtained equations yields

$$\underline{I}_{LA1} = \underline{I}_{LA2}. \quad (6.17)$$

After substitution of (6.17) into equations (6.15), (6.16) and algebraic transformations we obtain the following

$$\underline{I}_{LA1} = \underline{I}_{LA2} = \underline{I}_{LA0} = \underline{I}_{LA} / 3. \quad (6.18)$$

Decomposition of boundary condition  $\Delta \underline{U}_{LA} = 0$  into symmetrical components yields the equation

$$\Delta \underline{U}_{LA1} = -(\Delta \underline{U}_{LA2} + \Delta \underline{U}_{LA0}), \quad (6.19)$$

Equations (6.18) and (6.19) describe boundary conditions via voltage symmetrical components on phase A for this type of fault.

According to generated by (6.18) and (6.19) integrated equivalent circuit for phase A (Fig. 6.5) we can determine current symmetrical components as

$$\underline{I}_{LA1} = \underline{I}_{LA2} = \underline{I}_{LA0} = \underline{E}_{A\Sigma} / (\underline{z}_{1res} + \underline{z}_{2res} + \underline{z}_{0res}). \quad (6.20)$$

Voltage drops of negative and zero sequences are defined by (6.3). Positive sequence voltage are determined by

$$\Delta \underline{U}_{LA1} = \underline{E}_{A\Sigma} (\underline{z}_{2res} + \underline{z}_{0res}) / (\underline{z}_{1res} + \underline{z}_{2res} + \underline{z}_{0res}). \quad (6.21)$$

Symmetrical components of mode's parameters in phase A allows estimation of currents and voltages at fault point or any random point of network using equations (6.1)-(6.3). The latter case requires transformation of integrated equivalent circuit relative to point considered. Equivalent circuit shown in Fig. 6.5 is presented relative to fault point.

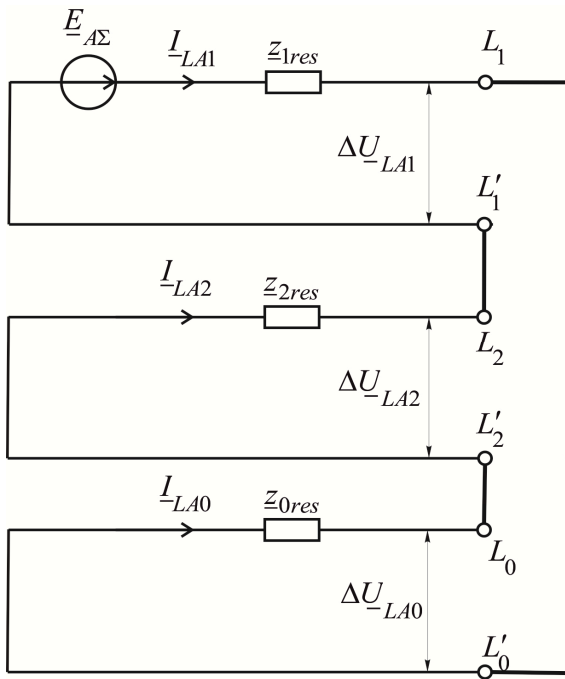


Fig. 6.5. Integrated equivalent circuit for phases B and C open fault

#### 6.4. Connection of unequal resistors in network phases

The reason for longitudinal asymmetry in electric supply network is connection of impedance in one or two phases i.e. phase load unbalance (Fig. 6.6).

**Connection of impedance  $\underline{z}$  into one phase** is described at the point of asymmetry by the following boundary conditions:

$$\left. \begin{aligned} \Delta \underline{U}_{LA} &= \underline{I}_{LA} \underline{z}; \\ \Delta \underline{U}_{LB} &= 0; \\ \Delta \underline{U}_{LC} &= 0. \end{aligned} \right\} \quad (6.22)$$

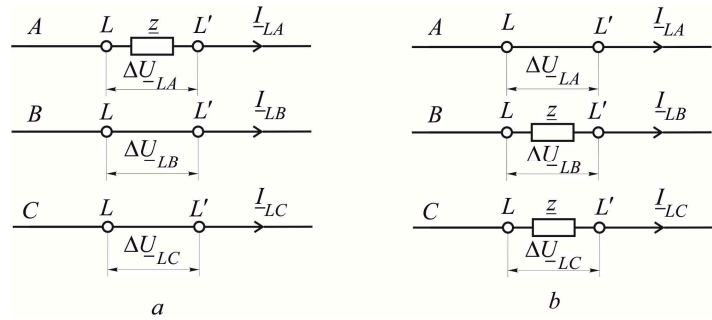


Fig. 6.6. Longitudinal asymmetry under connection of impedances in phases: a – in one phase; b – in two phases

For analysis of this mode we introduce in phase voltage  $\Delta \underline{U}_{LA}$  instead of impedance  $\underline{z}$  (Fig. 6.6, a). Under  $\underline{z} \rightarrow \infty$  boundary conditions (6.22) turns to particular case of boundary conditions for phase  $A$  open fault (6.4), because

$$\underline{I}_{LA} = \Delta \underline{U}_{LA} / \underline{z} \Big|_{\underline{z} \rightarrow \infty} = 0. \quad (6.23)$$

This allows generalizing longitudinal asymmetry mode analysis under  $\underline{z} \neq \infty$ .

Under decomposition of boundary conditions (6.22) into symmetrical components relative to special phase  $A$ , equation (6.5) obtained before remains valid, and, besides

$$\Delta \underline{U}_{LA} = (\underline{I}_{LA1} + \underline{I}_{LA2} + \underline{I}_{LA0}) \underline{z}. \quad (6.24)$$

Simultaneous transformation of (6.3), (6.5) and (6.24) yields expressions for generated integrated equivalent circuit (Fig. 6.7, a):

$$\underline{I}_{LA1} = (\underline{E}_{A\Sigma} - \Delta \underline{U}_{LA1}) / \underline{z}_{1res}; \quad (6.25)$$

$$\underline{I}_{LA1} = \Delta \underline{U}_{LA1} (3 / \underline{z} + 1 / \underline{z}_{2res} + 1 / \underline{z}_{0res}). \quad (6.26)$$

To estimate positive sequence current, transform integrated equivalent circuit as it is shown in Fig. 6.7, b. Additional impedance plugged at the point of asymmetry is defined using (6.26) as

$$\underline{z}_{LL1} = 1 / (3 / \underline{z} + 1 / \underline{z}_{2res} + 1 / \underline{z}_{0res}). \quad (6.27)$$

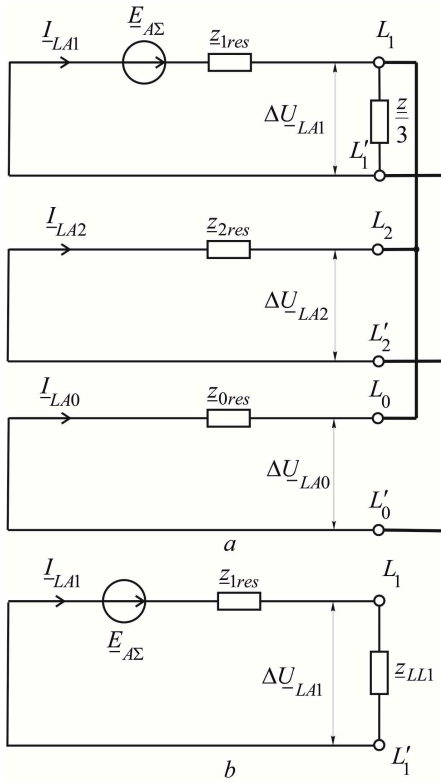


Fig. 6.7. Integrated equivalent circuit (a) and its equivalent transformations (b)

Expression (6.27) under  $\underline{z} \rightarrow \infty$  yields (6.8), describing open-phase fault.

Symmetrical components of currents and voltages, expressed using impedance  $\underline{z}_{LL1}$ , can be found by (6.7), (6.11), (6.12) and (6.3), (6.5). On the basis of currents and voltages symmetrical components in phase  $A$ , one can determine operating parameters of all phases at the fault point, as it was given above.

**Connection of impedance  $\underline{z}$  into two phases** of three-phase network is described by the following boundary conditions (Fig. 6.6, b) for longitudinal asymmetry point:

$$\left. \begin{aligned} \Delta \underline{U}_{LA} &= 0; \\ \Delta \underline{U}_{LB} &= \dot{I}_{LB} \underline{z}; \\ \Delta \underline{U}_{LC} &= \dot{I}_{LC} \underline{z}. \end{aligned} \right\} \quad (6.28)$$

Equations (6.28) can be transformed using symmetrical components of currents and voltages of special phase  $A$ :

$$\Delta \underline{U}_{LA1} + \Delta \underline{U}_{LA2} + \Delta \underline{U}_{LA0} = 0; \quad (6.29)$$

$$a^2 \Delta \underline{U}_{LA1} + a \Delta \underline{U}_{LA2} + \Delta \underline{U}_{LA0} = (a^2 \underline{I}_{LA1} + a \underline{I}_{LA2} + \underline{I}_{LA0}) \underline{z}; \quad (6.30)$$

$$a \Delta \underline{U}_{LA1} + a^2 \Delta \underline{U}_{LA2} + \Delta \underline{U}_{LA0} = (a \underline{I}_{LA1} + a^2 \underline{I}_{LA2} + \underline{I}_{LA0}) \underline{z}. \quad (6.31)$$

Their solution for voltage drops of positive, negative and zero sequence yields:

$$\left. \begin{aligned} \Delta \underline{U}_{LA1} &= (2 \underline{I}_{LA1} - \underline{I}_{LA2} - \underline{I}_{LA0}) \underline{z} / 3; \\ \Delta \underline{U}_{LA2} &= (-\underline{I}_{LA1} + 2 \underline{I}_{LA2} - \underline{I}_{LA0}) \underline{z} / 3; \\ \Delta \underline{U}_{LA0} &= (-\underline{I}_{LA1} - \underline{I}_{LA2} + 2 \underline{I}_{LA0}) \underline{z} / 3. \end{aligned} \right\} \quad (6.32)$$

Thus, symmetrical components of signals in phase  $A$  can be expressed by equation sets (6.3) and (6.32).

Equations (6.3) and (6.32), transformed using symmetrical components of current in phase  $A$  are

$$\left. \begin{aligned} \underline{I}_{LA1} (2 + 3 \underline{z}_{1res} / \underline{z}) - \underline{I}_{LA2} - \underline{I}_{LA0} &= 3 \underline{E}_{A\Sigma} / \underline{z}; \\ -\underline{I}_{LA1} + \underline{I}_{LA2} (2 + 3 \underline{z}_{2res} / \underline{z}) - \underline{I}_{LA0} &= 0; \\ -\underline{I}_{LA1} - \underline{I}_{LA2} + \underline{I}_{LA0} (2 + 3 \underline{z}_{0res} / \underline{z}) &= 0. \end{aligned} \right\} \quad (6.33)$$



Solution of equations set (6.33) yields expressions for estimation of positive, negative and zero sequences currents:

$$\left. \begin{aligned} \underline{I}_{LA1} &= \underline{E}_{A\Sigma} / (\underline{z}_{1res} + \underline{z}_{LL1}); \\ \underline{I}_{LA2} &= \underline{E}_{A\Sigma} (\underline{z} - \underline{z}_{LL1}) / [(\underline{z}_{1res} + \underline{z}_{LL1})(\underline{z} + \underline{z}_{2res})]; \\ \underline{I}_{LA0} &= \underline{E}_{A\Sigma} (\underline{z} - \underline{z}_{LL1}) / [(\underline{z}_{1res} + \underline{z}_{LL1})(\underline{z} + \underline{z}_{0res})], \end{aligned} \right\} \quad (6.34)$$

where additional impedance in equivalent circuit of positive sequence, plugged at the point of asymmetry, is determined by

$$\underline{z}_{LL1} = \underline{z} \left( \frac{\underline{z}\underline{z}_{2res}}{\underline{z} + \underline{z}_{2res}} + \frac{\underline{z}\underline{z}_{0res}}{\underline{z} + \underline{z}_{0res}} \right) / \left( \underline{z} + \frac{\underline{z}\underline{z}_{2res}}{\underline{z} + \underline{z}_{2res}} + \frac{\underline{z}\underline{z}_{0res}}{\underline{z} + \underline{z}_{0res}} \right). \quad (6.35)$$

Voltage drops of positive, negative and zero sequences can be calculated considering equation sets (6.34) and (6.3) by formulae

$$\left. \begin{aligned} \Delta \underline{U}_{LA1} &= \underline{E}_{A\Sigma} \underline{z}_{LL1} / (\underline{z}_{1res} + \underline{z}_{LL1}); \\ \Delta \underline{U}_{LA2} &= -\underline{E}_{A\Sigma} (\underline{z} - \underline{z}_{LL1}) \underline{z}_{2res} / [(\underline{z}_{1res} + \underline{z}_{LL1})(\underline{z} + \underline{z}_{2res})]; \\ \Delta \underline{U}_{LA0} &= -\underline{E}_{A\Sigma} (\underline{z} - \underline{z}_{LL1}) \underline{z}_{0res} / [(\underline{z}_{1res} + \underline{z}_{LL1})(\underline{z} + \underline{z}_{0res})]. \end{aligned} \right\} \quad (6.36)$$

Validation of these expressions is accomplished by checking equality between sum of equations (6.36) and first equation for boundary conditions (6.28).

By summing equations (6.3) and expression (6.35) we can generate integrated equivalent circuit of special phase in case of connection impedances  $\underline{z}$  in two phases (Fig. 6.35).

The expression is

$$\underline{E}_{A\Sigma} = \underline{I}_{LA1} \underline{z}_{1res} + \underline{I}_{LA2} \underline{z}_{2res} + \underline{I}_{LA0} \underline{z}_{0res}. \quad (6.37)$$

Using this scheme we can determine symmetrical components of *A*-phase values at the point of fault or any random point. Currents and voltages in other phases are determined by (6.1) and (6.2).

So, analytical analysis of each type single longitudinal asymmetry is accomplished by obtaining design ratios between phase values and their symmetrical components, and also by synthesis of integrated equivalent circuit based on them. Integrated equivalent circuit may be used for further computer-aided analysis, if necessary.

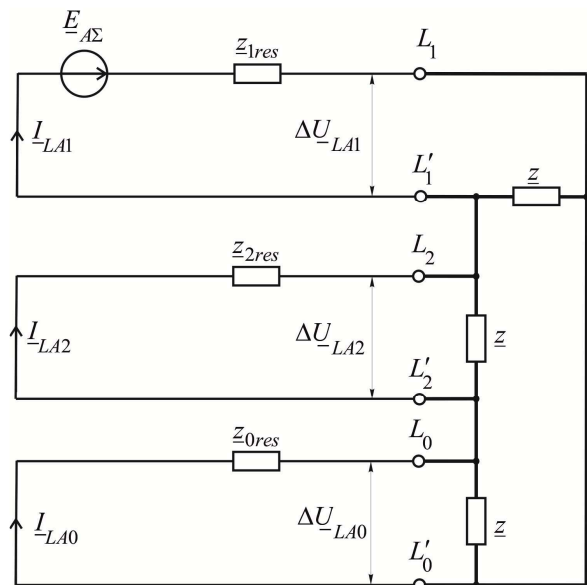


Fig. 6.8. Integrated equivalent circuit for special phase *A* in case of connection impedance  $\underline{z}$  in phases *B* and *C*

### 6.5. Double ground connection

Double ground connection refers to complex kind of fault, posing a simultaneous lateral and longitudinal asymmetry in separate subcircuits. Failures can appear at any instant in different points of network.

Analysis of symmetry breakdowns in two points of network (double asymmetry) is of practical interest for design and maintenance of industrial enterprises electric power supply networks. To this kind of asymmetry the following faults are referred: double ground connection in three-phase network with insulated neutral and single-phase ground short circuit with simultaneous open-phase fault in networks with grounded neutral.

The quantity of unknowns under double asymmetry is 12, because each point of symmetry breakdown is described by three symmetrical components of current and three symmetrical components of voltage. To determine them, the same quantity of independent equations are to be generated.

According to boundary conditions, there are three equations describing ratios between symmetrical components of currents and voltages of different sequences that can be generated for each point of symmetry breakdown. And according to equivalent circuit for each sequence the equations describing ratios between symmetrical components of currents and voltages of the same sequence.

The system of 12 unknowns can be solved analytically, by modeling or using computer. The latter principles are the most preferable because they allow analyzing and calculating mode parameters not only at the point of symmetry breakdown, but also at any random point. Computers perform equations set solution in matrix form according to typical programs of their software. To analyze solution by means of analog computing devices it is necessary to generate an integrated equivalent circuit for particular kind of double asymmetry.

In case of multiple symmetry breakdown, it is not allowable to generate integrated equivalent circuit for separate sequences only by electric coupling because of possible disturbance of currents distribution in the circuit. So, equivalent circuits for separate sequences are united to common electrically coupled circuit only in the point of symmetry breakdown. At the other point of asymmetry equivalent circuits for separate sequences are coupled in integrated equivalent circuit via adapter transformers.

Making up integrated equivalent circuits with adapter transformers requires remaining boundary conditions valid. So, selection of special phases for each point of symmetry breakdown must correspond to transformation gains and phase shift between currents and voltages of separate sequences. Besides, an error, introduced by adapter transformers in calculations must be taken into account.

*Double ground short circuit in three-phase networks with insulated neutral* (Fig. 6.9, a). Consider it under conditions when ground short circuit occurred at point  $L$  of phase  $B$  and at point  $M$  of phase  $C$ . Boundary conditions for asymmetry:

under short circuit in point  $L$

$$\left. \begin{aligned} \underline{I}_{LA} &= 0; \\ \underline{I}_{LC} &= 0; \\ \underline{U}_{LB} &= 0, \end{aligned} \right\} \quad (6.38)$$

under short circuit in point  $M$

$$\left. \begin{aligned} \underline{I}_{MA} &= 0; \\ \underline{I}_{MB} &= 0; \\ \underline{U}_{MC} &= 0. \end{aligned} \right\} \quad (6.39)$$

Currents in grounded phases are interrelated by additional condition

$$\underline{I}_{LB} = -\underline{I}_{MC}. \quad (6.40)$$

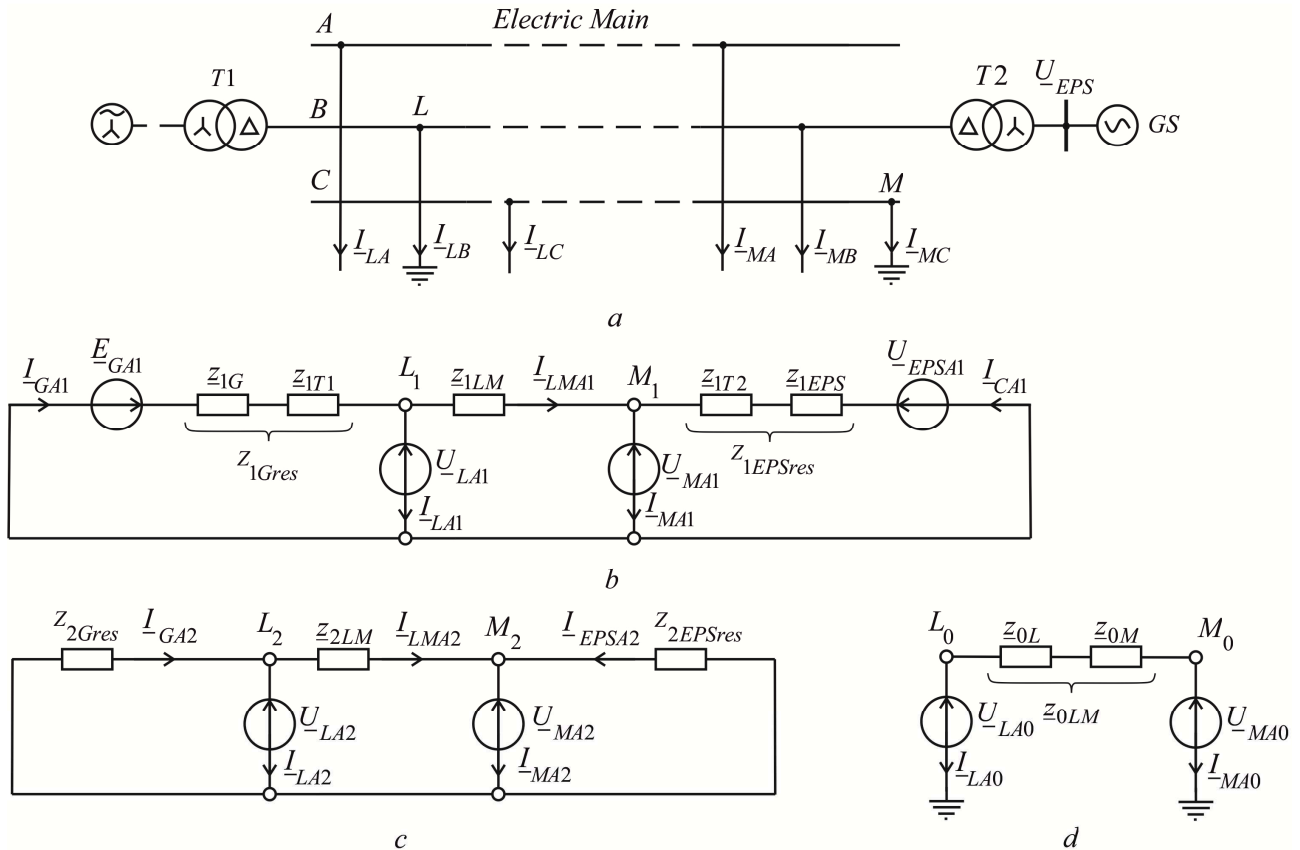


Fig. 6.9. Circuits for analysis of double ground short circuit in network with insulated neutral: a – design circuit; b – equivalent circuit for positive sequence; c – equivalent circuit for negative sequence; d – equivalent circuit for zero sequence

The analysis of correlation between dissimilar symmetrical current components and voltages at points of ground connection is given below.

For the point of connection to ground of phase  $B$  the boundary conditions are expressed as the following ratios between current and voltage symmetrical components ( $B$  is considered as a special phase):

$$\underline{I}_{LB1} = \underline{I}_{LB2} = \underline{I}_{LB0} = \underline{I}_{LB} / 3; \quad (6.41)$$

$$\underline{U}_{LB1} + \underline{U}_{LB2} + \underline{U}_{LB0} = 0. \quad (6.42)$$

Analogously, for the point of connection to ground of phase  $C$ , the boundary conditions (6.39) are expressed as the following ratios between current and voltage symmetrical components ( $C$  is considered as a special phase):

$$\underline{I}_{MC1} = \underline{I}_{MC2} = \underline{I}_{MC0} = \underline{I}_{MC} / 3; \quad (6.43)$$

$$\underline{U}_{MC1} + \underline{U}_{MC2} + \underline{U}_{MC0} = 0. \quad (6.44)$$

Comparison of components in equations (6.41) and (6.43) and condition (6.40) allows estimation of positional relationship of phasors of phase currents symmetrical components relative to ground contact points (Fig. 6.10) and of their interrelation:

$$\left. \begin{aligned} \underline{I}_{LB1} &= -\underline{I}_{MC1}; \\ \underline{I}_{LB2} &= -\underline{I}_{MC2}; \\ \underline{I}_{LB0} &= -\underline{I}_{MC0}. \end{aligned} \right\} \quad (6.45)$$

Fig. 6.10. Phasor diagrams of phases *B* and *C* currents' symmetrical components under double ground short circuit: a – at point *L*; b – at point *M*

Under simultaneous examination of short circuits at points *L* and *M* phase *A* is to be considered as special, because under complex fault it is under different conditions in comparison with phases *B* and *C*, grounded in according points. In this case, ratios (6.41)-(6.44) are:

$$a^2 \underline{I}_{LA1} = a \underline{I}_{LA2} = \underline{I}_{LA0} = \underline{I}_{LB} / 3; \quad (6.46)$$

$$a \underline{I}_{MA1} = a^2 \underline{I}_{MA2} = \underline{I}_{MA0} = \underline{I}_{MC} / 3; \quad (6.47)$$

$$a^2 \underline{U}_{LA1} + a \underline{U}_{LA2} + \underline{U}_{LA0} = 0; \quad (6.48)$$

$$a \underline{U}_{MA1} + a^2 \underline{U}_{MA2} + \underline{U}_{MA0} = 0. \quad (6.49)$$

For that, interrelation between symmetrical components of current in phase *A* at different points of short circuit is expressed by ratios, obtained by transformation of equations set (6.45):

$$\left. \begin{aligned} a \underline{I}_{LA1} &= -\underline{I}_{MA1}; \\ \underline{I}_{LA2} &= -a \underline{I}_{MA2}; \\ \underline{I}_{LA0} &= -\underline{I}_{MA0}. \end{aligned} \right\} \quad (6.50)$$

Design ratios between symmetrical components of currents and voltages of the same sequences can be obtained by analysis of their equivalent circuit (Fig. 6.9, b-d). Equivalent circuits of positive and negative sequences are transformed in order to single out grounded subcircuits with unknown symmetrical components of currents and voltages. Then, equipotential points (points of ground potential) are integrated and obtained delta-circuits are transformed into Y-connected ones (Fig. 6.11). EMFs in arms of transformed equivalent circuit for direct sequence (Fig. 6.11, a) are determined by formulae

$$\underline{E}_{LA} = [\underline{E}_{GA1} (\underline{z}_{1EPSres} + \underline{z}_{1LM}) + \underline{U}_{EPSA1} \underline{z}_{1G,res}] / \underline{z}_{1res}; \quad (6.51)$$

$$\underline{E}_{MA} = [\underline{E}_{GA1} \underline{z}_{1EPSres} + \underline{U}_{EPSA1} (\underline{z}_{1G,res} + \underline{z}_{1LM})] / \underline{z}_{1res}, \quad (6.52)$$

where  $\underline{z}_{1res} = \underline{z}_{1G,res} + \underline{z}_{1LM} + \underline{z}_{1EPSres}$ .

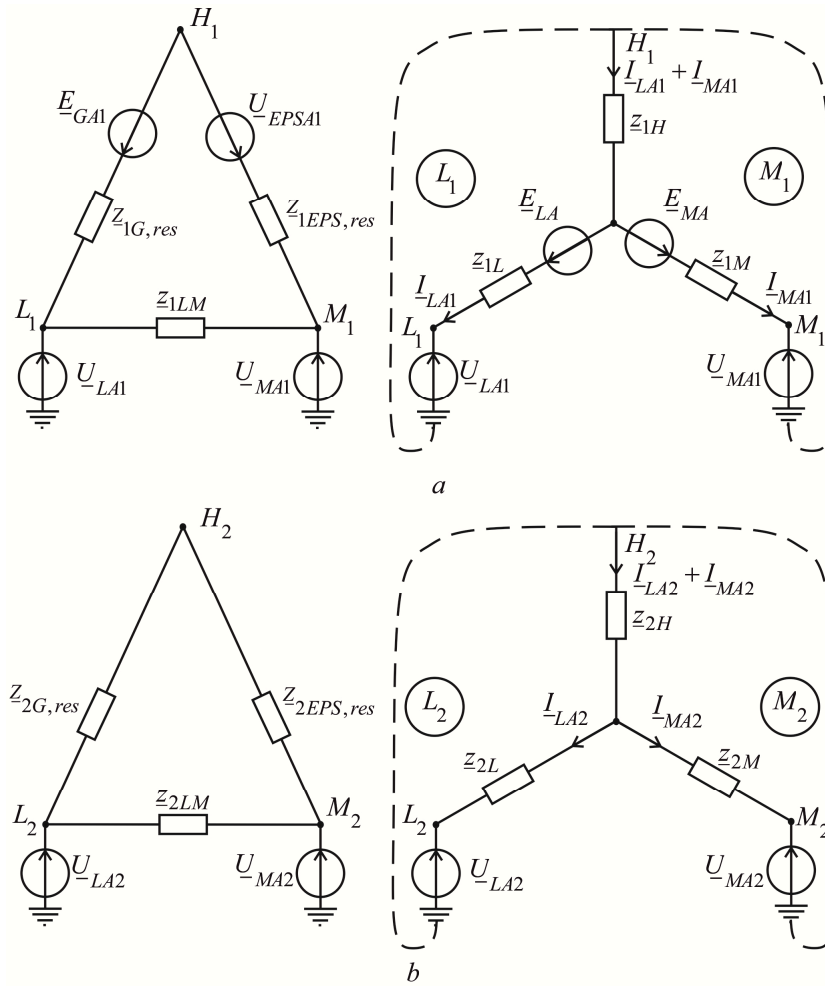


Fig. 6.11. Transformations of equivalent circuits:  
a – for positive sequence; b – for negative sequence.

Impedances of arms in Y-shaped equivalent circuit for positive sequence are calculated by formulae (their structure are the same for negative sequence):

$$\underline{z}_{1H} = \underline{z}_{1G,res} \underline{z}_{1EPS,res} / \underline{z}_{1res}; \quad (6.53)$$

$$\underline{z}_{1L} = \underline{z}_{1G,res} \underline{z}_{1LM} / \underline{z}_{1res}; \quad (6.54)$$

$$\underline{z}_{1M} = \underline{z}_{1LM} \underline{z}_{1EPS,res} / \underline{z}_{1res}. \quad (6.55)$$

Ratios between symmetrical components of mode parameters for the same sequences at the points of ground connection are described by the following equations sets:

in equivalent circuit for positive sequence (Fig. 6.11, a: loops  $L_1$  and  $M_1$ )

$$\underline{U}_{LA1} = \underline{E}_{LA} - \underline{I}_{LA1}(\underline{z}_{1L} - \underline{z}_{1H}) - \underline{I}_{MA1}\underline{z}_{1H}; \quad (6.56)$$

$$\underline{U}_{MA1} = \underline{E}_{MA} - \underline{I}_{LA1}\underline{z}_{1H} - \underline{I}_{MA1}(\underline{z}_{1M} + \underline{z}_{1H}); \quad (6.57)$$

in equivalent circuit for negative sequence (Fig 6.11, b; loops  $L_2$  and  $M_2$ )

$$\underline{U}_{LA2} = -\underline{I}_{LA2}(\underline{z}_{2L} + \underline{z}_{2H}) - \underline{I}_{MA2}\underline{z}_{2H}; \quad (6.58)$$

$$\underline{U}_{MA2} = -\underline{I}_{LA2}\underline{z}_{2H} - \underline{I}_{MA2}(\underline{z}_{2M} + \underline{z}_{2H}); \quad (6.59)$$

in equivalent circuit for zero sequence (Fig. 6.9, d)

$$\underline{U}_{LA0} = -\underline{I}_{LA0}\underline{z}_{0L}; \quad (6.60)$$

$$\underline{U}_{MA0} = -\underline{I}_{MA0}\underline{z}_{0M}. \quad (6.61)$$

Symmetrical components of currents and voltages at the points of double ground contact are described by the system of independent equations (6.46), (6.48), (6.49), (6.50), (6.56)-(6.61). In order to solve it one must reduce the number of unknowns by expressing symmetrical currents components via positive sequence currents using phasor diagrams (Fig. 6.10) for each point of asymmetry as:

$$\underline{I}_{LA2} = a\underline{I}_{LA1} = -\underline{I}_{MA1}; \quad (6.62)$$

$$\underline{I}_{LA0} = a^2\underline{I}_{LA1} = -a\underline{I}_{MA1}; \quad (6.63)$$

$$\underline{I}_{MA2} = a^2\underline{I}_{MA1} = -\underline{I}_{LA1}; \quad (6.64)$$

$$\underline{I}_{MA0} = a\underline{I}_{MA1} = -a^2\underline{I}_{LA1}. \quad (6.65)$$

Then equations set, considering (6.62) - (6.65) will take the form:

$$\left. \begin{aligned} a\underline{I}_{LA1} + \underline{I}_{MA1} &= 0; \\ a^2\underline{U}_{LA1} + a\underline{U}_{LA2} + \underline{U}_{LA0} + a\underline{U}_{MA1} + a^2\underline{U}_{MA2} + \underline{U}_{MA0} &= 0; \\ \underline{U}_{LA1} &= \underline{E}_{LA} - \underline{I}_{LA1}[\underline{z}_{1L} + (1-a)\underline{z}_{1H}]; \\ \underline{U}_{LA2} &= -\underline{I}_{LA1}[a\underline{z}_{2L} - (1-a)\underline{z}_{2H}]; \\ \underline{U}_{LA0} &= -\underline{I}_{LA1}a^2\underline{z}_{0L}; \\ \underline{U}_{MA1} &= \underline{E}_{MA} - \underline{I}_{MA1}[\underline{z}_{1M} + (1-a^2)\underline{z}_{1H}]; \\ \underline{U}_{MA2} &= -\underline{I}_{MA1}[a^2\underline{z}_{2M} - (1-a^2)\underline{z}_{2H}]; \\ \underline{U}_{MA0} &= -\underline{I}_{MA1}a\underline{z}_{0M}. \end{aligned} \right\} \quad (6.66)$$

Equations set (6.66) can be solved for positive sequence current  $\underline{I}_{LA1}$  by substituting of all its equations in the second one:

$$\underline{I}_{LA1} = (\underline{E}_{LA} - a^2\underline{E}_{MA}) / \underline{z}_{res}^{(LM)}, \quad (6.67)$$

where

$$\underline{z}_{res}^{(LM)} = \underline{z}_{11} + \underline{z}_{21} + \underline{z}_{0L} + \underline{z}_{1M} + \underline{z}_{2M} + \underline{z}_{0M} + 3\underline{z}_{1H} + 3\underline{z}_{2H}. \quad (6.68)$$

Symmetrical currents components are evaluated considering (6.68) and (6.62) - (6.66) as:

- for grounded arm in point  $L$  except current  $\underline{I}_{LA1}$ , which is determined by (6.67),

$$\left. \begin{aligned} \underline{I}_{LA2} &= (a\underline{E}_{LA} - \underline{E}_{MA}) / \underline{z}_{res}^{(LM)}; \\ \underline{I}_{LA0} &= (a^2\underline{E}_{LA} - a\underline{E}_{MA}) \underline{z}_{res}^{(LM)}; \\ \underline{U}_{LA1} &= \underline{E}_{LA} - (\underline{E}_{LA} - a^2\underline{E}_{MA})[\underline{z}_{1L} + (1-a)\underline{z}_{1H}] / \underline{z}_{res}^{(LM)}; \\ \underline{U}_{LA2} &= -(\underline{E}_{LA} - a^2\underline{E}_{MA})[a\underline{z}_{2L} - (1-a)\underline{z}_{2H}] / \underline{z}_{res}^{(LM)}; \\ \underline{U}_{LA0} &= -(a^2\underline{E}_{LA} - a\underline{E}_{MA})\underline{z}_{0L} / \underline{z}_{res}^{(LM)}; \end{aligned} \right\} \quad (6.69)$$

- for grounded arm in point  $M$

$$\left. \begin{aligned} \underline{I}_{MA1} &= -(a\underline{E}_{LA} - \underline{E}_{MA}) / \underline{z}_{res}^{(LM)}; \\ \underline{I}_{MA2} &= -(\underline{E}_{LA} - a^2\underline{E}_{MA}) / \underline{z}_{res}^{(LM)}; \\ \underline{I}_{MA0} &= -(a^2\underline{E}_{LA} - a\underline{E}_{MA}) / \underline{z}_{res}^{(LM)}; \\ \underline{U}_{MA1} &= \underline{E}_{MA} - (a\underline{E}_{LA} - \underline{E}_{MA})[(a^2 - 1)\underline{z}_{1H} - \underline{z}_{1M}] / \underline{z}_{res}^{(LM)}; \\ \underline{U}_{MA2} &= (a\underline{E}_{LA} - \underline{E}_{MA})[(a^2 - 1)\underline{z}_{2H} + \underline{z}_{2M}] / \underline{z}_{res}^{(LM)}; \\ \underline{U}_{MA0} &= (a^2\underline{E}_{LA} - a\underline{E}_{MA})\underline{z}_{0M} / \underline{z}_{res}^{(LM)}. \end{aligned} \right\} \quad (6.70)$$

Using (6.69) and (6.70) one can determine currents and voltages in subcircuits shorted at points  $L$  and  $M$ . Expressions obtained must fit boundary conditions (6.38) and (6.39). Short circuit currents in faulted phases are determined by (6.46) and (6.47), which for this case are

$$\underline{I}_{LB} = -\underline{I}_{MC} = 3a^2\underline{I}_{LA1} = 3\underline{I}_{LA0}. \quad (6.71)$$

Phases' currents at other points of electric network are expressed via symmetrical currents components in those branches considering distribution of short circuit currents symmetrical components. Distribution of short circuit current sequence into circuit branches is accomplished for each point of short circuit under absence of short circuit current at other point considering coefficient of current distribution. Current of each sequence in subcircuit is determined as superposition of components, corresponding to ground currents components for each fault point, flowing in this subcircuit, i.e.

$$\underline{I}_{Ai} = c_i^{(L)} \underline{I}_{LAi} + c_i^{(M)} \underline{I}_{MAi}. \quad (6.72)$$

Here:  $\underline{I}_{Ai}$  - is current of  $i$ -th sequence, flowing in considered subcircuit of network;  $c_i^{(L)}$  - is coefficient of current distribution for subcircuit, determined under  $\underline{I}_{*LAi} = 1$  and  $\underline{I}_{*MAi} = 0$ ;  $c_i^{(M)}$  - is coefficient of current distribution for subcircuit, determined under  $\underline{I}_{*LAi} = 0$  and  $\underline{I}_{*MAi} = 1$ .

Coefficients of current distribution for path “generator – fault point  $L$ ” in positive sequence equivalent circuit (Fig. 6.9,a) are evaluated by

$$c_i^{(L)} = (\underline{z}_{1LM} + \underline{z}_{1Cres}) / \underline{z}_{1res}; \quad c_1^{(M)} = \underline{z}_{1Cres} / \underline{z}_{1res}, \quad (6.73)$$

and positive sequence current – by formula

$$\underline{I}_{GA1} = \underline{I}_{LA1} [\underline{z}_{1LM} + (1-a)\underline{z}_{1EPSres}] / \underline{z}_{1res}. \quad (6.74)$$

Negative sequence current in considered subcircuit for negative sequence is evaluated by the same formula.

$$\underline{I}_{GA2} = \underline{I}_{LA1} [a\underline{z}_{2LM} - (1-a)\underline{z}_{2EPSres}] / \underline{z}_{2res}, \quad (6.75)$$

where  $\underline{z}_{2res} = \underline{z}_{2G,res} + \underline{z}_{2LM} + \underline{z}_{2EPSres}$ .

Zero sequence current does not flow in this subcircuit, because there is a transformer with windings connected as “Y- $\Delta$ ”, so  $\underline{z}_{0T} \rightarrow \infty$ . Phase currents in subcircuit are evaluated by:

$$\underline{I}_{GA} = \underline{I}_{GA1} + \underline{I}_{GA2} = \underline{I}_{LA1} \left[ \frac{\underline{z}_{1LM} + (1-a)\underline{z}_{1EPSres}}{\underline{z}_{1res}} + \frac{a\underline{z}_{2LM} - (1-a)\underline{z}_{2EPSres}}{\underline{z}_{2res}} \right]; \quad (6.76)$$

$$\underline{I}_{GB} = a^2 \underline{I}_{GA1} + a \underline{I}_{GA2} = \underline{I}_{LA1} \left[ \frac{a^2 \underline{z}_{1LM} + (a^2 - 1)\underline{z}_{1EPSres}}{\underline{z}_{1res}} + \frac{a^2 \underline{z}_{2LM} + (a^2 - a)\underline{z}_{2EPSres}}{\underline{z}_{2res}} \right]; \quad (6.77)$$

$$\underline{I}_{GC} = a \underline{I}_{GA1} + a^2 \underline{I}_{GA2} = \underline{I}_{LA1} \left[ \frac{a\underline{z}_{1LM} + (a - a^2)\underline{z}_{1EPSres}}{\underline{z}_{1res}} + \frac{\underline{z}_{2LM} + (1 - a^2)\underline{z}_{2EPSres}}{\underline{z}_{2res}} \right]. \quad (6.78)$$

Assuming that network components impedances of positive and negative sequences are equal, considering (6.71), expressions (6.76) - (6.78) can be simplified significantly. Simplified expressions for determination of currents symmetrical components in special phase  $A$  and currents in subcircuits (Fig. 6.9) are given in Table 6.1.

Phase voltages at different points of network are determined using symmetrical components of voltages at these points, which can be found by symmetrical components of currents in corresponding subcircuits.



### 6.6. Single-phase short circuit with simultaneous open-phase fault

Examine this type of longitudinal asymmetry in electric network with grounded neutral. Its design circuit is shown in Fig 6.12,a.

Boundary conditions for faults are:

- under break in phase  $A$  at point  $L$

$$\Delta \underline{U}_{LB} = 0; \Delta \underline{U}_{LC} = 0; \quad (6.79)$$

- under single-phase ground short-circuit of phase  $A$  at point  $K$

$$\underline{U}_{sA} = 0; \underline{I}_{sB} = 0; \underline{I}_{sC} = 0. \quad (6.80)$$

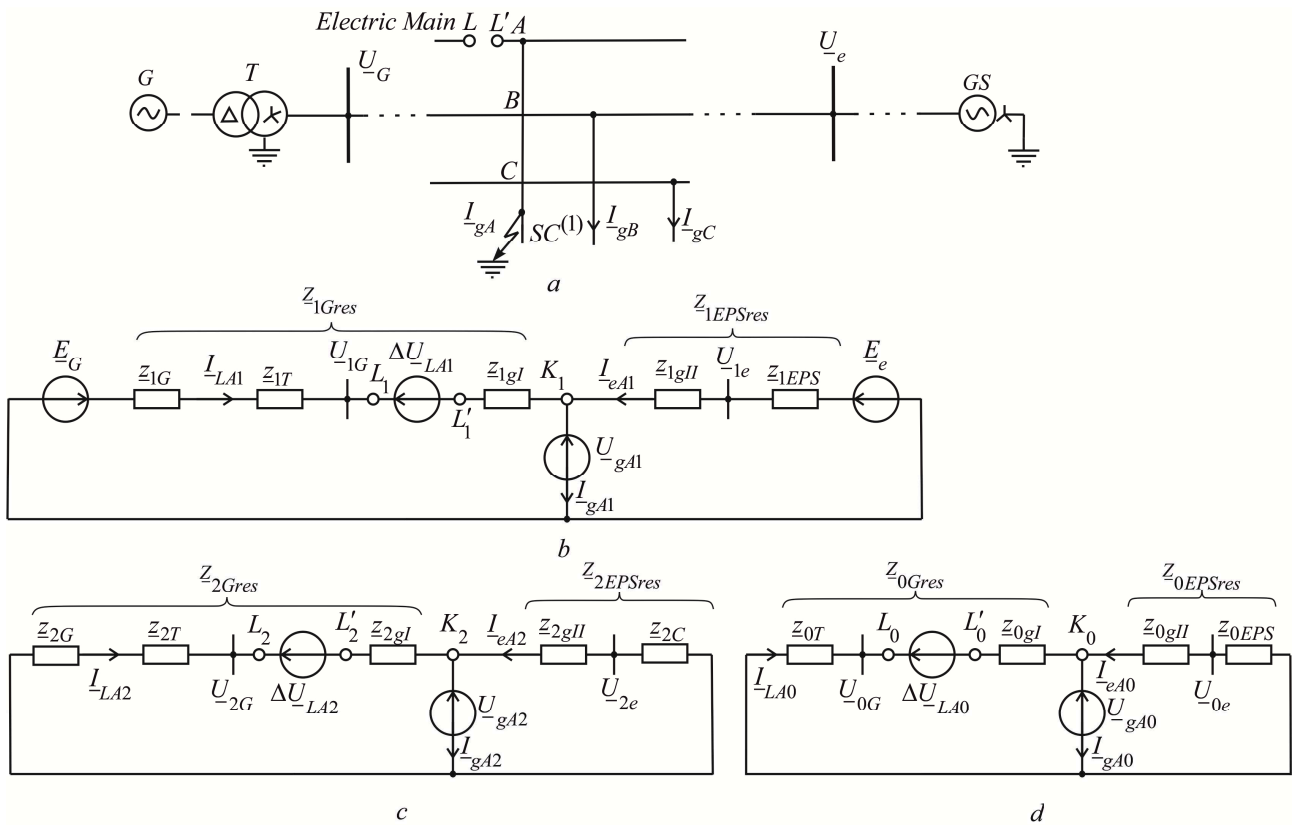


Fig. 6.12. Circuits for analysis of single-phase short circuit with simulteneous open-phase fault (phase A):

- a – design circuit; b – equivalent circuit for positive sequence; c – equivalent circuit for negative sequence; d – equivalent circuit for zero sequence

Assumed phase  $A$  to be special and using corresponding formulae we can obtain expressions for calculation of currents and voltages symmetrical components at fault points:

- at the point of open-fault in phase  $A$

$$\Delta \underline{U}_{LA1} = \Delta \underline{U}_{LA2}; \quad (6.81)$$

$$\Delta \underline{U}_{LA2} = \Delta \underline{U}_{LA0}; \quad (6.82)$$

$$\underline{I}_{LA1} + \underline{I}_{LA2} + \underline{I}_{LA0} = 0. \quad (6.83)$$

- at the point of single-phase short circuit

$$\underline{I}_{sA1} = \underline{I}_{sA2}; \quad (6.84)$$

$$\underline{I}_{sA2} = \underline{I}_{sA0}; \quad (6.85)$$

$$\underline{U}_{sA1} + \underline{U}_{sA2} + \underline{U}_{sA0} = 0. \quad (6.86)$$

Design ratios between symmetrical components of currents and voltages in faulted subcircuits of the same sequences can be obtained on the basis equivalent circuits for each sequences using the Second Law of Kirchhoff. To do that, a voltage sources must be introduced in faulted subcircuits in equivalent circuits for each sequence. Their magnitudes must be equal to voltage magnitudes of corresponding sequences (Fig. 6.12, b-d).

- According to equivalent circuit for positive sequence (Fig. 6.12, b) following equations are obtained:

$$\Delta \underline{U}_{LA1} = \underline{E}_G - \underline{E}_e - \underline{I}_{LA1} \underline{z}_{1G,res} + (\underline{I}_{sA1} - \underline{I}_{LA1}) \underline{z}_{1EPS,res}; \quad (6.87)$$

$$\underline{U}_{sA1} = \underline{E}_e + (\underline{I}_{LA1} - \underline{I}_{sA1}) \underline{z}_{1EPS,res} \quad (6.88)$$

- According to equivalent circuit for negative sequence (Fig. 6.12, c):

$$\Delta \underline{U}_{LA2} = -\underline{I}_{LA2} \underline{z}_{2G,res} + (\underline{I}_{sA2} - \underline{I}_{LA2}) \underline{z}_{2EPS,res}; \quad (6.89)$$

$$\underline{U}_{sA2} = (\underline{I}_{LA2} - \underline{I}_{sA2}) \underline{z}_{2EPS,res} \quad (6.90)$$

- According to equivalent circuit for zero sequence (Fig. 6.12, d):

$$\Delta \underline{U}_{LA0} = -\underline{I}_{LA0} \underline{z}_{0G,res} + (\underline{I}_{sA0} - \underline{I}_{LA0}) \underline{z}_{0EPS,res}; \quad (6.91)$$

$$\underline{U}_{sA0} = (\underline{I}_{LA0} - \underline{I}_{sA0}) \underline{z}_{0EPS,res} \quad (6.92)$$

So, mode parameters of faulted subcircuits are described by system of 12 independent equations (6.81)-(6.92). Its solution is given at Table 6.2. There are also expressions for estimation of symmetrical components of currents in arms of design circuit. The following symbolizations are used:

$$\underline{z}_{1res} = \underline{z}_{1G,res} + \underline{z}_{1EPS,res};$$

$$\underline{z}_{2res} = \underline{z}_{2G,res} + \underline{z}_{2EPS,res};$$

$$\underline{z}_{0res} = \underline{z}_{0G,res} + \underline{z}_{0EPS,res};$$

$$\Delta_A = \underline{z}_{1res} \underline{z}_{2res} + \underline{z}_{1res} \underline{z}_{0res} + \underline{z}_{2res} \underline{z}_{0res};$$

$$\Delta = \underline{z}_{EPS,res}^{(1)} \Delta_A - \underline{z}_{0res} \underline{z}_{EPS,1-2}^2 - \underline{z}_{2res} \underline{z}_{EPS,1-0}^2 - \underline{z}_{1res} \underline{z}_{EPS,2-0}^2.$$

Voltages at any desired point of network can be easily estimated on the basis of symmetrical components of currents in different network's subcircuits, EMFs of generators and power sources. Phase voltages and currents are estimated by subtraction of voltage drops between power source and given network's point from source's EMF by calculated symmetrical components of currents and voltages.

So, to analyze complex fault in electric supply system the following algorithm is used:

- 1) the design circuit of electric network is to be made up and faulted subcircuits is denoted;
- 2) boundary conditions are estimated for each point of symmetry breakdown;
- 3) special phase is defined;
- 4) boundary conditions for each point of symmetry breakdown are expressed via design ratios between the same symmetrical components of currents and voltages.
- 5) equivalent circuits for each sequence for special phase are generated and design ratios between currents and voltages of the same sequences are defined using them;
- 6) a system of independent equations is solved for estimation of unknown symmetrical components of currents and voltages in faulted subcircuits;
- 7) symmetrical components of currents and voltages in given branches and points of network are determined;
- 8) phase voltages and total phase currents in given branches and points of network are calculated.

**Test questions**

1. What are examples of longitudinal asymmetry in industrial enterprises' supply network?
2. What is the algorithm of longitudinal asymmetry analysis by symmetrical components method?
3. What is the integrated equivalent circuit for open-phase fault (impedance plugged in phase)?
4. What is the integrated equivalent circuit for plugged impedances in two phases (two-phases open-fault)?
5. What are the boundary conditions for double ground at different points of network and in phases?
6. What is the algorithm of complex faults analysis by method of symmetrical components?
7. What initial design conditions and concepts for longitudinal asymmetry are used?

**Topics for essay**

1. Evaluation of three-phase network mode parameters under blowout of fuses in one (two) phase(s)?
2. Differences between modes under two-phases open fault and connection equal impedances in them.
3. Peculiarities of current and voltages transients under open-phase fault with simultaneous ground short circuit and under faulted phase reclosure.

## **CHAPTER 7: TRANSIENTS UNDER SPECIFIC CONDITIONS**

- 7.1. Short circuits in the network of external electric power supply
- 7.2. Short-circuit to ground in the network with insulated neutral
- 7.3. Short circuits in the networks of increased frequency
- 7.4. Transients stipulated by peculiarities of production process
- 7.5. Processes taking place under capacitor banks commutation
- 7.6. Short circuits in DC networks

Test questions

Topics for essay

### 7.1. Short circuits in the networks of external electric power supply

Under short circuit in the feeding networks of 330 kV and more, the short circuit current higher harmonics, in addition to periodic and aperiodic components, can be selected. The higher harmonics are caused by distributed lateral capacitance of power transmission line. If lumped longitudinal capacitance is available in power transmission line, the short circuit current has also subharmonic component with decreased frequency to compare with industrial one. Calculation of short circuit current components is done by means of computers, taking into consideration power transmission line capacitance under short circuit in different network points.

Design circuit shown in Fig. 7.1 allows to determine general dependences for calculation of short circuit currents in such transmission lines. If series capacitance of the line is not available (short circuit in point  $SC_1$  across the source busses and in point behind the transformer  $T3$ ), total current in the place of three-phase short circuit has only periodical (of industrial frequency) and aperiodical components. These components and the surge current are calculated on formulae given above. In the same way short circuit current is calculated under the presence of longitudinal capacitance if short circuit arises behind some impedance (point  $SC_3$ ). In this case reactance  $x_s = -1/(\omega C_k)$  is taken into account in total impedance, where  $C_k$  - is longitudinal capacitance of the network.

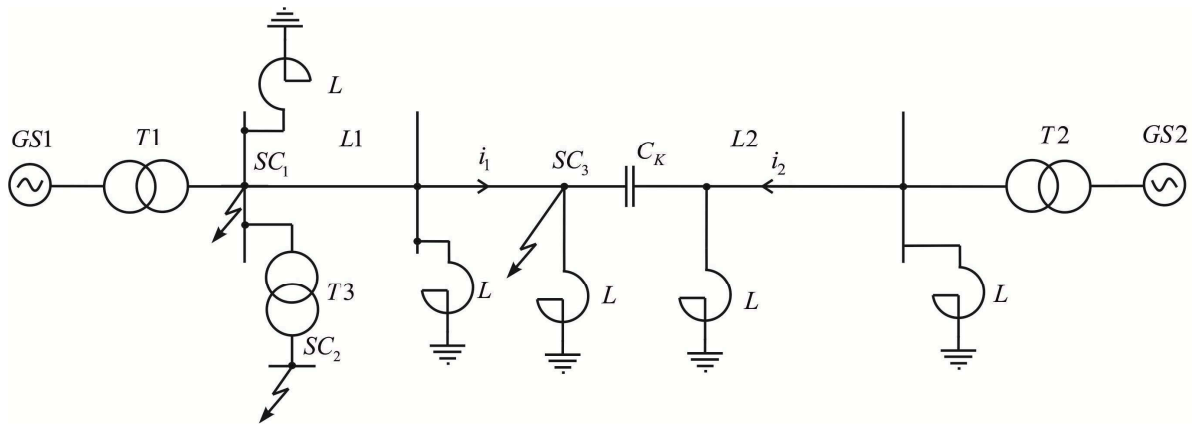


Fig. 7.1 Calculation circuit

Under three-phase short circuit in any point of power transmission line  $L2$  total current of short circuit consists of currents  $i_1$  and  $i_2$  sum which constituted by power sources located at the opposite sides of the longitudinal capacitance. Situation short circuit point in respect o the capacitance and sources are taken into consideration as well. Short circuit current generated by generators (point  $K_3$  and source are at one side concerning longitudinal capacitance) consists of periodical component of industrial frequency, and periodical and aperiodic components of supplementary source which are represented by the capacitance. This current is calculated by the relationships obtained for the case of longitudinal capacitance absence. The short circuit current, generated by the power system source which is located behind the longitudinal capacitance (points  $SC_1$  and  $SC_2$ ), includes periodical component of industrial frequency and subharmonic component stipulated by network capacitance. In the general case the subharmonic component is determined according to formula

$$i_{s,max} = \sqrt{2}E''_{GS} \exp(-t/T_s) \cos \omega_s t / (\sqrt{3}x_{2res}) \quad (7.1)$$

where  $x_{2res} = \omega_s L_{2res} - x_s$  - is the resulting reactance;  $T_s = 2L_{2res} / r_{2res}$  - is the time constant of subharmonic current damping;  $L_{2res}$ ,  $r_{2res}$  - are resulting values of the inductance and resistance of

all circuit elements located on the side of short circuit point as the longitudinal capacitance;  $\omega_s = 1/\sqrt{L_{2res}C_k}$  - is the angular frequency of the subharmonic current.

The surge current generated by a source connected to the point of short-circuit through the lumped longitudinal capacitance

$$i_{srg} = \sqrt{2}E''_{GS} \left[ 1 + \exp(-t_{srg}/T_s) \right] / (\sqrt{3}x_{2res}), \quad (7.2)$$

$$t_{srg} = \pi / (\omega - \omega_s)$$

where  $t_{srg}$  - is the instant of surge current arising ( $t_{stable} > 0,01$  due to presence of the longitudinal capacitance).

Total current of three-phase short circuit is approximately determined by the expression:

$$i_{max} = \left[ i_{F1max} + i_{F2max} + i_{s0max} \exp(-t/T_s) \right] \times \\ \times \sin\left((\omega - \omega_s)t/2\right) + i_{a0} \exp(-t/T_s), \quad (7.3)$$

where  $i_{F1max}$ ,  $i_{F2max}$  - are accordingly maximum values of industrial frequency periodic component of currents of the power plant generators and the power system:

$$i_{F1max} = \sqrt{2} \cdot E''_G / (\sqrt{3}x_{1res});$$

$$i_{F2max} = \sqrt{2} \cdot E''_{GS} / (\sqrt{3}x_{2res});$$

$i_{s0max} = \sqrt{2} \cdot E''_{GS} / (\sqrt{3}x_{2res})$  - is initial value of the subharmonic current amplitude;

$i_{a0} = \sqrt{2} \cdot E''_{GS} / (\sqrt{3}x_{1res})$  - is initial value of the generator aperiodic current component;

$x_{1res}$ ,  $x_{2res}$  - are resulting reactances of circuit elements till the place of short circuit from correspondent sources  $G$  and  $GS$ ;  $T_a = L_{1res}/r_{1res}$  - is time constant of the generator aperiodic current component damping. Surge short-circuit current is calculated by formula

$$i_{srg} = \sqrt{2} \cdot E''_G \cdot [1 + \exp(-t_{srg}/T_a)] / (\sqrt{3} \cdot x_{1res}) + \\ + \sqrt{2} \cdot E''_{GS} \cdot [1 + \exp(-t_{srg}/T_C)] / (\sqrt{3} \cdot x_{2res}). \quad (7.4)$$

In power transmission line with voltage of 330 kV and higher the periodic short circuit current component decreases on magnitude at distance from power source increases. Free current components of total short circuit current grow.

Separation of a synchronous generator or a group of generators from electric power system usually takes place under short circuit cutoff. Under automatic reclosing of generators after some time interval at persistent short circuit, the initial current of repeated short circuit can be higher than the initial current of the first fault that should be taken into consideration while choosing or testing apparatuses by conditions of emergency operation.

The calculation conditions are determined by reactive power which the generator "throws off" during the interval of automatic reclosing, the parameters of the used excitation control system, duration of the first short circuit and its electric remoteness and duration of the automatic reclosing interval. Growth of reactive power throwing off by the generator, approximation of the external impedance to the value of 0.6...0.7 per units relative the generator rated parameters and increase of initial emergency period are accompanied with increase of ratio of repeated short circuit initial

current to the initial current of the first fault. Initial current of repeated short circuit is not higher than that of the first fault if:

- a hydrogenerator disconnection from electric power system takes place after no longer than 0.5 s and a turbogenerator after no longer than 1 s (from the instant of emergency operation mode occurrence);
- generators are provided with valve excitation system in the cases of reclosing interval is not larger than 0.7 s from the instant of turbogenerator cutoff or than 1 s for hydrogenerators. Then, at selection or testing apparatuses and conductors for the short circuit conditions, current of the first fault is assumed having the calculated value.

Generator can be equipped with non fast-operating excitation system (electric machine exciter) with excitation voltage under forcing being in accordance with the generator excitation at no-load. Therefore, if emergency operation mode lasts more than 1 s, and short circuit remoteness is 0.6...0.7 per units, and generator completely throws off reactive power, the initial current of repeated short circuit exceeds the first fault current by 20...25 %. Under these conditions, despite the interval of automatic reclosing, initial value of short circuit current at the repeated circuit closing is assumed in the capacity of calculated initial value of short circuit current. The latter is by 25 % greater than initial current of the first fault.

## 7.2. Short circuit to ground in the networks with insulated neutral

This type of short circuit is typical for networks with small current of short circuit to ground. Processes taking place in such networks are rather complicated, and strongly depend on parameters and circuit design and from resistance of the short circuit current path.

Zero sequence impedance in networks with voltage of 6 to 35 kV is mainly determined by network elements capacitance relatively to ground. That is why closing to ground at networks of 6 - 35 kV are accompanied with small fault currents which are less than the load current. It is especially typical for overhead transmission lines of 6 – 10 kV in which capacitance to ground is relatively small. In networks with cable lines and with extended overhead transmission lines with voltage of 6 - 35 kV significant values of short circuit currents to ground can take place.

For improvement conditions of arc quenching, and prevention short circuit to ground transition in interphase short circuit in the networks under consideration, arc-suppression coils are mounted by means which fundamental harmonic of capacitive short circuit to ground current is compensated. Consequently the resulting fault current in such networks under steady state is reduced sharply. In the case of closure to ground (Fig. 7.2, a) the current flowing through faulted connection equals the total current of undamaged elements which is defined by capacitance and insulation resistance to ground of each of them and by reactance of arc-suppression coil (if available).

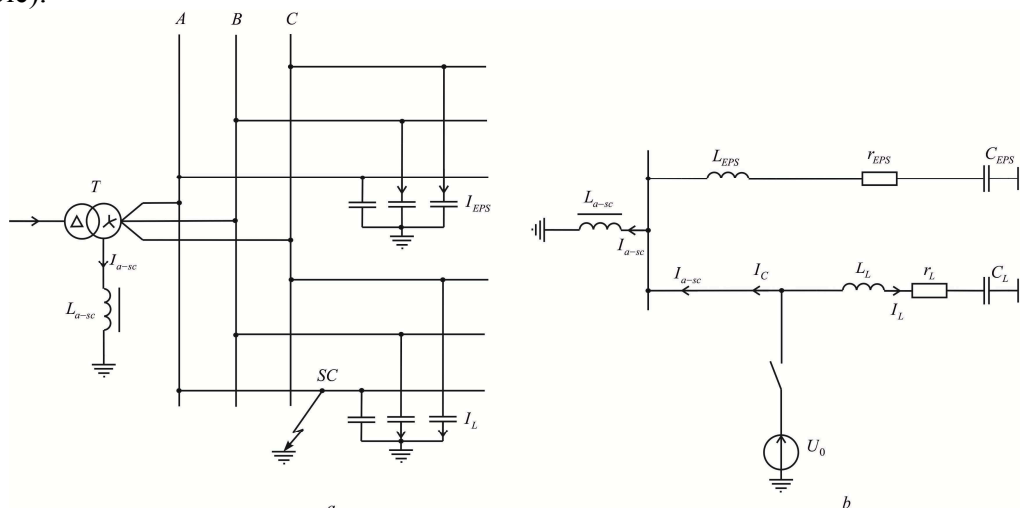


Fig. 7.2. An example of one-phase short-circuit in power network: a-design circuit; b-zero sequence equivalent circuit

In uncompensated networks the fundamental current harmonic on faulted and non-faulted connections are oppositely directed that is explained by placement the zero sequence voltage source in the point of short circuit (Fig. 7.2, b). Therefore current  $I_s$ , determined by unfaulted network capacitive reactance, flows in the faulted element in the direction of the buses, and in unfaulted one - in the opposite direction.

When arc-suppression coil  $L_{ac}$  is connected to the neutral of one of transformers within the faulted section, the phase of main harmonic of fault current will depend on ratio between values of capacitance of unfaulted sections and the coil inductance. If inductive component of faulted current prevails, the phase angle of reactive components of fault current are equal both within unfaulted (capacitive current is directed towards the line) and faulted ones (inductive current is directed towards buses) connections.

Magnitude and phase of currents of short circuit to ground are determined by voltage of zero sequence  $U_0$ . The largest  $U_0$  value will take place under short circuit to ground when the transition resistance is zero and equals the network phase voltage. Under short circuits via contact resistance the value  $U_0$  is determined by ratio between the zero sequence impedance and the contact resistance. Angle between voltage  $U_0$  and the current of short-circuit to ground is always the same, and is equal to network zero sequence impedance angle. Presence of contact resistance reduces value  $U_0$  and changes the angle relatively the phase voltage.

To analyze currents of one-phase short-circuits to ground in networks with insulated neutral, consider correspondent equivalent circuits and phasor diagrams (Fig. 7.3). We make the following assumptions proceeding from targets and practical problems of a network emergency conditions calculation:

- capacitance of individual phases with respect to the ground distributed uniformly along wires substitute with equivalent lumped capacitances  $C_A = C_B = C_C = C$ , connected in the middle of the power transmission line;
- leakage conductance as well as resistance and inductive reactance of power transmission line which are small in comparison with capacitive reactance of phases relative the ground are not taken into consideration;
- impedances of the load and power transmission line (phase voltages) consider to be balanced.

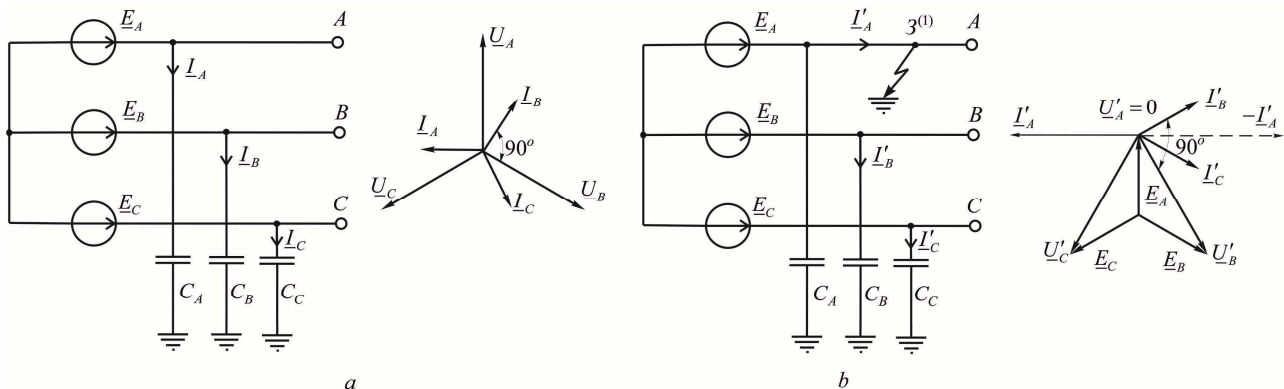


Fig. 7.3. Equivalent circuits and phasor diagrams of voltages and capacitance currents of network with insulated neutral in the conditions: a-normal; b-with single phase short circuit to ground

Capacitive phase currents  $I_A = j\omega C U_A$ ;  $I_B = j\omega C U_B$ ;  $I_C = j\omega C U_C$  are equal in magnitude and lead corresponding voltages by  $90^\circ$ . Under ordinary operation condition, sum of



currents is equal to zero (Fig. 7.3, a). Voltage of the neutral  $\underline{U}_0$  (between neutral point of the network and the ground) is determined by the expression:

$$\underline{U}_0 = (\underline{E}_A \cdot Y_A + \underline{E}_B \cdot Y_B + \underline{E}_C \cdot Y_C) / (Y_A + Y_B + Y_C) = 0, \quad (7.5)$$

where  $Y_A = Y_B = Y_C = Y = j\omega C$  - are capacitive susceptance of the phases under ordinary operation.

If short circuit to ground takes place in the point of fault in absence of transition resistance, the emergency phase  $A$  receives the ground potential (Fig. 7.3, b). Therefore,  $\underline{U}_A = 0$ ;  $Y_A = \infty$ . The neutral voltage  $\underline{U}_0$  becomes equal  $\underline{E}_A$ . Voltages of unfaulted phases with respect to the ground are increased by  $\sqrt{3}$  times and are

$$\underline{U}'_B = \sqrt{3} \cdot E_A \cdot \exp(-j \cdot 150^\circ); \quad \underline{U}'_C = \sqrt{3} \cdot E_A \cdot \exp(j \cdot 150^\circ)$$

Capacitive phase currents also increase by  $\sqrt{3}$  times. Leading the currents  $\underline{U}'_B$  and  $\underline{U}'_C$  by  $90^\circ$ , these currents are summed up in the ground and return through the faulted phase which current is equal to the ground current:

$$\underline{I}'_A = \underline{I}_{gA}^{(1)} = 3 \cdot \underline{I}_0 = -(\underline{I}'_B + \underline{I}'_C) = 3 \cdot j\omega C \underline{E}_A, \quad (7.6)$$

where  $\underline{I}_0$  - is zero sequence current at the short circuit to ground.

Currents  $\underline{I}_{gA}^{(1)}$  and  $\underline{I}_0$  lead EMF  $\underline{E}_A$  by  $90^\circ$  and are determined by capacitances of energizing system phases of the given voltage as well as by value of  $\underline{E}_A$ . For this reason, the current at short circuit to ground is greater in branched networks with significant capacitance. So, the total current  $\underline{I}_{g\Sigma}$  in the point of short circuit, when a phase of one of several power transmission lines is closed to ground, is defined by capacitive currents of all the lines and is

$$\underline{I}_{g\Sigma} = 3\underline{I}_{0\Sigma} = 3j\omega C_\Sigma \underline{U}_{ph} \quad (7.7)$$

Here  $C_\Sigma$  - is the total capacitance of conductors belonging to one phase of all the power transmission lines, at that  $C_\Sigma = C_{sp} l$ , where  $C_{sp}$  - is capacitance of conductors of one phase of the network relative the ground F/km;  $l$  - total length of conductors of the network phase. The current of short circuit to ground for a network with cable lines can also be determined with the help of empirical formula:

$$I_{g\Sigma} = \sum_{i=1} (95 + 2,84 \cdot q_i) \cdot U_{rated} \cdot l_i / (2200 + 6 \cdot q_i) \quad (7.8)$$

where  $U_{rated}$  - is the network line-to-line rated voltage in kV;  $l_i$  - is length of the cable line in km;  $q_i$  - is the cable cross section area in  $\text{mm}^2$ .

Evaluation of emergency modes of closing to ground in networks with insulated neutral is of fundamental importance for a number of industrial enterprises and objects with respect to power supply reliability, the equipment electrical safety of and design. This is most typical for mining enterprises which are powerful and critical power consumers. There are specific features of mining industry (complicated mining and geological conditions, dust and explosion hazard) which distinguish them from conventional enterprises.

As an example, consider faults in mine electric networks. To limit formation of open sparking in underground workings arising at switching overvoltage and to avoid false operation of leakage protection due to deterioration of electrical installations insulation, these networks are fed from special isolating or triple-wound transformers of 35-110/6/6 kV.

The majority of faults in mine power network is the result of untimely elimination of malfunctions in electrical equipment, and erroneous actions of maintenance personnel. Under exploitation coal dust and moisture are deposited on mine equipment owing to what leakage currents can arise contributing to the emergence of short circuits with dangerous sequences under specific conditions. Mechanical damage of electric power equipment is the main reason of short circuit into networks of underground mines. Mine armoured cables and especially flexible ones are most susceptible to damage, single-phase and line-to-line leakage occur in them more often. In conditions of underground mine workings short circuits can cause the underground fires being very dangerous for people. Under specific situations short circuits stipulate violation of safe properties of mine electric equipment. Protection from short circuits is one of the main means providing explosion-proof and fire-proof of electric equipment.

In underground mine networks single-phase to ground as well as line-to-line faults can arise. Taking into account particular danger of these short circuits, the existing safety regulations allow application in underground mine workings only transformers having insulated neutral. Neutral point grounding is permitted only on mines surface at voltage of 0.4 kV. According to the regulations the maximum short circuit power on buses of central underground substation must not exceed 50 per cent of the breaking power and be not more than  $100 \text{ MV} \cdot \text{A}$ .

In view of the above, currents of single-phase short-circuit to ground can't be significant. In networks of underground mines, closing of a phase to the ground or to the frame of electrical equipment can cause electric injury of people even under small currents. Therefore such short circuits in mines, unlike the networks of common industrial enterprises, must be immediately disconnected by means of special leakage. Such type of protection eliminates transition of single-phase into two-phase short circuits, or into two-phase short circuits to ground.

The current of short circuit to ground always contain components having frequency exceeding the industrial frequency. At the instant closing to ground the transient arises. In it there are two stages. The initial stage is characterized by electromagnetic waves propagation in both directions from the fault location. At this stage frequency of transient components is high (up to 100 kHz), and process duration is extremely short. At further stage the course of transient is approximately the same that in circuits with lumped parameters. Roughly the transient can be estimated using zero

sequence equivalent circuit to which disturbance voltage  $u_0(t)$  is applied. The character of short circuit currents and network voltage alternation is represented by curves in Fig. 7.4. Transient lasts no longer than 10 ms and its frequency is hundreds of Hz. The transient is most pronounced (Fig. 7.4,a), if initial value of voltage  $u_0(t)$  is accordant in the steady state with maximum value of voltage on resulting capacitance  $U_c$  (Fig. 7.2, b). It is possible if short circuit to ground occurs at the instant at which the voltage of damaged phase passes the maximum value (the most common case).

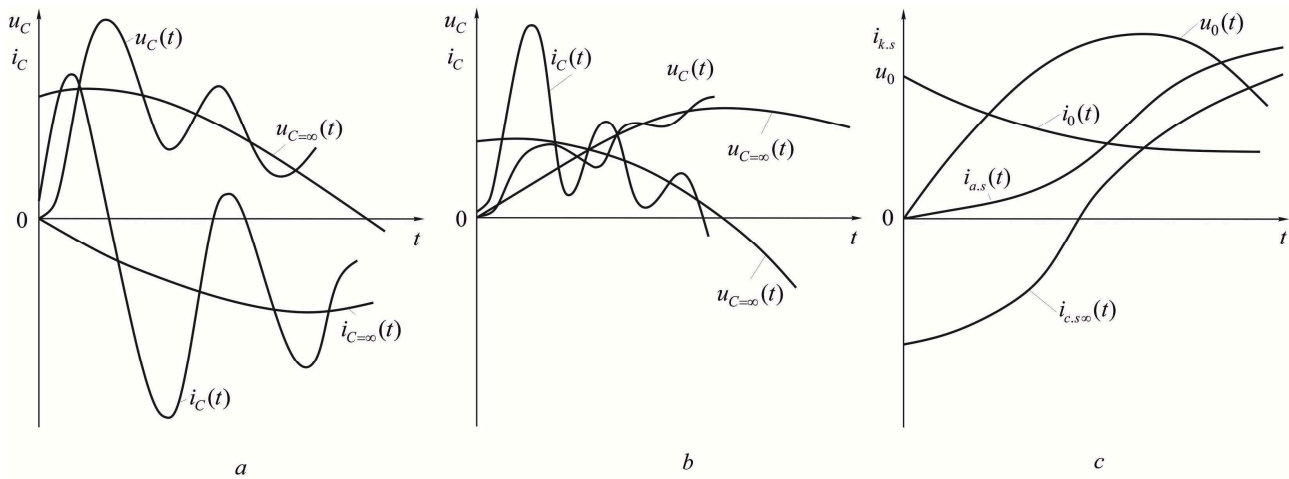


Fig. 7.4. Curves of transient current and voltage at applying the disturbance voltage  $u_0(t)$

If initial value  $u_0(t)$  corresponds to passing voltage  $u_0(t)$  through zero, the currents of transient currents are decreased (Fig. 7.4, b). Transient currents amplitudes can be by dozens times greater than stable currents of short circuit to ground. Approximately the ratio of amplitudes can be assumed equal to the ratio of the transient frequency to the industrial frequency.

In networks at compensation the capacitive component of short circuit current to ground, the arc suppression coil does not influence practically the high-frequency components of transient (Fig. 7.4, c). Here tuning frequency is close to 50 Hz at rather accurate compensation of capacitive current. Arc-suppression coil can stipulate the emergence of non-periodic component of current to ground. Its time constant depends on the coil quality factor and the current amplitude not exceeding the rated value. Therefore, in these networks the resulting current of short circuit to ground contains both high-frequency and non-periodic components.

It was established and confirmed by practice that higher harmonics are practically always present in the short circuit current, accounting from 5 to 15 per cent of the current fundamental value. The short circuit current harmonic content depends on the network type, and on short circuit conditions and vary in a wide range.

### 7.3. Short circuits in the networks of increased frequency

Electromagnetic processes in higher frequency network (up to 10 000 Hz) under emergency are calculated on the same conditions and formulae as it is done for industrial frequency networks (50 Hz). Skin effect as well as proximity effect is observed at higher frequencies. Therefore, the current is distributed over the wire cross-section non-uniformly. Depth of current penetration into the conductor of nonmagnetic material is determined by expression

$$\Delta = 5030\sqrt{\rho/f}$$

where  $\rho$  - is the specific resistance of the conductor, Ohm·cm;  $f$  - is the current frequency, Hz.

The penetration depth  $\Delta$  decreases at increase of the frequency and conductor cross-section area. At that, resistance and inductive reactance increase as well, and this stipulates decrease of permissible continuous current of load and the increase of voltage drop. In conductors with cross-section of 10mm<sup>2</sup> and less skin effect is insignificant and it is not taken into consideration while calculating higher frequency networks.

Proximity effect determines alternative current redistribution in conductor if another conductor with current or conductor with induced current comes near it. The effect takes place under any pattern of conductors' cross-section shape.

At frequency increase, the dimensions and mass of ferromagnetic cores of electric equipment and transformers decrease. Power networks of higher frequency are more metal-intensive than networks of industrial frequency because conductors in them have larger cross-section area. Generally, electric installations of higher frequency are lighter and smaller than analogue devices of industrial frequency. To feed networks and equipment of higher frequency the thyristor frequency converters are used.

The main task of emergency operation modes in the networks of higher frequency calculation and analysis is to determine their parameters at line-to-line and single-phase to ground short circuits to the frame (to ground), at overloads, voltage dips or the power supply short-term disconnection. Power networks of higher frequency are performed as busways and cable lines as well as insulated wires laid open or in steel pipes. Conductors for them are usually chosen according to permissible heating and voltage drop, and then are checked on short circuit currents.

Emergency operation modes in higher frequency networks are calculated taking into account corresponding power distribution circuit of radial, bulk or combined bulk power system. Radial system is applied for separate consumers of power over 20 kW if feeding point is located approximately in the load center. In the case of bulk system, one main feed supplies several using installations of comparatively small capacity (less than 20 kW) connected to it in different points with higher frequency power. In radial systems, cables or wires laid in tubes are used, in bulk systems busways are applied.

#### 7.4. Transients stipulated by peculiarities of production process

Power electrotechnology equipment is widely used in different branches of national economy. Their use prospects are rather promising. As power consumers they have a number of specific peculiarities manifesting in wide diversity of operation modes and transients in power supply systems of enterprises as a result of sharply variable, impulsive, non-sinusoidal and unbalanced load, voltage variation, electromagnetic disturbances in networks etc. Voltage fluctuations are caused by sharply variable and impact loads which are characteristic, first of all, for arc steel melting furnaces and electric welding equipment. Besides, they can be created by such consumers as valve converters feeding drives of rolling mills, electric traction equipment etc.

As it is known voltage fluctuations are defined as the voltage magnitude variation at rate not less than one per cent of the rated voltage per second. Voltage fluctuations influence the operation of lighting units, radio- and TV equipment, automatic and control systems, and computer facilities. Under non-linear load, higher harmonics of current are generated which overload capacitor banks causing extra power losses. Voltage oscillations result in worsening performances of power equipment and power networks as well as in undesirable variations in mechanical characteristics of motors.

Now consider as an example the peculiarities of transients in some electric installations.

*Electric arc steel melting furnaces* are considered as consumers with cyclic sharply variable operation mode. Metal melting is one of periods of their load diagram characterized by the greatest irregularity (strong and frequent random current oscillations). That can be explained by operational short circuits. In this period 50 to 80 per cent of energy consumed per a metal founding is spent, and impact effect in network is arisen. A great number of disconnections from the network in the course of melting due to working operations is typical for arc furnaces.

In networks with arc furnaces all voltage variations can be considered as its fluctuations. Random voltage fluctuations in such networks often exceed allowable limits. These fluctuations are stipulated by current oscillations in the furnace as a result of short circuit and the arc break, and periodic nature of the process control as well as electric arc peculiarities causing current oscillations at frequency of 2 to 10 kHz and magnitude of  $\pm 15\%$ . Voltage fluctuations are 2 to 4 per cent at voltage of 110 and 35 kV, and 3 to 12 per cent at 6 and 10 kV. The fluctuations frequency is 0.5 to 1 Hz.

The estimation of the impact effect of arc steel furnaces is made by methods used in analysis of electric circuits. It is necessary to create a power supply system of enterprise taking into account the influence of this equipment. Due to random nature of arc furnaces load parameters variation and of the voltage variations range, it is necessary to use the methods of random processes theory to analyze and normalize them. Analyzing voltage oscillations in networks with parallel connection arc furnaces, it is necessary to take into account their mutual electromagnetic influence.

Technological disconnections of arc furnaces taking place as a rule after the furnace arcs interruption are accompanied with switching surges on the windings of transformer being switched off. As a number of such disconnections and surges is significant it is necessary to take them into account at development the equipment for electric furnace installation.

Mainly, the level of overvoltages is determined by a circuit-breaker speed. Switching surges also depend on operation mode of transformers. The highest levels of surges (7-fold) are possible when idle transformers are disconnected. Two-phase loaded transformers disconnection under currents close to the no-loaded unit currents is the most unfavorable operation mode from the viewpoint of significant (5-fold) surges. Surges are calculated with the help of methods being used in electrical engineering.

Ratio of magnetizing current inrushes at transformers switching on reaches  $4I_{\text{rated}}$  (Fig. 7.5). Current inrushes are reduced sharply when at switching to lower voltage step, and also in the course of the furnace heating.

Impulse character of load diagrams of *electric welding installations* causes the dips in the envelope voltage curve which shape depends on the shape of individual pulses of the welding current. Variations of current and voltage on terminals of butt-welding machine with power of 600 kVA are shown in Fig. 7.6.

Laws of voltage dips variations correspond to laws of individual load diagrams and are described by probabilistic methods. Voltage dips depth is determined by the power of electric welding installation and the feeding source as well as parameters of the supply network. Installations of spot and projection welding cause voltage dips up to 7 per cent. The most voltage dips are observed on terminals of butt and multiple-spot installations (up to 19 per cent). Average

frequency of voltage dips is  $f_{D,av} = n/t_{D,av}$ , where  $n$  - is the number of current pulses per average welding cycle  $t_{D,av}$ .

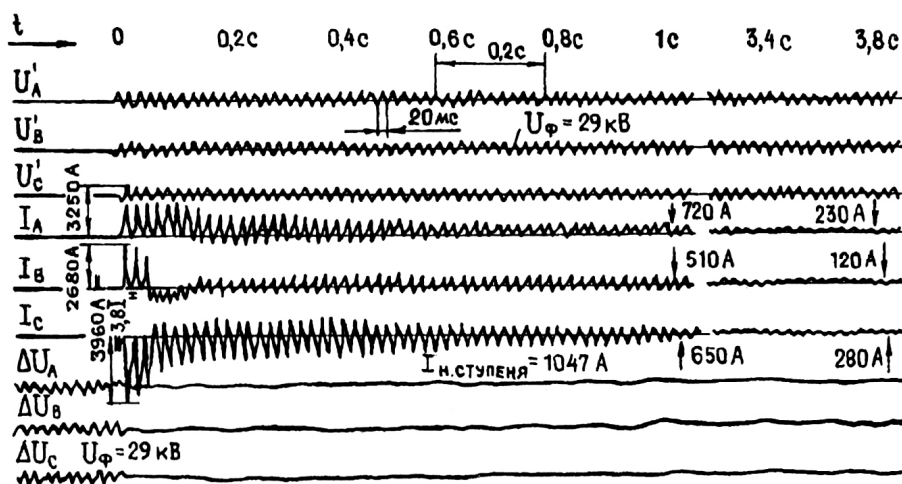


Fig. 7.5. Transient in phase currents and voltages in a supply network at no-load operation mode of the transformer of 45 MVA feeding furnace ДСП-200

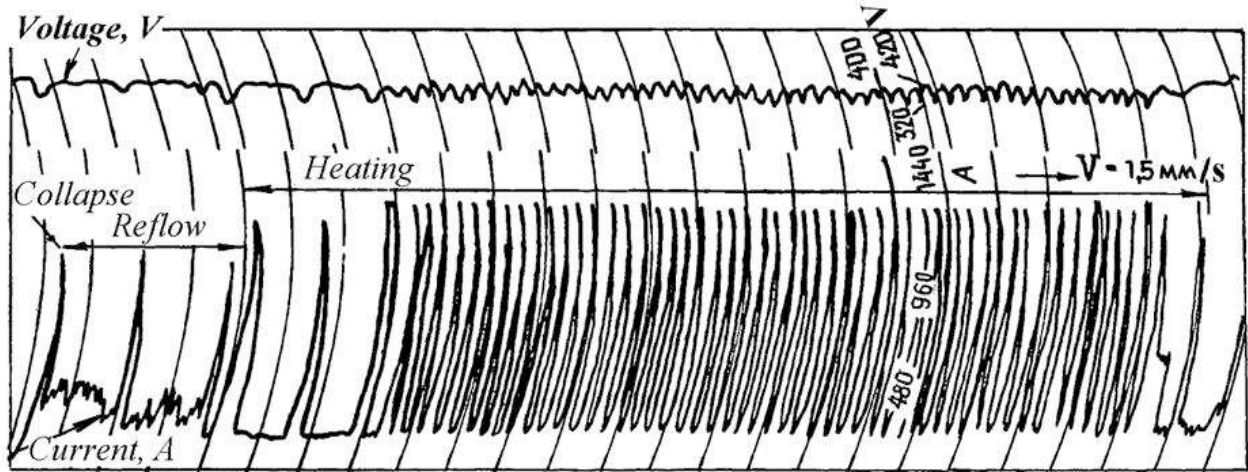


Fig. 7.6 Transient in welding current and voltage of the butt welding installation of 600 kV·A

In networks providing power supply of welding installation groups, the voltage dips have random nature. The greatest values of the dips (up to 20 per cent) are observed in networks which supplying butt and multiple-spot plants, and the least dips are in networks supplying arc welding installations (nor more than 3 per cent). Maximum voltage dips frequency in group load diagrams are 1.8 to 4 Hz. Besides electric welding plants produce voltage fluctuations which average

frequency is  $f_{D,av} = 2n/t_{D,av}$ . Voltage fluctuations are various: from periodic square oscillations to random Markov's process. Maximum voltage fluctuations frequency in the group networks is 5 to 12 Hz. The main part of oscillation spectrum energy is concentrated within the frequency range of 2.5 to 3 Hz. Electric welding plants operate in pulse mode, and it is necessary to take into account the emergence of transients in the consumed current. Under asynchronous switching on the maximum value of transient current can be 3 times larger than the rated current. Duration of the process is three to six periods. Great values of transient currents result in increase of maximum values of voltage dips and extra distortion of voltage sine curve due to appearance constant component and even harmonics in the current curve. Operation modes of electric welding installations are controlled by means of thyristor converters.

Aperiodic voltage oscillations caused by impact load can be reduced to periodic ones being equivalent on energy or average power during the process monitoring time. Equivalent range of periodic  $p$ -th oscillation in per cent is

$$\delta V_e = \sqrt{\sum_{r=1}^n \delta V_r^2 / n}$$

where  $n$  - is the number of oscillations per time  $T$ .

Expression to determine  $\delta V_e$  suitable for use in design practice can be written in another form using values of surges of reactive powers  $\delta Q_p$  and short circuit power on the buses to which sharply variable loads are connected:

$$\delta V_e = \frac{\sqrt{\sum_{r=1}^n \delta Q_r}}{S_s}$$

Inequality  $\delta V_e \leq \delta V_{add}$  is the condition of voltage oscillations permissibility.

### 7.5. Processes taking place under capacitor banks commutations.

Capacitor banks are mainly mounted in load nodes of networks with voltage of 6 to 110 kV and are used mainly to control generation of reactive power. This provides essential improvement the electric power supply system energetic performances. Besides, capacitor banks can make possible to solve in addition problems of operation modes stability increase, short circuit currents limitations, and the voltage control.

Processes of capacitor banks commutation have specific features. Their connection and disconnection is accompanied with variation of many parameters of operation mode. During transients significant surges of current and voltage can arise which are dangerous for both networks and commutation apparatus, and for the banks. Switch installed in the banks circuit connects and disconnects great currents and accomplishes commutation much more often than a switch of overhead transmission line. Besides, surges of transient current are larger under connection of capacitor banks than under lines connection. Therefore, increased requirements are put to the switches intended for operation in circuits of capacitor banks. At a capacitor bank\* switching on, the high frequency current of transient are superposed with industrial frequency current. In the case of unfavorable voltage phase in the instant of bank connection the speed of current rise is the same as under switching on short circuit. The transient current magnitude is considerably less than of the short circuit current but it can reach the value of several kA. Even larger current inrushes are possible under parallel connection of banks but such currents are specially limited by reactors.

Comparatively frequent connections and disconnections of a capacitor bank (once or twice within a day) make heavier the influence of current on the switch. As a rule it results in greater chance of its failure (welding of contacts, their great wear, inadmissible pressure rise in arc quenching devices, etc.) Processes of current variation under connection of a single-phase circuit with lumped capacitance are known from course of Electrical Engineering. Under connection of three-phase capacitor bank in a network with grounded neutral (at the voltage of network of 110 kV) the processes are similar to single-phase one. At that, it is necessary to neglect impedance of ground current return circuit, or to assume that connection of the capacitor bank phases took place simultaneously though this can't be satisfied in many of the networks.

In a factual three-phase circuit the inrush current amplitudes can differ from the calculated for a single-phase circuit owing to transient current damping, availability of several components of various frequency, and current emergence in different phases not at the same time. Even insignificant time difference of current emergence can influence the transient due to difference in instantaneous values of phase voltage applied to contacts, and unavoidable time difference of contacts closing. As periods of high frequency currents are small, even small time difference of currents emergence can cause great currents in the earlier switched on phases at switching on the second or third phase, and their mutual influence becomes significant.

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\* Processes under capacitor banks commutation in networks where other banks aren't available are considered here..

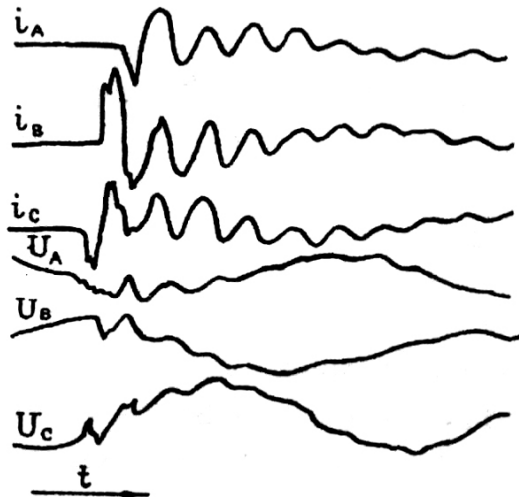


Fig. 7.7. Oscillogram of phase currents and voltages at the transient caused by a capacitor bank switching on

Numerous tests prove that despite a number of the noted factors which can influence the divergence in transient current maximum magnitudes under capacitor bank connection in design single-phase and real three-phase circuits, the current magnitudes are practically coincide. Variations of phase currents and voltages under capacitor bank connection are shown in Fig. 7.7. Rate of transient current rise, if high frequency components are available, can reach great values (up to 50 A/ms) to be dangerous for the switch. To reduce it either stray capacitance of a capacitor bank is decreased or reactor between switch and bank is connected.

When capacitor banks are switched on the transients are accompanied by overvoltage. Theoretically the surge factor equals 2, and it is possible under closure at the instant of maximum

voltage. Practically this coefficient does not exceed 1.9.

At investigation of circuit breakers with natural oil blast, current and voltage inrushes were usually greater at the bank disconnection accompanied with repeated arc initiation than at its connection to the circuit. In this case, current values depend on the bank power and characteristics of the network, and the overvoltage depends on the number of repeated arc ignitions in the period of disconnection. Connection the resistors for banks discharge or use for this aim a voltage transformer influences only the process of connection if grounding device is connected constantly. At switching off and on of the capacitor bank being connected to the substation via a long cable, additional oscillation processes resulting in significant overvoltage in the network arise.

When capacitor bank is disconnected by means of low-oil-content circuit breaker, the repeated break-down or arc ignition is not available, and time of the arc burning reduces. Continuous presence of pressure in arc quenching devices of the circuit breaker facilitates that. Use of such circuit breakers for capacitor banks switching seems very promising.

Use of grounding resistances in the neutral of capacitor bank can reduce both surges under repeated initiations of arc and voltages restoring on the constants of circuit breaker by 15 to 20 per cent.

## 7.6. Short circuits in DC networks

Direct current is used for power supply of electrolysis installations of non-ferrous metallurgy and chemical industry, vacuum arc and graphite electric furnaces, of installations for electrochemical processing of metals and for electroplating, of electric transport, electric drives, charging devices. Supply systems of direct current for circuits of signaling and dispatching control are usually of low power.

Semiconductor rectifiers using uncontrolled or controlled rectifying elements (diodes or thyristors) are applied as converters of alternating to direct current.

Three-phase bridge circuits, and six-phase zero circuit with balancing reactor, and three-phase zero circuit of are used for rectifying units. Low-power rectifying units have the three-phase zero circuit. In the three-phase bridge circuit (Fig. 7.8, a) the primary and secondary windings of energizing transformer have either star or delta connection. Each phase of secondary winding is connected to positive and negative poles of direct current trough valves. In the six-phase zero circuit (Fig. 7.8, b) the primary winding of energizing transformer have a star or delta connection, and the secondary has two reversed stars which zero points are connected through the balancing reactor. The middle point of the balancing reactor is the negative output terminal of the rectifier.



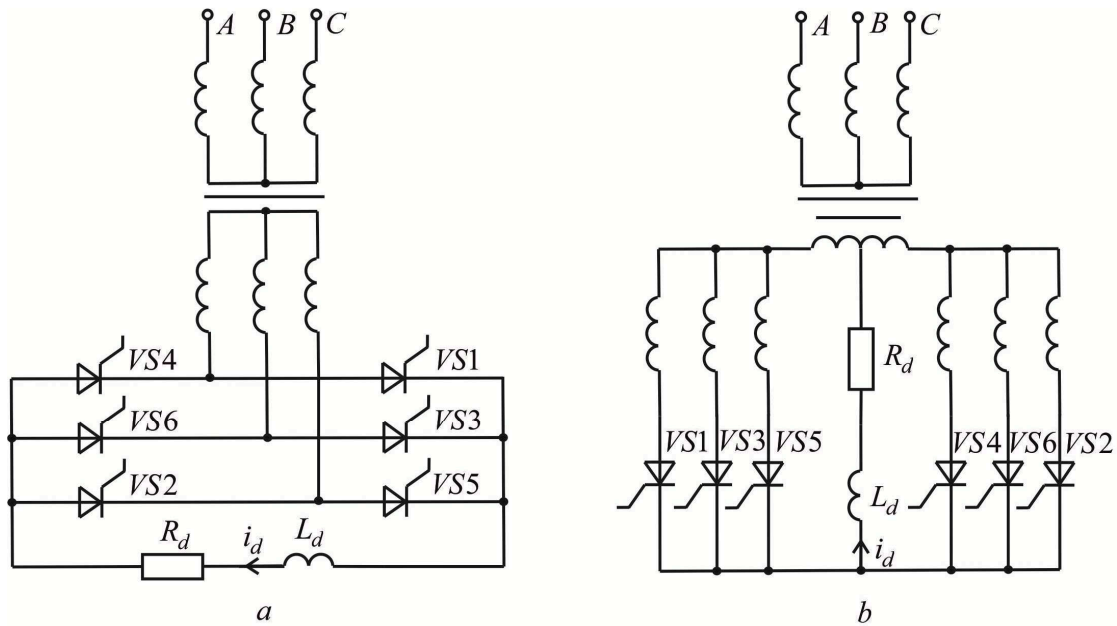


Fig. 7.8. Circuits for rectification of three-phase alternating current: a-three-phase bridge rectifier; b- six-phase rectifier with balancing reactor

In three-phase zero circuit the secondary of the transformer have either star or zigzag connection with zero point. In the first case primary winding must have delta connection and in the second case star connection.

Under short circuit the processes in all networks of direct current are similar. Short circuits in these networks mainly have the same reasons as in networks of alternative current. Emergency operation modes are possible under inadmissible overloading, failure of some elements of rectifier power circuit or faults in the rectifier control system and automatic control system. These operation modes calculations are necessary to select parameters of power circuit of the rectifying unit and the protective means.

Emergencies can be divided into external and internal (power circuit elements malfunction, failure of one of bridge valves). External break-down can cause failure of semiconductor valves and propagation of internal accident.

Consider emergency operation modes for two circuits of rectification: three-phase bridge circuit and six-phase zero circuit with balancing reactor. Such an approach to analysis and calculations of emergency operation modes is applicable to the three phase zero rectification circuit with account of its parameters and characteristics. The following assumptions are taken:

- parameters of power elements of rectifier are linear;
- non-linearity of the rectifier regulation characteristic due to changing the number of commutating valves in the course of short circuit is taken into account only at great duration of the transient and small remoteness of short circuit point;
- three-phase system of supplied voltages is symmetrical and the system is balanced;
- magnetizing currents of transformers and corresponding capacitances of electric equipment elements are insignificant;
- the accident arises at steady state of the supply network.

These assumptions provide obtaining results which accuracy meets the demands of practice.

Current of external rectifier short circuit in the course of transient when its load current equals  $I_d$  is:

$$i_s = I_s \cdot [1 - \exp(-tR_d / L_d)] + I_d \cdot \exp(-tR_d / L_d). \quad (7.9)$$

Besides, proceeding from the electromagnetic transient processes and possible conditions of short circuit, the rectifier regulation characteristic consists from linear and elliptic parts.

The steady short-circuit current depends on rectifier circuit, its operation mode, and parameters of the circuit. Under small values of the fault current (remote short circuits), alternate operation of two or three valves and linear rectifier regulation characteristic (valve commutation angle is  $\gamma < 60^\circ$ , operation mode 1), we have for rectifying circuits under consideration accordingly:

$$\begin{aligned} I_{s(a)} &= 1,35U_{2L} (1 \pm \Delta u_s / 100) / [r_{\text{res}(a)} + 0,955(x_s + x_T / n)]; \\ I_{s(b)} &= 0,675U_{2L} (1 \pm \Delta u_s / 100) / [r_{\text{res}(b)} + 0,239(x_s + x_T / n)], \end{aligned} \quad (7.10)$$

where  $U_{2L}$  is secondary line voltage of the transformer feeding the rectifier, B;  $\Delta u_s$  is deviation of supply network voltage, %;  $n$  - is the number of rectifiers working parallel.

Under great values of the fault current (short circuit is close to the rectifier), simultaneous operation of three valves and elliptic regulation characteristic of the rectifier (angle  $\gamma = 60^\circ$ , operation mode 2), we have:

$$\begin{aligned} I_{s(a)} &= 1,17U_{2L} (1 \pm \Delta u_s / 100) / \sqrt{r_{\text{res}(a)}^2 + [3(x_s + x_T / n) / 2]^2}; \\ I_{s(b)} &= 0,585U_{2L} (1 \pm \Delta u_s / 100) / \sqrt{r_{\text{res}(b)}^2 + [3(x_s + x_T / n) / 2]^2} \end{aligned} \quad (7.11)$$

Resistances of rectifiers under short circuit are determined by expressions:

$$\begin{aligned} r_{\text{res}(b)} &= 3r_T / (2n) + r_{\text{ext}} + r_{\text{arc}}, \\ r_{\text{res}(b)} &= 3r_T / (2n) + r_{\text{ext}} + r_{\text{arc}}, \end{aligned} \quad (7.12)$$

inductance at operation mode 1 is

$$\begin{aligned} L_{\text{res}(b)} &= 0,955(x_s + x_T / n) / \omega + L_{\text{ext}}, \\ L_{\text{res}(b)} &= 0,239(x_s + x_T / n) / \omega + L_{\text{ext}}, \end{aligned} \quad (7.13)$$

and at operation mode 2

$$\begin{aligned} L_{\text{res}(a)} &= 3(x_s + x_T / n) / (2\omega) + L_{\text{ext}}, \\ L_{\text{res}(b)} &= 3(x_s + x_T / n) / (8\omega) + L_{\text{ext}}. \end{aligned} \quad (7.13,a)$$

Arc resistance is  $r_{\text{arc}} = 0,01 \dots 0,015$  Ohm. Currents of short circuit on the rectifier buses arc resistance  $r_{\text{arc}} = 0$  for the circuits being considered can be calculated by formulae:

$$\begin{aligned} I_{s(a)} &= 0,78U_{2L} (1 \pm \Delta u_s / 100) / \sqrt{r_T^2 + (x_s + x_T)^2}; \\ I_{s(b)} &= 1,56U_{2L} (1 \pm \Delta u_s / 100) / \sqrt{r_T^2 + (x_s + x_T)^2}, \end{aligned} \quad (7.14)$$

where reactances  $x_s, r_T, x_T$  (Ohm) are determined by expressions:

$$x_s = U_{2L}^2 \cdot 10^{-6} / S_s;$$

$$r_T = \Delta P_M U_{2L}^2 10^{-3} / S_T^2;$$

$$x_T = U_{2L}^2 10^{-5} \sqrt{u_s^2 - (100 \Delta P_M / S_T)^2}$$

where  $S_s$  - is short circuit power on the buses alternating current;  $\Delta P_M$  - power loss in the transformer windings under short circuit.

Transition from the operation mode 1 to the operation mode 2 takes place when the fault current rectifiers equal:

$$I_{s,F(a)} = 0,26U_{2L} (1 \pm \Delta u_s / 100) / (x_s + x_T);$$

$$I_{s,F(b)} = 0,26U_{2L} (1 \pm \Delta u_s / 100) / (x_s + x_T) \tag{7.15}$$

If short circuit currents values are less or greater than  $I_{s,F}$ , it is necessary to use accordingly expressions (7.10) and (7.11) for their determination.

Current of internal short circuit caused by breakdown of a valve of three-phase bridge rectifier are determined as follows.

Using the transformer parameters, the ratio  $x_{(a)} / R_{(a)}$  where  $x_{(a)} = \omega L_{(a)}$  and  $R_{(a)}$  are total inductive reactance and resistance of one phase is determined, and maximum basic current of the circuit is calculated

$$I_{b \max} = \frac{\sqrt{2} U_{2ph}}{\sqrt{x_{(a)}^2 + R_{(a)}^2}} \tag{7.16}$$

Instantaneous values of fault current of thyristors and diodes using the calculation curves in Fig. 7.9 and 7.10 are  $i_s = i_* \cdot I_{b \max}$ . Cases with blocking control pulses (is used for the rectifier protection) and without it are considered. Curves A in the figures refer to the currents of a broken valve; curves B – to the currents injecting the valve coming into operation; curves C – to the currents of a valve coming out of operation. It is seen that the fault current amplitude and duration of it flow through broken valve are larger than through non-damaged valves providing injection.

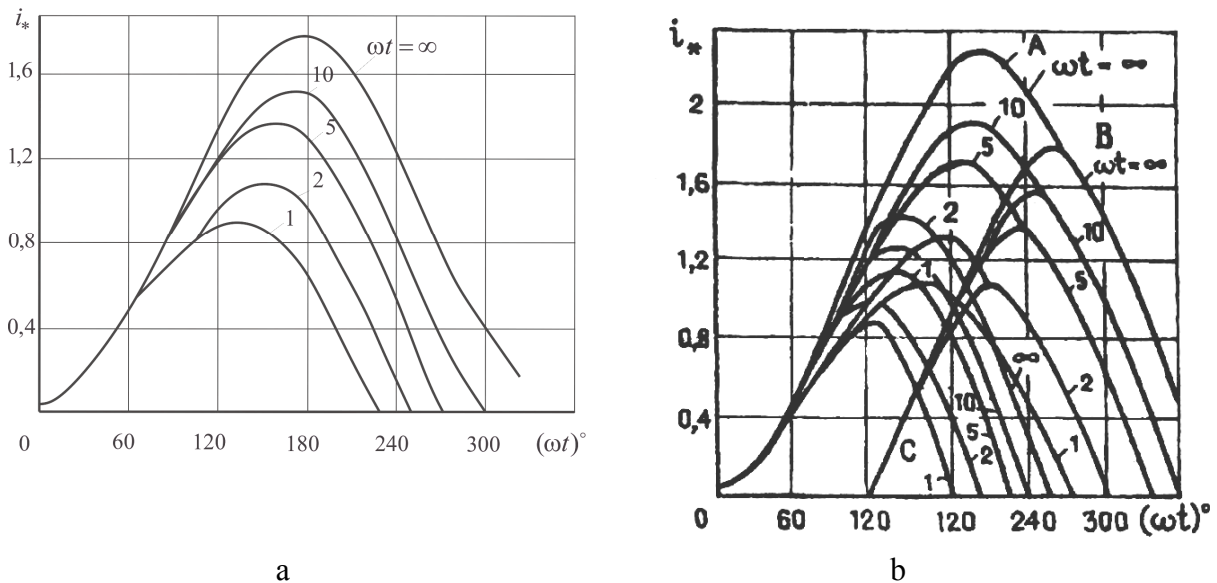


Fig. 7.9. Curves for calculation of instataneous values of fault current of thyristors under internal short circuit at control pulses blocking: a) before the next commutation; b) after the first commutation

Using graphical dependences shown in Fig. 7.11, the relative value of thermal equivalent, which is subsequently transformed into absolute value, is determined by the formula:

$$W = I_{b\max}^2 \cdot A_T / \omega$$

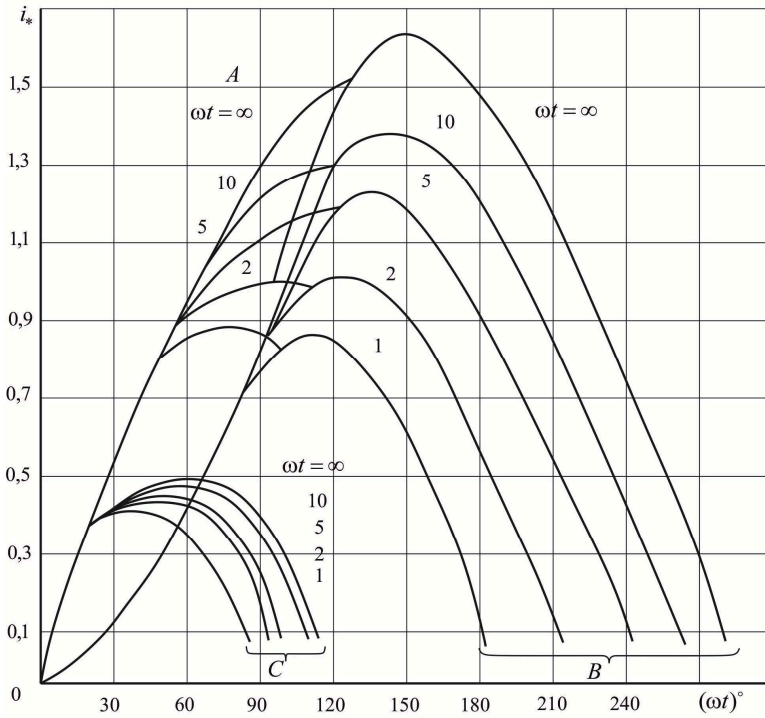


Fig. 7.10 Curves for calculation of instantaneous values of fault currents of diodes at internal short circuit in the instant of maximum backward voltage application

As it is seen, breakdown of a valve at the instant of its commutation completion is being the most severe in comparison with breakdowns in any other instant from the onset of the accident. This is because the backward voltage impacts the broken valve during the major part of the feeding voltage period. Other things being equal uncontrolled rectifying circuits with diodes have larger values of the currents and of time they flow than rectifiers with thyristors.

Power sources can be connected with short circuit point via direct current power transmission line. Taking these sources into account in calculation of short circuit currents carried out with the aim of selection or testing the apparatuses and conductors depends on the short circuit location

and the type of converter unit. When short circuit point is located in the circuit of the rectifier alternating current, the latter has to be introduced in the positive sequence equivalent circuit as fixed load (fixed shunt). Parameters of the shunt are determined by prior load of power transmission. The rectifier does not introduced in negative sequence equivalent circuit. Zero sequence equivalent circuit includes only rectifier unit transformers with connection  $Y_0 / \Delta$ .

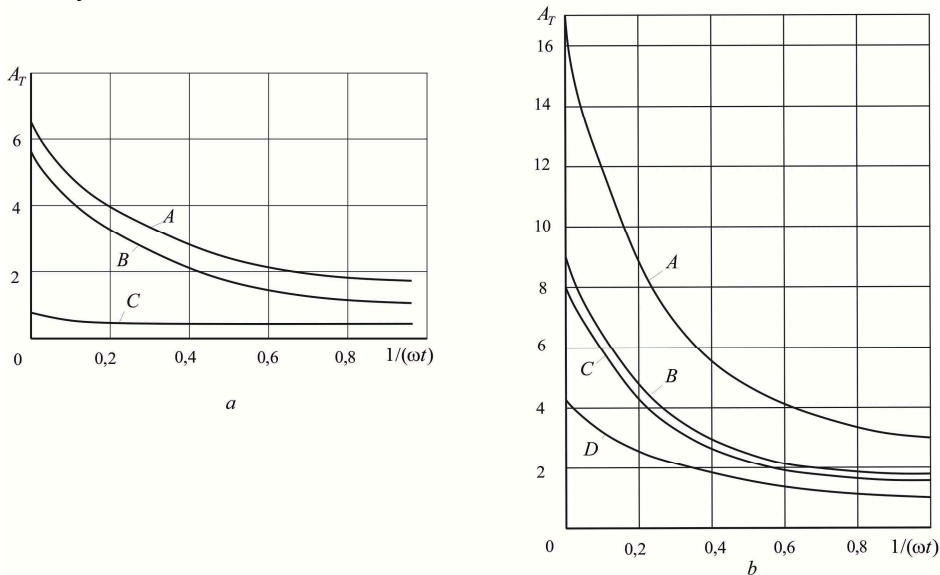


Fig. 7.11. Curves of thermal equivalent relative value at inner short circuit in rectifier after the first broken valve commutation (A) and for the injection circuit thyristors (C and D): a – for a non-controlled rectifier; b – for a controlled rectifier

If the point of short circuit is located in the inverter circuit of alternating current, the latter is introduced in positive sequence equivalent circuit as a shunt. Impedance of the shunt  $Z_{SH}$  depends on location of short circuit and design time instant. At location of short circuit close to the inverter (instability in the form of commutation failure is possible) for the instant 0.02 s the value of  $Z_{SH}$  is assumed based on the previous load. For the following instants  $Z_{SH} = jx_T$ , where  $x_T$  is total reactance of the inverter transformers.

If there is no commutation failure of the inverter at the short circuit, then for any instant of time

$$Z_{SH} = U_{res} Z_{ld} / U_{res(0)}$$

where  $U_{res}$  is residual voltage on inverter buses at short circuit (dc transmission line is not taken into consideration);  $Z_{ld}$  is shunt impedance, defined by previous load;  $U_{res(0)}$  is residual voltage on the inverter buses before short circuit. The inverter is not included to the equivalent circuit of zero sequence. Zero sequence equivalent circuit comprises only the inverter transformers with connection  $Y_0 / \Delta$ .

When short circuit leads to inverter control pulses removal then for any instant of time (up to the next electric transmission switching on)  $Z_{SH} = \infty$ .

### Test questions

1. What are the features of short circuit in supply networks with voltage of 330 kV and higher?
2. What are general dependences used in calculations of short circuit currents in power supply networks?
3. What components has total current of three-phase short circuit for conditions shown in Fig. 7.1?
4. What are the features of short circuit calculation in the networks of higher frequencies?
5. What does characterize short circuits to ground in power networks of 6 – 35 kV?
6. For what purpose are arc suppressing coils used? How do they affect the processes at short circuits to ground?
7. How important is the assessment of operation modes arising at short circuit to ground in the networks with insulated neutral?
8. What are the peculiarities of transients that occur in circuits of technological equipment?
9. What are the special features of transients at switching the capacitor banks?
10. What are the conditions of short circuits emergence in direct current networks?
11. What are operation modes of rectifiers under possible short circuits?
12. In what way is the transition from operation mode 1 to operation mode 2 performed at calculations short circuit in rectifiers?
13. In what way are internal short circuits of rectifier caused by breakdown of rectifying element calculated?

### Topics for essay

1. Calculations of emergency operation modes in direct current networks.
2. Estimation of emergency operation modes in the networks of higher frequency.
3. Danger of ground closure in electric power networks.
4. Calculation of processes under capacitor banks commutations in electric power networks.

## **CHAPTER 8 LEVELS OF CURRENTS AND POWER OF SHORT CIRCUIT**

- 8.1. Quality of electromagnetic transients
- 8.2. Ways of short circuit currents limiting
- 8.3. Use of technical facilities for short circuit current level optimization
- 8.4. Short circuit current level optimization
- 8.5. Short circuit currents levels coordination
- 8.6. Transients in power-supply systems of enterprises in the context of problems of electromagnetic compatibility

Test questions

Topics for essay

### 8.1. Quality of electromagnetic transients

Electric power system is the main electric power supply source for industrial enterprises. Trend to increase the number and power of electric energy sources of the power system, approaching the power sources to consumers and increase synchronous and induction motors number and power in load nodes mean simultaneous increase in levels of power and short circuit currents on the buses of step-down substations and in distribution electric networks of power supply systems. This stipulates increase the requirements to electromagnetic and thermal stability of electric power supply system elements, as well as to switching equipment, relay protection and system automation operation.

While designing an electric power supply system it is necessary to coordinate the requirements to their elements with possible transients. Analysis of electromagnetic transients with their quality evaluation is necessary for designing and proper operation of the electric power supply system.

Transient quality coordinated with requirements to the electric power supply system and its elements is characterized by qualitative and quantitative indices. The latters are usually functions expressing dependence of the operation mode parameters on transient duration or its maximum values and contain specific information. Those qualitative performances of electromagnetic transients, which occur when ordinary operation of the electric power supply system passes into fault, are evaluated by the properties, characteristics, and consequences indicated below.

1. **Transient duration.** This is time during which the electric power supply system changes from one stable operation to another stable condition. Duration of processes close to aperiodic ones can be evaluated by time interval  $t_{dr} \leq 3T_a$  using the value of equivalent time constant of electric network  $T_a$ . Under transients resulting from ordinary operation sudden failure, try usually to reduce the period of fault operation. When estimated duration of fault operation (short circuit) is evaluated, this time interval is made up of minimum time of protection relays operation  $t_{P,min}$  and own time of commutation apparatus disconnection  $t_{P,br}$ :

$$\tau = t_{P,min} + t_{P,br}$$

Possible failures of operation (connection, disconnection, short circuit, starts. etc.) are calculated for each type of electric power equipment with the aim to compare with allowable period of transient duration which can be limited by technical and technological conditions, safety requirements, overheating etc.

2. **Character of transient**, which is evaluated on current time variation. The character of electromagnetic transient depends on power of electric energy sources, electric networks parameters, availability of generators automatic excitation control and installation in the networks automatic reclosure means.

Transient character of operation parameters variation can be aperiodic, oscillatory with constant or aperiodic magnitude, or monotonic one. Qualitative estimation of transient magnitude character is used at calculation short circuit operation parameters by the amplitude. At consideration demagnetizing effect of generators stator reaction, the power sources under short circuit are conventionally divided for the sources of unlimited and limited capacity. It depends on their electric remoteness of short circuit point (see chapter 3).

Quantitative assessments of transient character are performed using the damping factor of short circuit current periodic component

$$\gamma_{F\tau} = I_{F\tau} / I_{F,t=0} \tag{8.1}$$

and the damping factor of short circuit current aperiodic component

$$\gamma_{at} = i_{a\tau} / i_{a(t=0)}. \tag{8.2}$$

3. **Dangerous for electric power supply system equipment consequences** are estimated by following indices of transient short circuit current:

- electrodynamic stability of electric power supply system elements (it is checked on the surge current under three-phase short circuit);
- thermal stability (it is estimated on the maximum thermal impulse of current under either three-phase or two-phase short circuit)

$$B_s = \int_0^t i_s^2(t) dt. \quad (8.3)$$

4. **Influence of fault transient parameters** on normal operation of electric power supply system and its elements affects the properties of electric power reflecting relevant power quality indices in general-purpose electric power supply systems (Standard 13109-97).

Voltage deviations across the terminals of electric consumers under faults  $\Delta U = (U - U_{\text{rated}}) / U_{\text{rated}}$  can exceed maximum permissible values; the area of emergency state can be determined in the electric power supply system such voltage deviation values.

Voltage fluctuations are characterized by the peak-to-peak voltage variation  $\Delta U_t$  and the flicker dose  $P_t$ . The peak-to-peak voltage variation is proportional to reactive power surge, and inversely proportional to the short-circuit power:

$$\Delta U = (\Delta P x_{\text{res}} / r_{\text{res}} + \Delta Q) / S_s.$$

For example, The intensity of voltage flicker, as an example, for electric arc furnace is determined by the expression

$$P_t = k_t S_{\text{rated,T}} / S_s, \quad (8.4)$$

where  $k_t$  - is a coefficient, which value depends on furnace type, kind of load and features of its operation;  $S_{\text{rated,T}}$  - is the apparent power of furnace transformer;  $S_s$  - is short circuit apparent power in the point of the furnace connection.

In a system of electric power supply, when voltage deviations and fluctuation take place, the banks of compensating capacitors are constantly under transient conditions of recharging or partial discharging. These transients are accompanied with increased heating of capacitor banks due to increase of current in their circuit:

$$\frac{I}{I_{\text{CB,rated}}} = \sqrt{1 + \frac{3\alpha S_s \sigma_{*(\text{rated})\delta U}^2}{\omega Q (1 + \sigma_{*(\text{rated})\Delta U}^2)}}.$$

Here  $I_{\text{CB,rated}}$  - is the capacitor bank rated current,  $\alpha$  - is the damping coefficient of voltage fluctuation correlation function;  $\sigma_{*(\text{rated})\delta U}^2$  - is the relative root-mean-square value of  $\Delta U_t$ ;  $\sigma_{*(\text{rated})\Delta U}^2$  - is the relative root-mean-square value of voltage deviation;  $k_\Sigma = x_{\text{res}} / r_{\text{res}}$  - is a ratio of reactive and active components of total impedance of capacitor banks connected to the power source;  $Q$  - is the capacitor banks reactive power.

Non-sinusoidal shape of voltage is characterized by the distortion factor of sine voltage curve  $k_U$ , which is directly proportional to the apparent power of converter  $S_{\text{conv,a}}$  and inversely proportional to the short circuit power

$$k_U = S_{\text{conv,a}} / S_s \leq 0,05. \quad (8.5)$$



Voltage unbalance is characterized by the voltage unbalance factors of negative sequence and zero sequence. For example, the unbalance factor of negative sequence  $K_{2U}$  is directly proportional to the power of single-phase load  $S_{ph,ld}$  and inversely proportional to the short circuit power

$$K_{2U} \approx S_{ph,ld} / S_s \leq 0,02. \quad (8.6)$$

Frequency deviations are directly proportional to active power surge of consumers with sharply variable load and inversely proportional to the short circuit apparent power

$$\Delta f \approx (\Delta P / \Delta t) / (2\pi S_s). \quad (8.7)$$

#### 5. *The cost of extra* measures for improvement of transient characteristics

For electric power supply systems of large enterprises short circuit currents at points of consumption have so great values that it is impossible to avoid their restriction or installation of more expensive elements. Solving and realization this task require extra capital investments in the power supply system.

As it is seen, quality of electromagnetic transients influences the operation of electric power supply system and electric power consumers in different ways. Currents and power of short circuit underlie the quantitative assessment of all above indicators. Typically, there is a contradiction at assessment their required level. From the point of view of cost of the system elements reduction and their operation conditions facilitation, it is desirable to decrease levels of short circuit current and power. On the contrary, providing the required power quality for consumers their high level is needed. This defines formulation of a task of finding a compromise selection of the electromagnetic transient quality indicators values.

## 8.2. Ways of short circuit currents limiting

Levels of short circuit currents and powers characterize the design conditions of electric power supply equipment performance under fault operations. They determine selection of the buses cross section areas, wires and cables, necessary switching gears, electromagnetic and thermal stability of current-carrying parts and design of electric equipment. The selection of electric equipment on the factors of fault operation means not only more specific requirements to the technical performances, but also indicates correspondent increase of its cost.

When electric power supply system is designed, the technical and economic problem concerning the reducing of currents and short circuit power to values permissible by the parameters of electric equipment to be economically expedient is solved.

In the process of electric power supply system maintenance, which is accompanied by their development and new electric power sources inclusion, the problem of current levels and short circuit power arises if they exceed technical performances of electric equipment installed. To solve it, different measures connected with short circuit currents limitation are used: the increase of electric impedance of short circuit current path, the localization of its power sources under fault operation and disconnection of faulted electric network during time interval  $t < 1/(4f)$  (it is  $t < 5$  ms for  $f = 50$  Hz current frequency).

Such method include: the selection of design and circuit of electric power supply system; stationary and automatic division of electric network; selection of its operation modes; selection of switching schemes; use of equipment with higher electric impedance; use of high-speed switching gear; variation of network elements neutral operation and electromagnetic transformation of system parameters operation.

*The design circuits of electric connections of electric power supply system elements* are chosen at the stage of its design or reconstruction. Following items should be the basis for that:

- longitudinal division of networks with similar voltage level when they are located at the territorially different sites of electric power supply units and their connection by means of higher voltage network (Fig. 8.1,a);

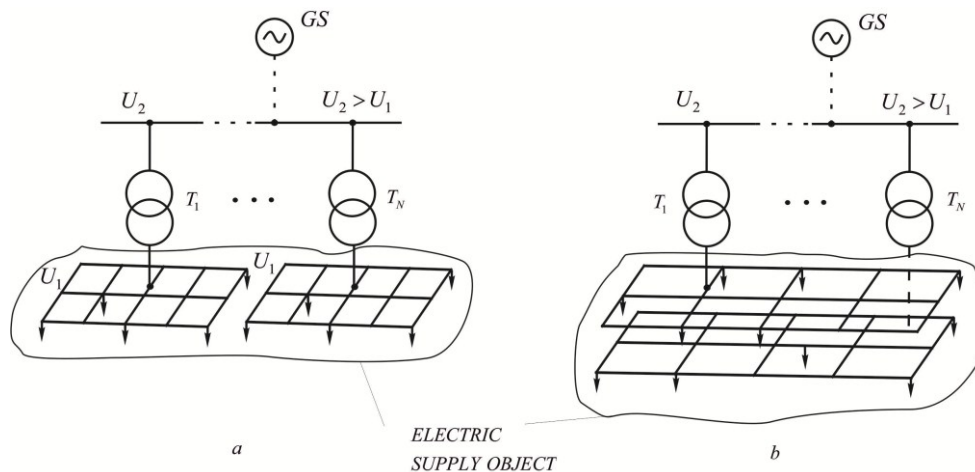


Fig. 8.1. Electric network division: a – direct axis; b – quadrature

- lateral division of networks with similar voltage if they have common territory but are connected by means of higher voltage network (Fig. 8.1, b);
- separate feeding of load nodes by electric power sources and their parallel operation, it is done by means of connections in electric network and block connection “generator - transformer - line”;
- use of substation breaking up and stage-by-stage formation of electric power supply system by means of high voltage deep terminal;
- use of separate operation of electric power supply system elements (power transmission lines, transformers) at each stage of electric power distribution;
- use of open-loop distributive network with wide implementation of current conductors, bus and cable bus ways. It gives ability to use current-limiting feature of the network itself.

While making-up the electric power supply circuits it is necessary to follow a number of important requirements made to electric power supply systems:

***Maximum approach of power sources to consumers***

Centralized electric power supply by one or several receivers is used for prolonged operations. It is typical for industrial enterprises electric power supply system presence of several power sources feeding short circuit point: own electric power sources, such as generators of thermal power stations connecting the point with district power system by means of substations; synchronous capacitors as well as synchronous and induction motors went to generator mode.

The share of each the source share in feeding of the short circuit point depends on its power and electrical remoteness. Approaching of the main power source means decrease of intermediate transformations in the system of electric power supply, and increase of network elements with higher voltages, and as a result, the less operating and short circuit currents. To provide stand-by facilities all power sources of enterprise are connected by current pathways, cable or overhead transmission lines of secondary feeding voltage. Along with elements reservation, connections at the secondary voltage prevail, because they give ability to have less level of short circuit currents.

***Sectioning of all electric power distribution steps of power supply system***

This demand is closely connected with the choice of quantity and power of transformers of the main step-down substations and transformer units, feeding transmission lines capacity. Such a design of electric power supply system permits to increase electric impedance of the network to short circuit current, avoid the break-down progress and localize the short circuit area.

***Electric network design and geometry selection*** (radial, bus ways, and radial and bus way) have to be substantiated (side by side with such key factors as reliability, losses of power and energy and non-ferrous metal etc) by the grade of conductors sections use as well. They must be chosen on short circuit current.

***The use of stepwise current limitation*** in electric power supply circuit when devices or network elements having current-limiting properties are installed at several consecutive stages of electric power distribution. From this viewpoint, electric power supply circuits of coal mines are the

most typical, in which the first stage of short circuit power limitation is the current reduction at the primary step-down substation of the mine, and the second one - at the terminals feeding the load of underground consumers, where short circuit power is limited to 50 ... 100MV·A.

Stationary or automatic division of networks usually takes place in the system of external electric power supply, and is connected with the increase of quantity and power of electric power energy sources both in power system and at own thermal power stations. The necessity to divide the network emerges when short circuit currents level in the load nodes exceeds permissible level on the parameters of electric equipment being in use. The network division greatly affects on operation stability, reliability of electric system performance and losses of power in networks as well.

Stationary division of primary circuit of network (Fig. 8.2,a) is done under normal operation in such a way not to let the maximum level of short circuit current in a separate load node exceed the permissible on the parameters of installed equipment (Fig. 8.2,b,c).

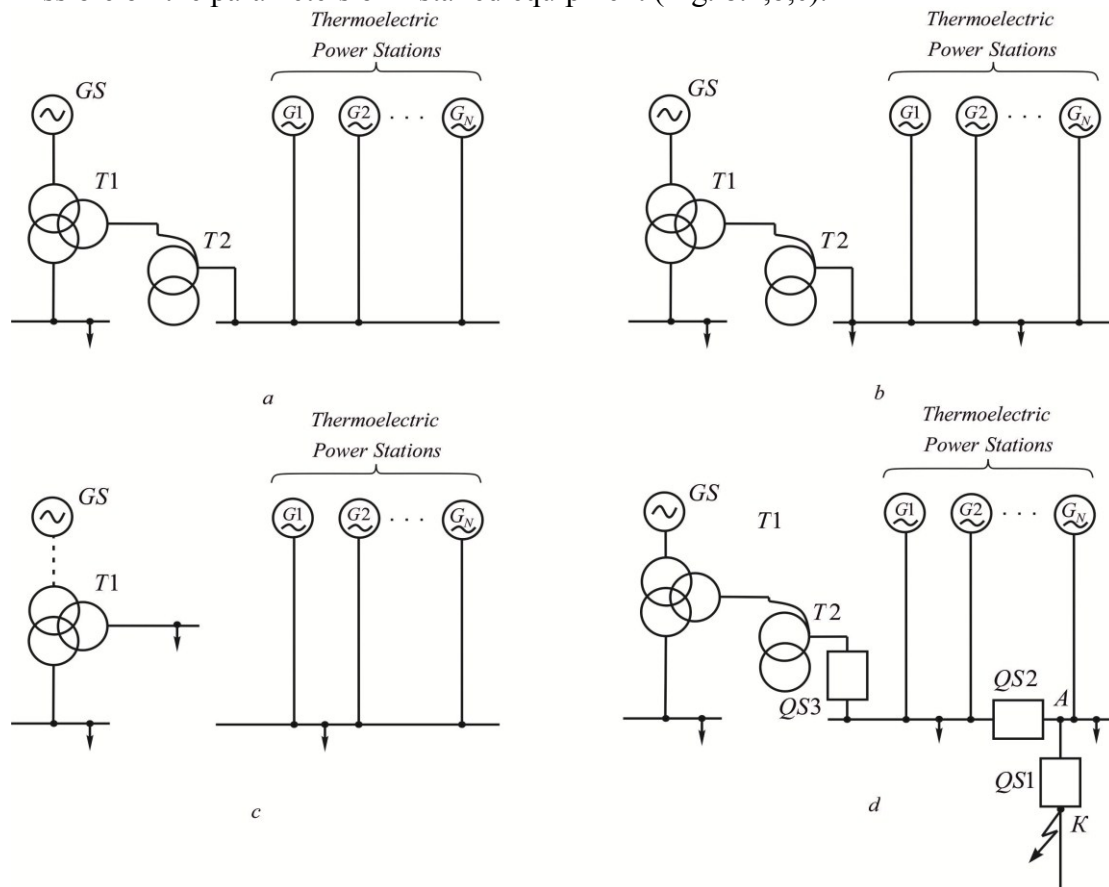


Fig. 8.2. Division of electric power supply system networks: a-initial circuit; b, c-stationary; d-automatic

Automatic division of network is done under fault operation by means of short circuit area consecutive localization (Fig. 8.2,d). Under short circuit at the connection  $A$ , a part of faulted area power sources is disconnected (with the help of circuit breaker  $QS2$  or  $QS3$ ), and then circuit breaker  $QS1$  is used to disconnected the connection. Consecutive disconnection of short circuit current feeding sources lets to use switching gears having less disconnecting ability comparing to gears correspondent to actual level of short circuit currents.

Choice of network maintenance mode is closely connected with circuit designs. In the electric power supply system with prolonged operation the separate performance of power transformers of the main step-down substations as well as transformer units is recommended. Together with separate performance of electric power sources the subdivision of substations, and sectioning of all energy stages permit to obtain the greatest impedance of short circuit current. Under normal operation each section of distributive devices performs independently, and necessary grade of electric power supply interrupted operation is provided by sectional switches closing done either by

operative attendants or if devices of automatic connection of reserve are used. Ring circuits of electric power supply are used with broken sectional switch at one of substations.

The above recommendations concerning operation choice are not always admissible if large power consumers with sharply variable impact load are available. Under limited power of electric supply the large power consumers stipulate active and reactive loads surges which cause voltage oscillations. To provide admissible minimum voltage level in feeding network of such power consumers, the following means are effective: short circuit currents levels increase (div.8 4), use of higher voltages, and separate electric power supply.

*Electric power supply system's feeding circuits* while designing the external electric power supply are chosen on the basis of short circuit actual power by the power system, necessary degree of electric power supply uninterrupted operation and a list and territorial location of power consumers. Besides, it is necessary to estimate short circuit currents generated by synchronous and induction motors, and the ability of electric power supply system further progress. The circuits of electric connections must meet the demands of reliability, simplicity and economy.

The choice of circuit of electric connections of the main step-down substation as the linkage node between electric power system and distributive network of electric power supply system is the most important factor for short circuit current levels in the power supply system. The more is the power of step-down transformer with electric power system, the more are short circuit currents at the buses of secondary voltage of the main step-down substation. To reduce them, it is necessary to subdivide the substations of electric power supply system on power or to use the circuits of electric connections limiting short circuit currents level at the buses of secondary voltage. The variations of electric connections circuits are shown in Fig. 8. 3. It is recommended to use them if single rated power of transformers increase.

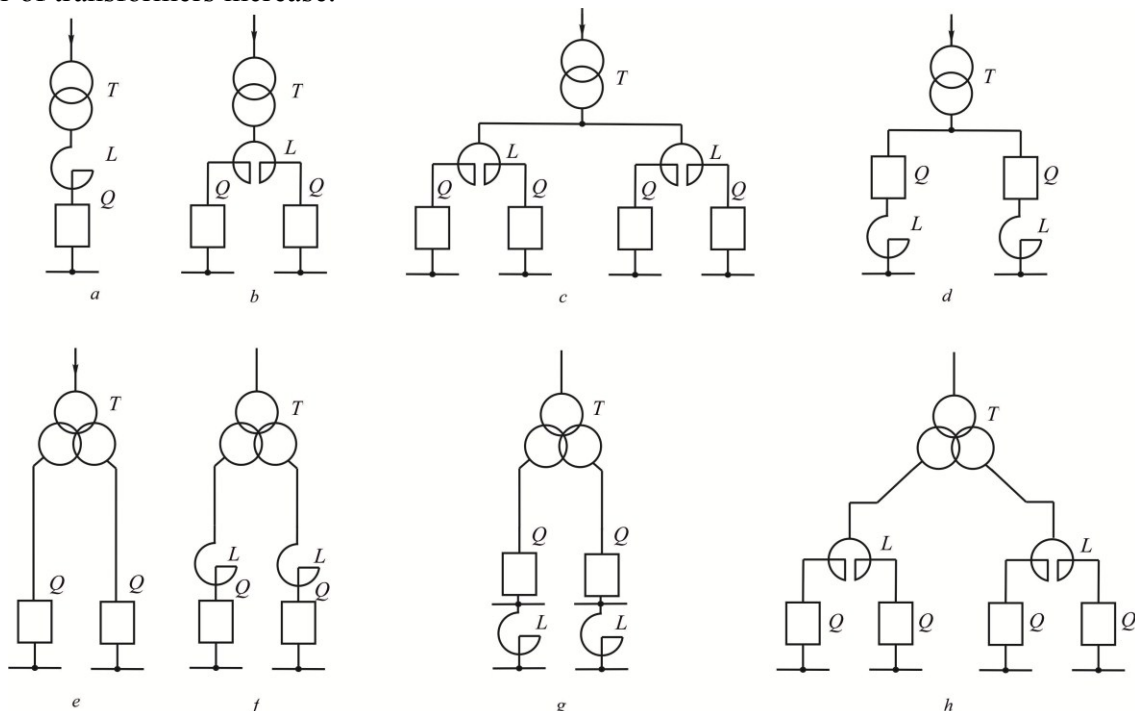


Fig. 8.3. Variations of electric connections circuits of the main step-down substations with short circuit current limitations on the buses of secondary voltage under different single power of transformers:

- a – 25–40 MV·A; b – 32–63 MV·A; c – 63–80 MV·A; d – 63–100 MV·A;  
 e – 25–80 MV·A; f – 40–80 MV·A; g – 63–80 MV·A; h – 100 MV·A

When electric connections are chosen it is necessary to pay attention to configuration of the circuits connected to buses of secondary voltage of the main step-down substation. By means of distributive network the area of short circuit is fed by synchronous and induction motors which went to generating mode. Sectioning at all stages of electric power distribution decreases short circuit current value generated by these local sources. It is necessary to account the change of

electric connections circuit under operations when prolonged performance with connected sectional devices (for the heaviest duty) is approved.

It is expedient to use such connections in the circuits of electric power supply of enterprises as: the line of feeding voltage - transformer of the main step-down substation, the line of feeding voltage - transformer of the main step-down substation a current conductor of distributive voltage, and the line of distributive voltage substation transformer low voltage busway etc.

*The use of electric equipment with higher resistance* provides installation of both common and specific elements. While designing the electric power system one can purposefully choose network elements with great reactance and resistance, make variations in quantity and power of transformers, use transformers with higher relative voltage of short circuit, overhead transmission lines and current conductors with increased distance between phases, extended bus ducts, etc. Specific electric equipment includes transformers and autotransformers with split secondary windings, single and double chokes (reactors), current limiting devices of resonance, transformer or choke types, which function is to increase the impedance for the current exceeding operation current value.

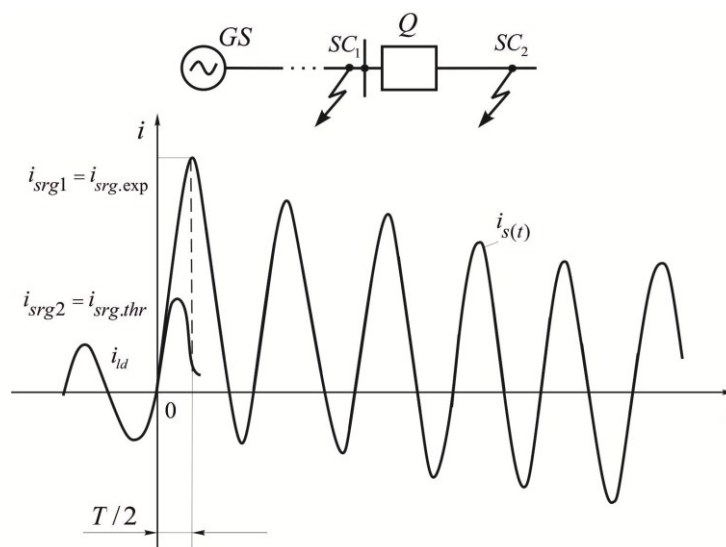


Fig. 8.4. The limitation of electro-dynamic effect of short circuit current from by means of commutation apparatus Q

Current-limiting function of switching gear arises under response rate comparable with the period of current variation. Besides, they limit effect on magnitude and duration of short circuit current disconnection. Electrodynamics effect of short circuit current reduces when apparatus which operation time is less than 5 ms are used (Fig. 8.4). Inertialess fuses, thyristor breakers with forced commutation, explosive action limiters of maximum current and definite types of automatic units for the voltage up to 1 kV can be used as such apparatuses. If short circuit lasts more than 5 ms, the heating action of current is shown which can be reduced by faulted circuit disconnection speed up as well.

*The variation of neutral operation of electric network elements* is rather essential factor of limitation of short circuit current which takes place in loops “conductors-ground”. Grounding of neutrals through circuits with extra resistances results in zero sequence equivalent impedance increase. The same target is also attained if transformers in the node points are substituted with transformers of the same power with connection “star-star”. Grounding of neutral facilitates solving of other important design problems (levels of insulations, safety demands, levels of overvoltage, reliability) but increases the short circuit current to ground. That’s why the problem of choice and variation of the network or its elements neutral operation must be solved in a complex way by means of feasibility study.

*Electromagnetic transformation of electric power supply system mode parameters* (the load mode) means transmission of power to consumers, accompanied with current rectification, inverting, regulation of current frequency, and also transformation of three-phase system into

single-phase, into current system, etc. Such transformations of electric power parameters in power supply systems are one-sided. They are mainly accomplished for feeding of special groups of consumers (rolling mills, welders, industrial and civil transport). Facilities that maintain such operations are decoupling devices eliminating energizing of short circuit point by following them load nodes and local sources (in emergencies return power transmission into the feeding network is excluded).

At transformation of the voltage system into the current system, the short circuit mode is non-emergent, but normal one (for groups of welders; networks feeding arc furnaces; secondary circuits of relay protection). The same transformation of mode parameters is used for power transmission and distribution in mine transport system with contactless electric locomotives.

Thus, limiting of short circuit currents and powers can be accomplished by correct design of power substations and electric networks. Selection of limitation principle is an ambiguous task. It comprises a set of measures solving jointly a number of tasks for selection of electric equipment parameters and operating modes for providing best technical and economical efficiency.

### 8.3. Use of technical facilities for short circuit current level optimization

The use of different means concerning short circuit currents limiting in addition to decisions on design and operation of structure and circuits of elements and modes of electric power supply systems stipulates application of specific technical means as well. The latter are specific electric equipment which directly limits value or duration of short circuit current influence, or is used in circuits of elements which perform this duty. Such means are:

- apparatuses and devices providing automated networks division;
- power transformers and autotransformers with specific design and connection of phase wirings;
- current-limiting elements and devices;
- current-limiting switching gears;
- devices which modify operation of power transformers neutral.

*Automatic division of network* can be used in external electric power supply of enterprises in networks with voltage of 35 kV and higher. Such an operation is applied with the use of means of emergency automation and switching gear to be installed at the powerful connections between distribution sections and on the terminals.

The means of emergency automation cover protection responding to short circuit, devices of automatic successive disconnection of switching gear (Fig. 8.2,d), devices of automatic frequency off-loading, automatic frequency off-loading, automatic reclosure, automatic load transfer. The system of means of short circuit currents successive disconnection must be highly reliable and fast-operating both under short circuit disconnection and initial operation recovery. Its switching gear must withstand without damage full through short circuit current and switching on for short circuit current of an injured connection. This system has such disadvantages:

- decrease of reserve of static stability at post-emergent operation due to imbalance of power of sources and loads in divided parts of electric power supply circuit;
- long recovery period of initial power setting after switching off short circuited branching.

To divide the network automatically it is necessary:

- to analyze the pattern of calculation for possible short circuits;
- to estimate balance of power in electric circuit parts intended for dividing;
- to take into consideration variation in electric network configuration;
- to calculate the time of successive short circuit current disconnection, and time of circuit recovery for normal operation;
- to estimate the reserves of post-emergent operation static stability;
- to evaluate reserve stability of postdamage mode and coordinate short circuit currents levels with technical parameters of switching gear.

Power transformers and autotransformers can be designed taking into account necessity of short circuit currents limitation. The design peculiarity of a transformer is used to limit short circuit power on the buses of secondary voltage. It is known that short circuit relative voltage is determined by rated voltage and throughput power of transformer which stipulate geometric dimensions of its windings. Under concentric arrangement of windings two-winding transformers have low voltage winding inside, and high voltage one outside, three-winding transformers have medium voltage winding between windings of low and high voltage. Arrangement of windings, diameter and dimensions of passage between windings influence the value of short circuit relative voltage. The passage is less between medium voltage and high voltage windings of three-winding-transformers than between low voltage and high voltage windings of two winding transformers.

Decrease of short circuit power by means of step-down transformers (Fig. 8.5) is determined by expressions:

for two-winding transformer

$$S''_{s2} / S''_{s1} = 1 / (1 + u_s S''_{s1} / 100 S_{T, rated}); \tag{8.8}$$

for three-winding transformer

$$\left. \begin{aligned} S''_{s2} / S''_{s1} &= 1 / (1 + 2u_{s, H-L} S''_{s1} / (100 S_{T, rated})); \\ S''_{s3} / S''_{s1} &= 1 / (1 + 2u_{s, H-M} S''_{s1} / (100 S_{T, rated})). \end{aligned} \right\} \tag{8.9}$$

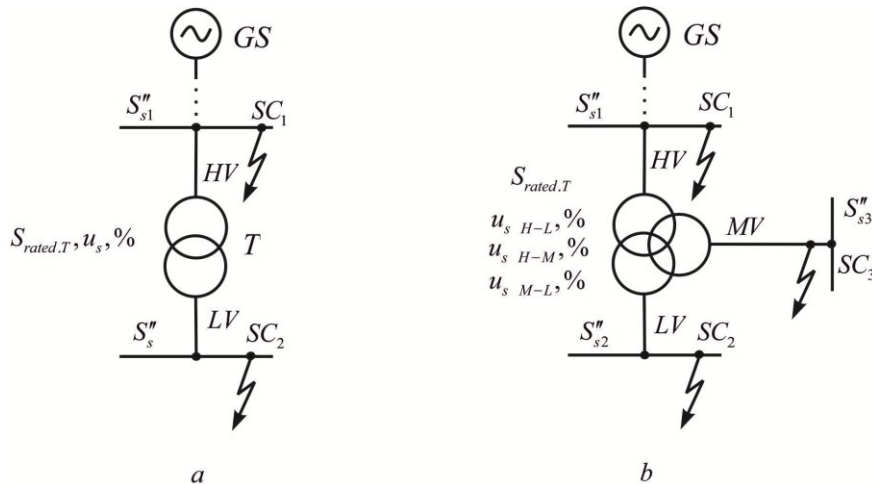


Fig. 8.5. To short circuit power level limitation by means of transformer parameters: a-two-winding transformer; b-three-winding one

It results from (8.8) and (8.9) that short circuit power decrease depends on short circuit power between correspondent windings of transformers, their unit power, and short circuit power intake from electric power system. Two winding step-down transformers as well as three-winding transformers have a definite range of variation of impedance voltage: for two winding transformers with voltages of 35/6-10 kV and power 1-80 MV·A it is within 6.5...14.4%, for voltages of 110/6-10 kV and power of 2.5-400 MV·A – within 10.5...13.5%, for voltages of 150/6-10 kV and power of 2.5-250 MV·A-within 10.5...14.6 %, for voltages of 220/6-10 kV and power of 31.5-125 MV·A – within 10.6...14 %.

In order to limit level of short circuit power on the buses of secondary voltage, it is necessary according to (8.8 and 8.9) to choose transformers having heightened impedance voltage for correspondent windings, to diminish step-down substations having connections with powerful power system by power and to use separate operation of transformers.

Transformers and autotransformers with split low voltage winding can be used. Parts of the split winding are placed symmetrically relatively the high voltage winding. They have independent bushings and permit arbitrary distribution of load. A substantial role in limitation of short circuit currents belongs to connection of the transformer (autotransformer) windings. As the equivalent circuit for zero sequence includes only the paths through which the zero sequence currents flow, the

parts of the network behind the windings connected in delta are not present in this equivalent circuit.

Current-limiting reactors represent extra reactance connected in different points of electric network with voltage of 6-220 kV. Their destination is to reduce short circuit currents behind the reactor and to provide the needed level of residual voltage in the network nodes in front of the choke.

Depending of the place of a reactor connection it is distinguished current-limiting of connections (Fig. 8.6, a), terminals (Fig. 8.3 and 8.6, b), sections (Fig. 8.6, c) and their combinations (Fig. 8.6, d). Single and double reactors are distinguished on connection circuit. The difference between them is in availability of medium terminal making possible different connections and applications of the reactor parts. Current limiting effect of a reactor is characterized by its inductive reactance and rated current.

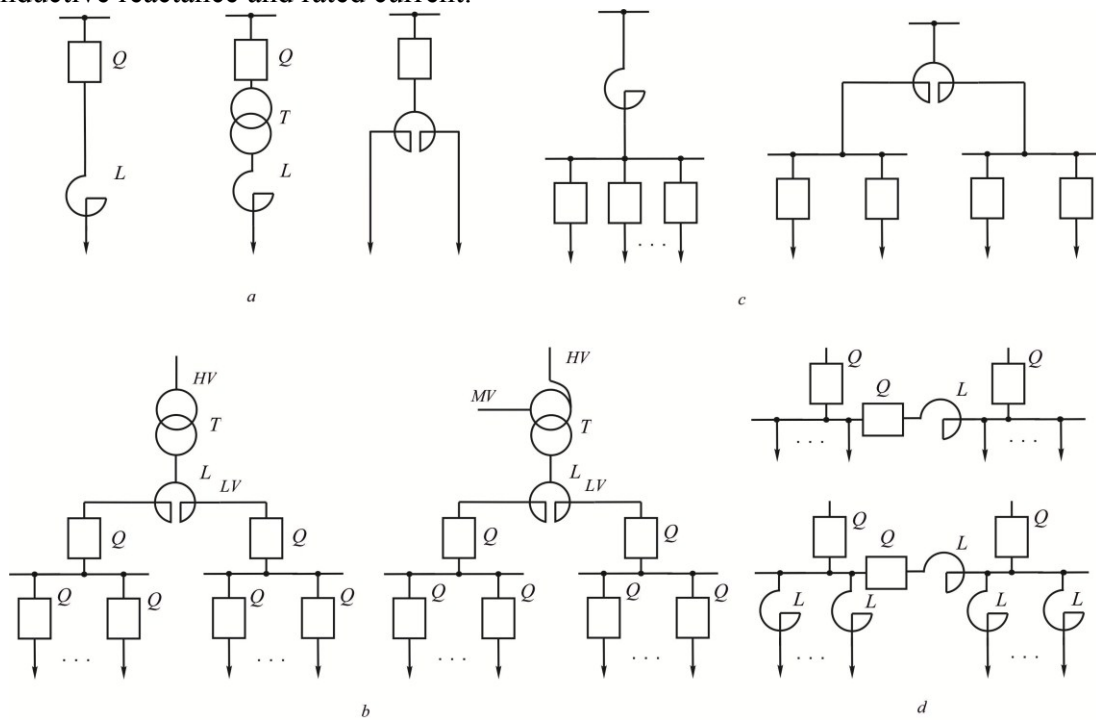


Fig. 8.6. Circuits of reacting: a-taps; b-terminals; c- sections; d- combined

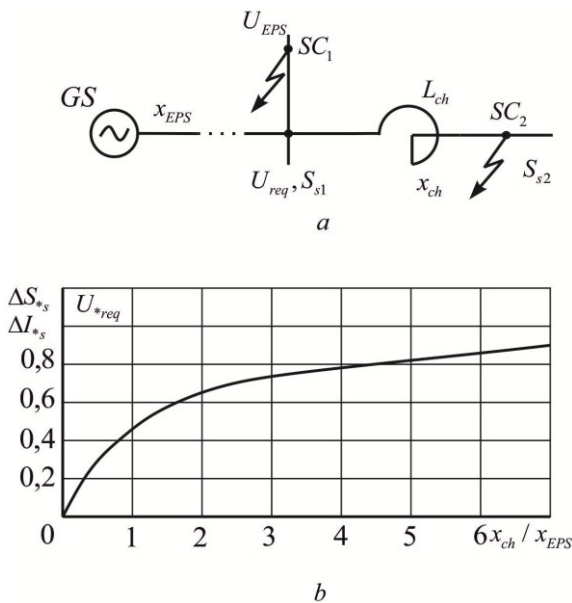


Fig. 8.7. To estimation of single choke efficiency: a – connection circuit; b – maintenance characteristics

To reduce costs it is necessary to use group reactors in the points of connection, on terminals, on inputs and in circuits of the main step-down substation instead of individual ones. Besides, fluctuations caused by load variations are possible in circuits containing group reactors in cases of reactors with great rated currents and reactance. This disadvantage is eliminated by means of double reactors installation with their branches uniform loading. In the case of sharply varied loading the voltage fluctuation is reduced in some considerable degree.

If specified value of voltage fluctuation is provided on terminals of one reactor branch, the sharply varied load can be connected to another branch. This load power is determined by the formula



$$S_{ld} = \delta V / (u_s / (100S_{T, rated}) - 50x_{rated} / U_{rated}^2). \quad (8.10)$$

Effectiveness of a single-circuit reactor depends on parameters of the network and the power of short circuit that flows to the point of connection from the power source (Fig. 8.7, a). Relative value of short circuit power and current are calculated by expression:

$$\Delta S_{*s} = (S_{s1} - S_{s2}) / S_{s1} \equiv \Delta I_{*s} = (I_{s1} - I_{s2}) / I_{s1}$$

which after the substantiation  $I_{s1} = I_b / x_{net}$ ;  $I_{s2} = I_b / (x_{net} + x_{ch})$  yields

$$\Delta S_{*s} \equiv \Delta I_{*s} = (x_{ch} / x_{net}) / (1 + x_{ch} / x_{net}).$$

The ratio  $x_{ch} / x_{net}$  has limited range of variation and depends on parameters of reactors which reactivity is within 3...16 %, as well as on parameters of network node link with feeding source (in per cent)  $x_{net} = 100S_{net} / S_s$ , where  $S_{net}$  - is current-carrying capacity of the network elements of district electric power system. Current-limiting performance of a reactor falls when power of consuming load points increases, and becomes higher when they are subdivided by quantity and power of transformers (Fig. 8.8).

The reactor is chosen by its rated voltage and current, and inductive reactance and checked for dynamic and thermal stability to the short circuit current. If necessary, it is checked for the level of residual voltage in previous network node.

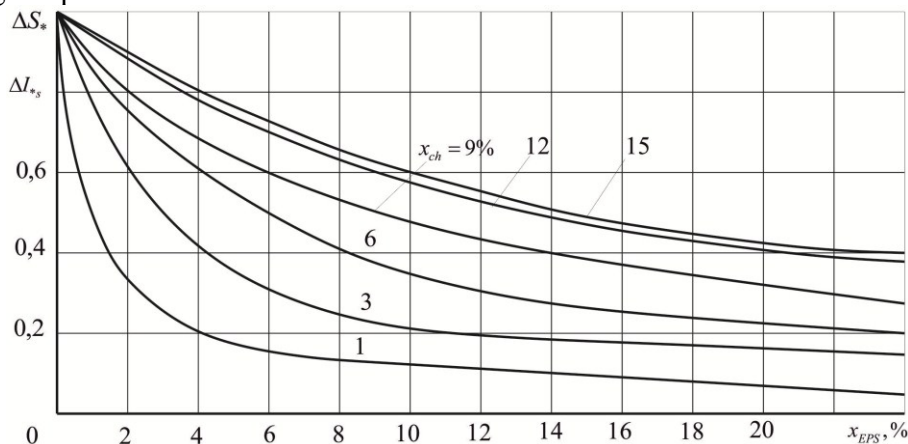


Fig. 8.8. The dependences of single-circuit reactor current-limiting effect on its reactance and electric system impedance

The first step at choosing the choke is determination of its inductive reactance. Proceeding from the short circuit power level behind the reactor it is necessary that  $S_{s2} = S_{s, req}$ , the required inductive reactance in per unit or in concrete units are accordingly calculated by formulae:

$$x_{*ch} = (S_b / S_{s2} - x_{*(b)net}) I U_b / (U_{net} I_b); \quad (8.11)$$

$$x_{ch} = x_{*ch} U_{net} / (\sqrt{3} I), \quad (8.12)$$

where  $I, U_{net}$  - are operating current and voltage of the network which correspond to continuous operation of the reactor.

The choke with the closest standard value of  $x_{ch, rated}$  is chosen on current, network voltage and inductive reactance, and is checked (if required) on the value of residual voltage

$$U_{res} \geq 0,6 U_{net}. \quad (8.13)$$

Level of residual voltage depends on the voltage ratio (Fig. 8.7.b)

$$U_{*res} = U_{res} / U_{net} = \sqrt{3} I_{s2} x_{ch,rated} / U_{net} = 1 / (x_{net} / x_{ch,rated} + 1) \quad (8.14)$$

or (in per cent)

$$U_{res} = x_{ch,rated} I_{s2} / I_{ch,rated}, \quad (8.15)$$

where  $x_{ch,rated}$  (%),  $I_{ch,rated}$  – are parameters of the chosen reactor;  $I_{s2}$  – is the short circuit current value meeting the standard inductive reactance of the reactor and its rated current.

If condition (8.13) is not satisfied, the new estimated value (in per cent) should be determined basing on the required level of residual voltage  $U_{*res,req}$  by formula

$$x_{ch} = 100 U_{*res,req} x_{*(b)net} I U_b / (U_{net} (1 - U_{*res,req}) I_b). \quad (8.16)$$

Basing on the value of  $x_{ch}$ , the reactor with the nearest standard reactance is chosen, and the short circuit current behind the choke is recalculated and the correspondence of its parameters to requirements of electrodynamic and thermal stability is checked.

Sectional reactors limit short circuit current on collecting buses and branching. To compare with line chokes they have smaller current-limiting ability, as they are calculated for higher rated currents taking place between sections if their separate performance is interrupted.

Sectional reactors are chosen on the rated voltage, and the greatest of operative currents of sections, and on inductive reactance. First, impedance of choke is taken, and then it is modified by means of check calculations till the permissible short circuit current value, corresponding the installed equipment parameters is obtained. The sectional chokes are not checked on values of electrodynamic and thermal stability.

Double chokes are characterized by branches inductance  $L_1 = L_2 = L_B$  and the mutual coupling factor of split winding parts

$$k_{cpl} = M / \sqrt{L_1 L_2} = M / L_B = \omega M / x_{rated}, \quad (8.17)$$

where  $M$  – is mutual inductance of choke winding parts.

An equivalent circuit of double choke looks like Y-circuit (Fig. 8.9, a) with  $x_1, x_2, x_3$  beam reactance. But depending on the reactor connection it performs in different current-limiting modes, and has unequal resulting impedances:

- under single-circuit operation (Fig. 8.9, b)

$$x_{ch} = x_{rated} (1 + k_{cpl}) - x_{rated} k_{cpl} = x_{rated}; \quad (8.18)$$

- under longitudinal operation (Fig. 8.9, c)

$$x_{ch} = x_{rated} (1 + k_{cpl}) + x_{rated} (1 + k_{cpl}) = 2x_{rated} (1 + k_{cpl}); \quad (8.19)$$

- under throughput operation (Fig. 8.9, d) with equal branch currents

$$x_{ch} = x_{rated} - x_{rated} k_{cpl} = x_{rated} (1 - k_{cpl}). \quad (8.20)$$

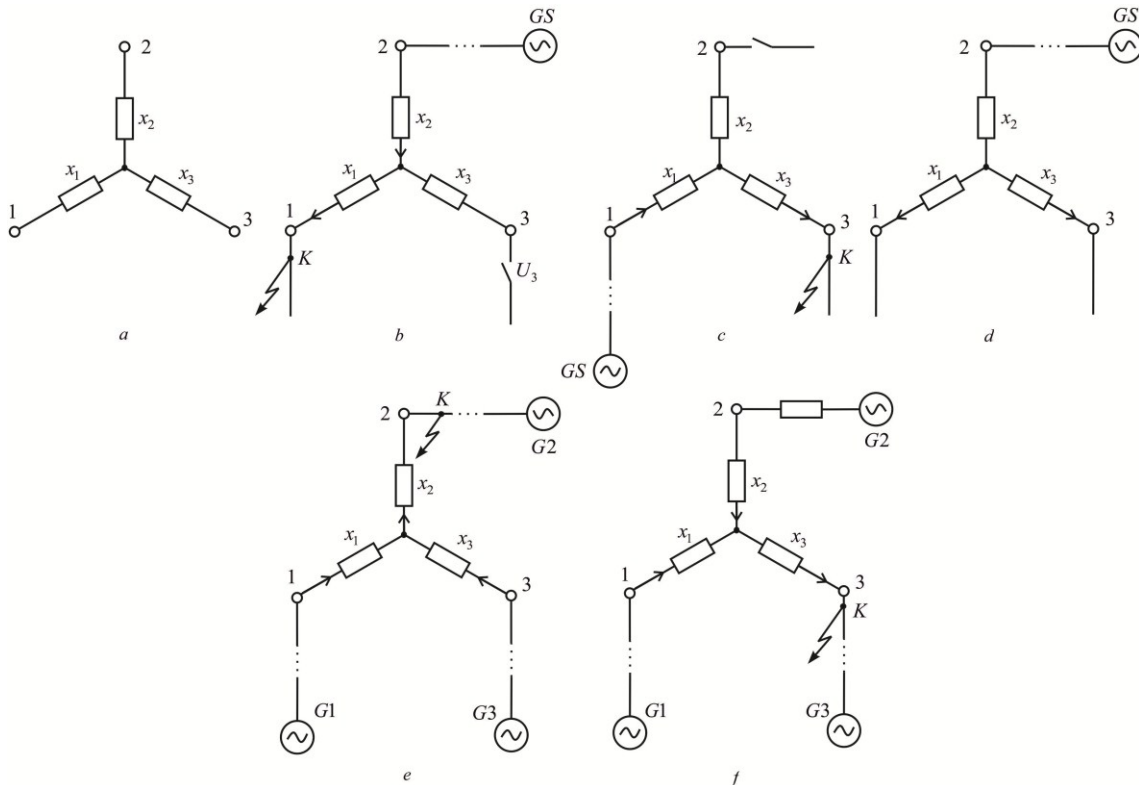


Fig. 8.9. Equivalent circuits for double choke at different connections: a – general base circuit; b – for single-circuit mode; c – for lateral mode, d – for throughput mode; e, f – for combined modes

Under combined operation the feeding sources are on the side of each choke branch. If short circuit takes place at the side of branch 2 the resulting choke reactance equals

$$x_{ch} = 0,5x_{rated} (1 - k_{cpl}) \quad (8.21)$$

If the short circuit is at the side of branch 1 or 3 (Fig. 8.9, f) then in the case of  $x_{2res} > k_{cpl}x_{rated}$

$$x_{ch} = x_{rated} (1 + k_{cpl}) (x_{2res} - k_{cpl}x_{rated}) / [x_{rated} (1 + k_{cpl}) + x_{2res} - k_{cpl}x_{rated} + x_{rated} (1 + k_{cpl})], \quad (8.22)$$

or when  $x_{2res} < k_{cpl}x_{rated}$

$$x_{ch} = x_{rated} (1 + k_{cpl}) \quad (8.23)$$

The choice of double choke is similar to the choice of a single-choke at  $x_{ch, rated} \equiv x_{rated}$ . Each the double-choke branch current must not be less than 0.675 of the transformer winding rated current or of total current of load. Besides, the uniformity of load distribution between branches of choke is supposed.

Possibility of voltage increase at the branch with smaller load in the single-circuit and throughput modes is disadvantage of a double chokes (Fig. 8.9, b, d). The voltage at such a branch has the EMF component caused by magnetic coupling of the choke winding parts which is induced by loaded branch short circuit current. Under the short circuit, the voltage appearing at another switched off branch equals

$$U_Z = \sqrt{3}x_{rated} (1 + k_{cpl}) I_{s1} \quad (8.24)$$

This voltage depends on the choke branch reactance, its rated current and the mutual coupling factor which values are within 0.4...0.63.

Availability of great impedance of current-limiting chokes results in extra loss of voltage, power and electricity supplied. Nonlinear characteristics of the choke impedance under which the impedance is the least at rated conditions and maximum at fault.

In regulated chokes the impedance is modified by magnetic flux biasing on control windings. Choke resulting impedance can also be controlled with the help of thyristor switchers at the parts of its winding.

In a saturable choke, the impedance increases as the result of its magnetic circuit saturation when short circuit current flowing through the choke increases.

*In current-limiting devices* of transformer type series connection of resistance into transformer primary winding circuit is used. Their resulting impedance is modified by means of transformer secondary winding operating condition control with the help of nonlinear resistors of thyristor switches.

Performance of current-limiting devices of resonance type is based on use of voltage resonance. Their impedance increase under short circuit is the result of the resonance conditions disturbance caused by variation of the frequency in the course of transient. The voltages resonance detuning is provided with the help of threshold elements, saturable chokes, and thyristor switchers and surge current limiters.

**Current-limiting switching gears** carry out the functions of limiting the short circuit currents maximum value and protection of electric equipment from the overcurrent by means of their fast disconnection. These include quick-break current-limiting fuses, surge current limiters, and special automatic circuit breakers for the voltage up to 1 kV.

Current-limiting fuses provide protection of electric equipment if

$$i_{\text{set,thr}} < i_{\text{set,exp}}, \quad (8.25)$$

where  $i_{\text{set,exp}}$  is expected short circuit current which would occur in the network if current-limiting fuses are not available (Fig. 8.4).

Current-limiting fuses are used in the networks with voltage up to 35 kV. They are characterized by the following features: the rated voltage, the fuse-holder current and the fuse-link current that should not exceed the rated current of the fuse-holder, as well as the largest and the least breaking currents, dependences of the fuse-link melting time  $t_{\text{fuse}}$ , the break time  $t_{\text{br}}$ , the current of limitation  $i_{\text{set,thr}}$  of expected short circuit current periodic component  $I_{\text{F}}$ .

Fuse current limiting effect is determined by the fuse-link rated current  $I_{\text{fuse, rated}}$  and also by values of periodic component and short circuit surge current taking place at absence the fuse (Fig. 8.10). If degree of current limitation is assessed with the factor of limitation

$$k_{\text{lim}} = i_{\text{set,thr}} / i_{\text{set,exp}}, \quad (8.26)$$

the latter is reduced when the fuse-link rated current increases, and takes the least value at the rated fuse-holder current.

As means of current limitation, fuses are comparatively cheap and simple. But they have disadvantages: the fuse-links are non-reusable, limited choice by fuse-link and holder scale, instability of current-time characteristic, poor comparability of their action with devices of protection and system automation, low reliability. Therefore, the range of their application is restricted to power supply circuits of unimportant consumers.

**Surge current limiters** like fuse-links are non-reusable. Their principle of operation is breaking the current due to destruction of current-carrying piece by pyrocartridge explosion. Operating signal for limiter actuation comes from external protection devices that monitor the short circuit current and its first-order derivative values. Short circuit current limitation is provided for about 0.5 ms at full network switching off for time up to 5 ms.

Surge current limiters are used in the networks of large current and voltage of 0.66-35 kV. In power supply systems of industrial enterprises they can be used with the aim of:

- choke shunting in normal operating mode in order to reduce voltage drop and power loss (Fig. 8.11, a-c);
- provision of parallel operation in switching circuits with electric equipment of insufficient stability on short circuit operation parameters (Fig. 8.11, d);
- power circuit construction of especially responsible load equipment (where interruption of power supply is not allowed) (Fig. 8.11, e);
- automated division of doubly-fed network (Fig. 8.11, f);
- power transformer neutral to ground disconnection at great values of current to ground.

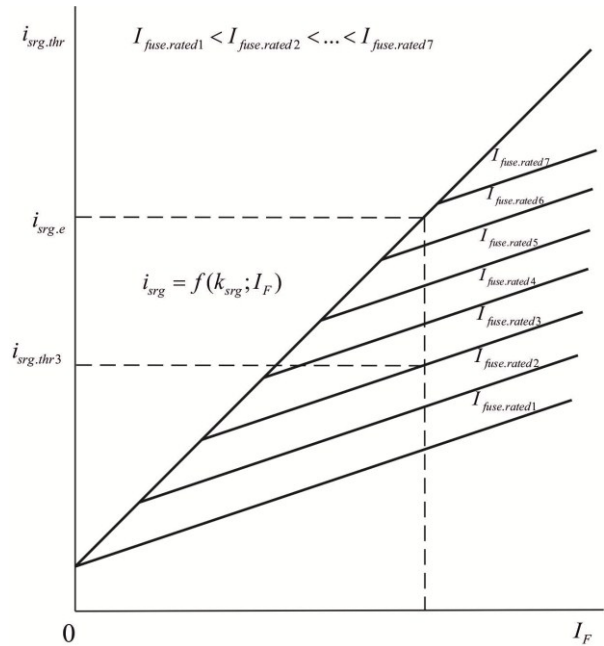


Fig. 8.10. Characteristics of fuse current limiting

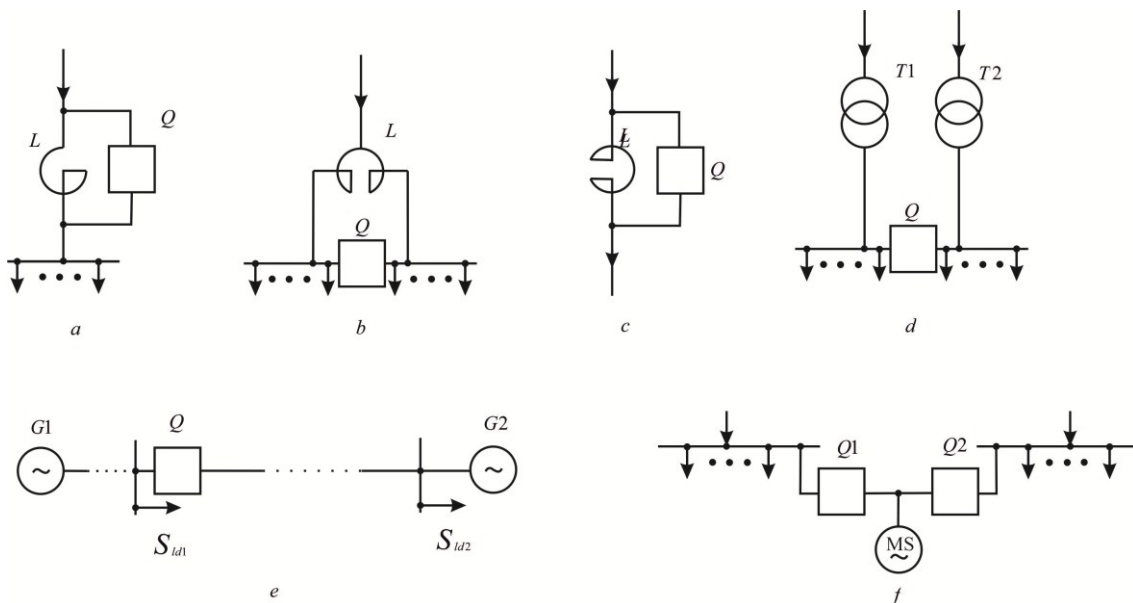


Fig. 8.11. Possible schemes of surge current limiters connection

Disadvantages of surge current limiters are high cost and complicated control.

**Automatic circuit-breakers**, used in networks of voltage up to 1kV, switch off the short circuit current for 0.2 – 0.6 s. This time is sufficient to provide protection of electric equipment

from short circuit current heating effect. On this reason electric networks protected by the automatic circuit breakers are not checked for thermal stability. There are special constructions of automatic circuit-breakers, such as a current limiting circuit-breaker and a circuit-breaker with a current limiter, providing decrease of amplitude of short circuit during the time of fault disconnection. The overcurrent reduction is provided by fast introduction of high impedance into the electric circuit. For that, the electric arc resistance that occurs between breaking contacts of the circuit-breaker or in special devices called the current limiters is used. Fast increase of the arc resistance is carried out by means of the breaker contacts opening under influence of electro-dynamic forces caused by the short circuit current or due to high operation speed of electromagnetic components. In current limiters, the short circuit current level is reduced by means of electric arc resistance to the value at which the circuit breaker operating simultaneously with the limiter can open the circuit.

Circuit breakers capacity to decrease the short circuit currents level is assessed with the help of the time-current and cut-off current characteristics.

The time-current characteristic is a curve giving the operating time, as a function of the prospective short circuit current. For current limiting circuit-breakers this characteristic (Fig. 8.12, a) has a section 1 that corresponds to the action of thermal overload release, the section 2 corresponding the action of magnetic overload release. For a circuit-breaker with a current limiter the characteristic has also additional section 3 corresponding to the action of the current limiter (Fig. 8.12, b). The boundary between sections of the thermal and magnetic releases action depends of selection of magnetic release setting  $I_1$ . The time-current characteristic of the current limiter is adjusted so that a breaker switches the circuit off independently when the current is less than  $I_2$ . When the currents exceeds  $I_2$ , first the limiter must operate, then the circuit-breaker switches off the current reduced by the limiter action.

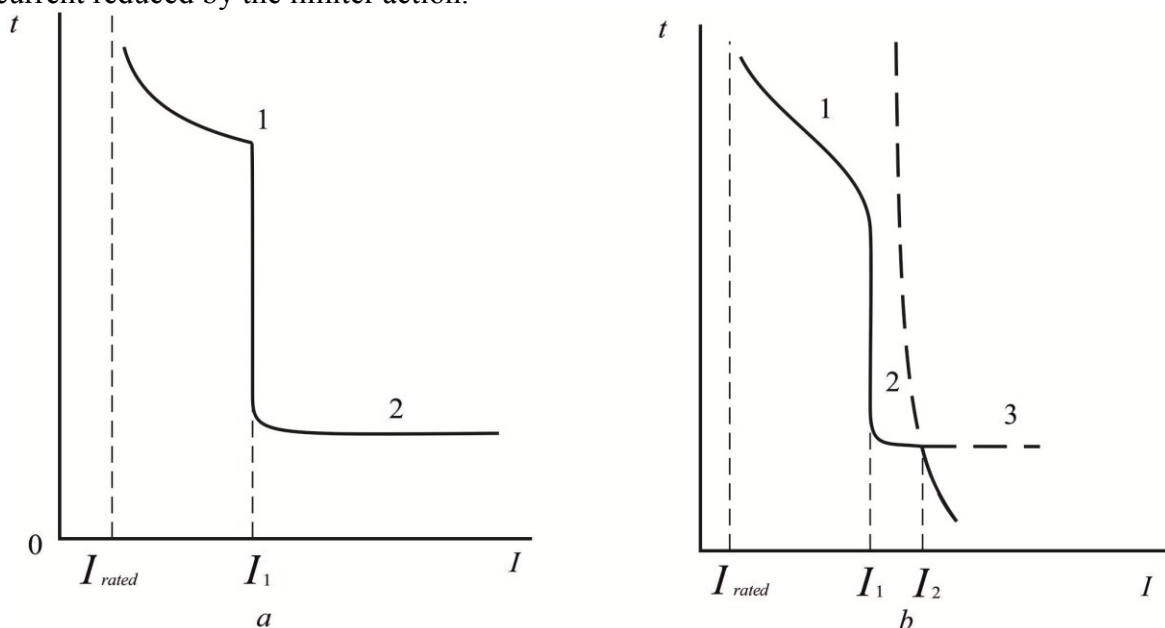


Fig. 8.12. Time-current characteristics of circuit-breakers: a – current-limiting circuit-breaker; circuit breaker with current limiter

The cut-off current characteristic gives a value of the cut-off surge current as a function of the prospective value of the short circuit current periodical component. On this dependence the voltage, power factor, fault circuit impedance components ratio for short circuit current effect (Fig. 8.13). It should be noted that with decreasing switching voltage a circuit-breaker current-limiting ability increases.

Current-limiting circuit-breakers have the following advantages in comparison with common automatic circuit-breakers:

- less dimensions and weight;

- lower cost for the same rated braking current;
- reduction of electrodynamic action of short circuit current by 10...30% and its thermal action by 5...10 %;
- better reliability parameters (wearing capacity, failure-free operation, safety).

Disadvantages of current-limiting circuit-breakers are difficulty in selectivity providing and impossibility of reutilization.

Devices, connected into circuit of power elements neutrals grounding arrangement fulfill different purposes. The neutral condition impacts on solution of many tasks of power supply such as improvement of a protection system performance, choice of conductors insulation thermal class, reduction of lightning and switching surge level, etc. Here is also the problem of limiting the neutral currents at the most frequently taking place faults to ground. According to results of study, the single-phase short circuit current can exceed the current at three-phase short circuit by 25%; therefore the latter should be calculated providing restriction of the single-phase short circuit influence.

In electric power supply systems, the network neutral condition is defined depending on the voltage level, values of capacitive fault currents to ground, safety requirements and working environment of enterprise. Common industrial networks with voltage less than 1 kV have solidly grounded neutral.

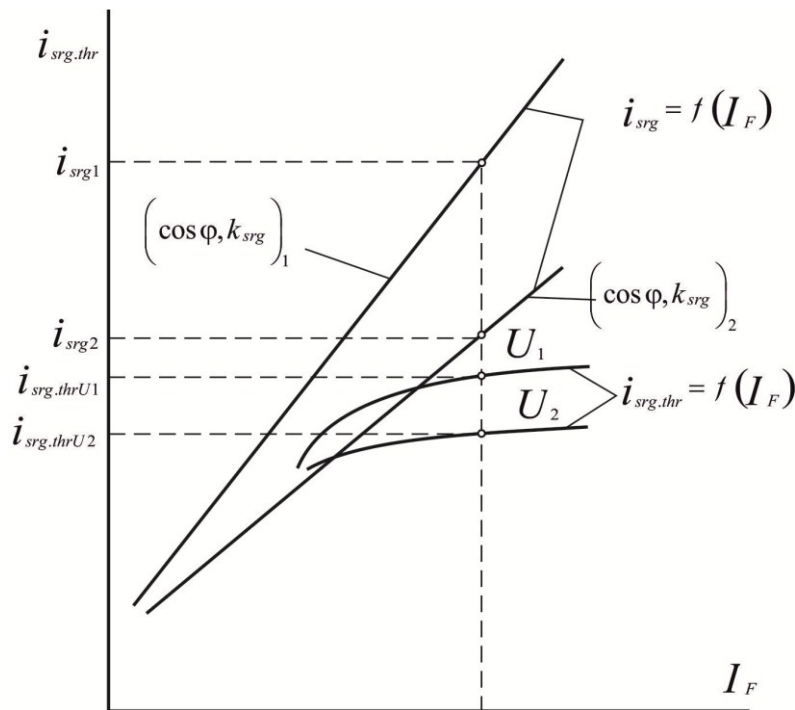


Fig. 8.13. Cut-off current characteristics of circuit-breaker under  $U_1 > U_2$ ,  $k_{thr1} > k_{thr2}$ ,  $\cos \varphi_1 < \cos \varphi_2$

In the cases when the determining factor is safety and prevention of explosion of surrounding explosive gas atmosphere, the networks up to 1 kV are made isolated. Networks of 6-35 kV have or isolated either arc-suppression-coil-grounded neutral. Effective grounded neutral is used in networks of 110 kV and more with great short circuit currents, when the factor

$$K_3 = U_{ph,g} / U_{ph, rated} \leq 1,4 \tag{8.27}$$

where  $U_{ph,g}$  - is the potential difference between the unfaulted phase and ground at the point of short circuit of other or two other phases;  $U_{ph, rated}$  is the potential difference of phase and ground before the short circuit.

Inequality (8. 27) is true when network parameters are related as

$$z_{0res} / z_{1res} \leq 3...4 \text{ or } x_{0res} / x_{1res} \leq 5, \quad (8.28)$$

where  $z_{1res}$ ,  $x_{1res}$  - are resulting impedance and reactance of positive sequence;  $z_{0res}$ ,  $x_{0res}$  - are resulting impedance and reactance of zero sequence.

To limit currents of short circuit to ground, power transformers neutrals are disconnected from the ground (Fig. 8.14, a); reactors with linear characteristic are connected to the neutral circuits of the network elements (Fig. 8.14, b); saturable reactors (Fig. 8.14, c), arc-suppression-coils or resistors (Fig. 8.14, d), non-linear resistors (Fig. 8.14, e) are used; the network is divided for galvanically isolated parts by means of transformers or by replacement autotransformers for transformers that widens capabilities of the neutral condition variation for the network parts (Fig. 8.14, f).

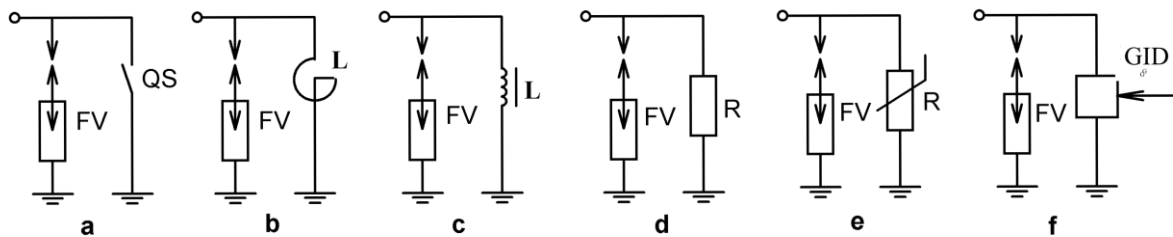


Fig. 8.14. Variations of neutral grounding system

The devices can be constantly connected into neutral grounding system, or be connected into the grounding circuit by means of switching devices, or vary it resulting impedance to ground at occurrence of a fault due to threshold element operation (saturable reactors, inductive-capacitive series circuits tuned on resonance).

Neutrals are grounded through the arc-suppression-coils in order of capacitive fault current to ground component compensation. The arc-suppression-coils have greater reactance than a single circuit net reactor, and its volt-ampere characteristic at voltage exceeding the rated value is non-linear. In the circuit that constituted by the arc-suppression-coil with non-linear inductive susceptance and equivalent capacitive network susceptance to ground connected in parallel, the resonance conditions can occur. Such a neutral condition should be taken into account if the fault capacitive current to the ground exceeds the following values:

- 10 A – in the networks of overhead lines of 6-20 kV having reinforced concrete or metallic supports and in all networks of 35 kV;
- 30 A – in the networks of 3 – 6 kV which have not the indicated supports;
- 20 A – in the networks of 10 kV, and 15 A – in the networks of 15-20 kV;
- 5A- in the connection circuit “generator-transformer” (on the generator side voltage).

At the ground fault current greater 50 A, installing not less than two arc-suppression-coils is recommended.

The highest short circuit currents to ground occur in networks with dead grounded neutral. When the network elements neutral is grounded through linear or non-linear resistor, the single-phase short circuit current periodic component is decreased by 20 - 30%. At this, the decay time of its periodic component is considerably reduced. At the same time, the protection equipment operating conditions are facilitated, and conditions to avoid intermittent short circuit to ground through arc appear though the neutral electric potential and potentials of unimpaired phases grow under short circuit.

The neutral grounding through resistor is equivalent to it dead grounding in the cases when overvoltage is caused by lightning or switching. Neutral grounding through reactors or through resonant current-limiting devices is intended to introduce inductive reactance into the neutral circuit at short circuit occurrence. The inductive reactance restricts the short circuit current more efficiently



than the resistance of the same value introduced into the neutral circuit, and also reduces the neutral voltage. But it is less efficient at lightning overvoltage.

In networks of 110 kV resistors are more efficient than reactors for short circuit current limitation. In this case, degree of the current reduction is restricted by allowable increase of the voltage on unimpaired phases (till  $1,4U_{ph, rated}$ ). In networks of 220 kV both resistors and reactors can be efficient depending on the network parameters. Grounding transformer neutral through resistor or reactor allows to reduce steady-state short circuit current to 50...80 % of short circuit current at dead grounded neutral without exceeding allowable overvoltage levels on the neutral and unimpaired phases.

In networks with isolated neutral or the neutral grounded through arc-suppression-coils, the short circuit current to ground is the smallest and is caused by the network conductance to ground and by degree of the capacitive current compensation with the help of a reactor. Therefore, the simplest way to limit currents of single-phase network fault to ground is disconnection the neutral and its power elements from the ground.

Short circuit current reduction by means of described methods is restricted by allowable increase of the neutral and unimpaired phases voltage in the course of fault, and by insulation thermal class. As transformers of voltage of 110 kV and more have low neutral insulation class, degree of the current restriction is reduced at increase of the network rated voltage. Disconnection the neutral of transformers of 330 kV and more from the ground is not allowed.

Choice of circuit and devices for grounding power elements of the neutral system for reduction of open-phase short circuit current to ground depends on the neutral condition throughout the network, its parameters and restrictions by the overvoltage levels that are defined by the range of rated voltage. At this, the following factors are also important: serviceability, reliability and uninterruptable power supply, finding cost effective border between decrease of the fault current to ground and allowable increase the level of different types of overvoltage.

#### 8.4. Short circuit current level optimization

An electric supply system is a small part of a power system for which determination of short circuit power level is carried out. If electric power supply system does not comprise own electric power sources, the highest short circuit power value is on the boundary with feeding power system. When there are own sources, maximum value of short circuit power is defined by the source power and the power of short circuit that comes from the power supply system as well as by electrical remoteness of power sources and the point of short circuit.

Level of short circuit power in load nodes depends on the structure of electric power supply system, its network parameters and load equipment structure. Defining optimal short circuit power level of every node requires fulfillment analysis of more performance indicators including technical and cost indicators of the equipment, conductors and current limiting devices, electricity consumers classes of service and supply reliability, the loading stability, the relay protection system serviceability, quality of the voltage delivered to consumers starting and self-starting of powerful motors, power and energy losses in the network and damages caused by power supply interruptions.

Effect of factors defining optimal short circuit power level is contradictory. From one side, the reduction of maximum short circuit power values in load nodes allows to install more simple and cheap equipment, to reduce conductors cross-section, to apply simple solutions of power distribution system using devices of system automatics providing automatic excitation control and automatic reclosure and to reduce losses caused by the fault currents and their localization. At this, expenses for additionally installed special equipment and current limiting devices are increased, different types of overvoltage and damages due to power supply interruptions are increased.

From the other side, providing of voltage level for motor starting and self-starting, reduction of voltage oscillations and deviations in load nodes with sharply impact load, voltage harmonics reduction, decrease of load unbalance effect, providing protection system reliability require keeping high values of short circuit power.

At the self-starting of powerful motors, the residual voltage in load nodes depends on the short circuit power and the network reactance (Fig. 8.15, a). Level of voltage oscillation is reduced with the node short circuit power increase in the case of sharply variable surge load (8.6). To fulfill requirements of (8.4), allowable power of the switched on converters depends of short circuit level at the load node (Fig. 8.15, b, c).

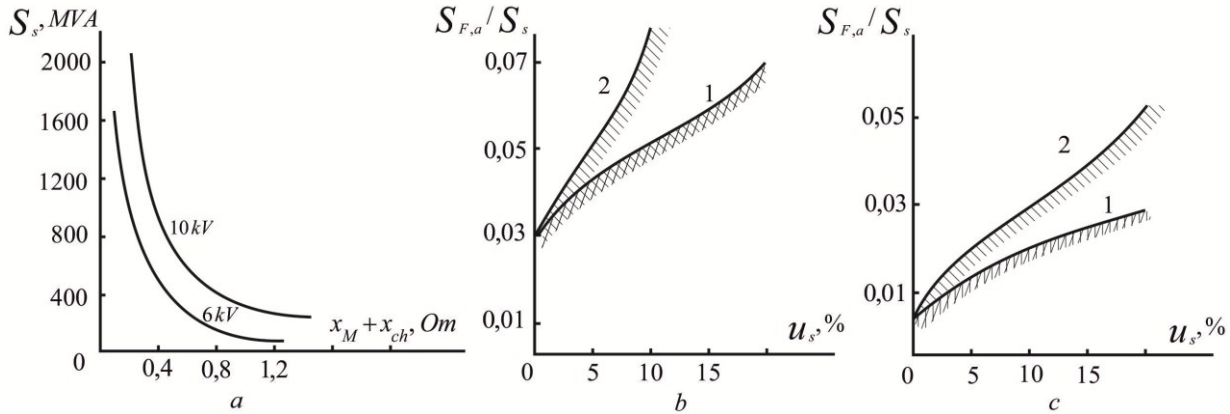


Fig. 8.15. Effect of short circuit power on electric power quality: a - residual voltage is preserved at starting motor, having reactance  $x_M$ , by means of reactor with reactance  $x_{ch}$ ; b - controllable converter with coefficient of transformer winding splitting equal 4 (curve1) and 0 (curve2) is connected; c - uncontrolled converter is used

At designing of electric power supply systems, the technical and economic task of reducing the short-circuit power in some nodes to optimal level can arise. In this task, the criterion function is

$$F(S_s) = \sum_{i=1} (K_i p_i + K_{\Delta i} p_{\Delta i}) + C_{\Sigma} + Y_{\Sigma}, \quad (8.29)$$

where  $K_i$  - is basic capital investment on electrical equipment of the power supply system equipment (transformers, distribution centers, electric apparatuses, power transmission lines);  $K_{\Delta i}$  - is additional investment on electric equipment (surcharges for special design of transformers and current-limiting switching devices), cost of current-limiting and arc-suppression reactors, current-limiting devices of different types as well as devices providing standardized values of electric power quality performances, starters and automatic excitation control devices of powerful motors;  $p_i, p_{\Delta i}$  - are total deduction ratios of main and additional capital investment on equipment construction;  $C_{\Sigma}$  - are total operational costs for main and special electric equipment and cost of electric energy loss;  $Y_{\Sigma}$  - is total expenses arising at reduction short circuit power level due to power supply interruption, motors in load nodes stability violation, decrease in power supply reliability and deterioration in the quality of electricity supplied to consumers.

The criterion for determining short circuit power optimum level is providing minimum of adjusted expenditures (8.29). To minimize criterion function of adjusted expenditures, it is advisable using the method of discrete optimization. This method allows passing from optimization the criterion function of adjusted expenditures of many discretely changing variables (8.29) to study of the function  $F(S_s)$  on extremum taking into account limitations by different parameters (discrete equipment power scale, scale of rated voltage, normalized indices of electric power quality, allowable power and energy losses, overvoltage level, maximum power of short circuit, etc.).

Availability of a large number of variables and limitations considerably narrows an opportunity of the global function minimum searching (8.29). Therefore, numerical mathematical models of short circuit power levels optimization which can be developed for particular nodes of electric power supply system with selection the group of the most essential variables that depend on short circuit operation parameters.

Optimum values of calculated short circuit current in networks of industrial enterprises should be determined considering two factors:

- providing of possibility to apply electrical apparatuses with better parameters and conductors of smaller cross section;
- keeping electric energy quality indices in the normalized limits.

For taking other factors into account, functional dependences between their quantitative indices and short circuit operation parameters should be found out.

### 8.5. Short circuit currents levels coordination

Coordination of short circuit current levels is matching of current values in different nodes of electric power supply system with electric equipment parameters or values of particular operation parameters. It is important mechanical and economical problem, affecting power and cost characteristics of electric power supply system.

At centralized electric power supply of industrial enterprises, cities and agricultural facilities, the problem arises when short circuit current has large values on the boundary with power system. The solution of the problem is consistent reduction of short circuit current levels caused by sources of electric power. Such coordination is carried out at an electric power supply system designing, its operation and at its further development or reconstruction.

At solving the problem of coordination in the course of designing, data on electric power sources and composition of consumers are initial data for the task solution. On the basis of a planned consumers' territorial location, the envisaged power sources having the given below characteristics are assumed predetermined:

- the installed power of generating sources of the industrial area with perspective of the system development for 8-10 years;
- the current or power at short circuit generated by the sources with perspective of development not less than for 5 years since the system setting to work;
- the operating voltage of the district power system;
- parameters of power transmission lines between the power system and electric power supply system.

Composition of the system consumers determines requirements to continuity of power supply, power quality, allowable duration of supply interruptions etc. Using these primary data, it is possible to establish the needed properties of the designed electric power supply system for its reliability, the voltage, power and energy losses, safety, flexibility, rigidity and stability, and also to identify options and determine parameters of equipment intended for use.

On the basis of quantitative values of the indicated parameters and characteristics of electric equipment to be installed in the nodes of power distribution, optimum short circuit current values can be defined. Taking them into account, it is necessary to substantiate the optimum structure of the power supply system, the number of points for connection to the power system, location of deep input substations, selection of the operating voltage, and number of power distribution stages.

On the stage of electric power supply structure operation the necessity to solve problems of short circuit current coordination arises at electric power supply circuit changes, capacity of the generating sources and capacity or transmission capacity or system elements increase, restrictions of operating conditions for main electric equipment, load density and loading the networks increase. Note that the obtained values of short circuit currents should be consistent with the parameters of the installed electric equipment and networks.

Coordination of short circuit currents is achieved by means of:

- stationary or automated network division;
- use of single-circuit or double-circuit current-limiting reactors or other current-limiting devices;
- application of switching devices of improved resistance to short circuit currents affect;

- disaggregation of transformer substations and their sections by means of use split-winding transformers or double-circuit reactors;
- change of network neutral operation by disconnection of some transformer neutral grounds, grounding neutrals through resistors, chokes or resonance current-limiting devices;
- electric separation of the network by installation isolation transformers.

Considering further widening and development of electric power supply system, the coordination of short circuit current levels has the same purpose that at the system operation. An additional rational measure is construction of new receiving centers providing connection to the power system and substations of deep input with lateral and longitudinal networks distribution for overcoming the electric load increase.

In all stages of short circuit current level coordination at different levels of power distribution, their analysis is used for substantiation of technical feasibility of creation new electric equipment and modernization the used equipment. Monitoring of short circuit currents in load nodes and their dynamic analysis is the most important factor of electric power supply reliability in the process of operation.

## **8.6 Transients in power-supply systems of enterprises in the context of problems of electromagnetic compatibility**

### **8.6.1. General information**

Any commutations taking place in electrical systems are followed by changes in storage of electric power concentrated in inductive and capacitive elements of electric equipment. The changes are accompanied by transients arising in the form of time variations of currents, voltages, and other parameters characterizing physical processes. With rare exceptions (electric arc, pulsed processes) relations between voltage and currents in transients known from the theory of electric circuits survive.

Until recent decades, electromagnetic processes in electric networks of industrial enterprises and power systems considered from the viewpoint of emergence of great violations of static and dynamic stability, of short-circuit currents occurrence etc. The problem concerning analysis and calculation of transients in power supply systems is a part of general problem of electromagnetic compatibility stated and structured by more general scientific discipline – the electromagnetic compatibility. Previous sections of the textbook consider electromagnetic transients within industrial power-supply systems at frequency of 50 Hz. At this, as a rule, interference of certain types of electric equipment does not take place. Electromagnetic field characterizes electromagnetic environment represented in the form of various electromagnetic interference. Therefore, certain types of electric equipment are either interference generators (interference emission sources) or objects of their impact that defines their electromagnetic compatibility.

Long-term effect of electromagnetic interference on electric equipment insulation may result in its faults and as a consequence in appearance of short circuit. Penetration of the electromagnetic interference into circuits of automatics, communication, and relay protection systems often results in malfunction of these systems operation, that is, in violation of electromagnetic compatibility. It follows by false operation of protection equipment, initiation of self-excited oscillations within the main grid of a power supply system, static instability, and other negative phenomena.

Therefore, investigation of electromagnetic transients within an enterprise power supply system should include not only calculations of fault currents and of parallel operation stability of power plants, connected to the electrical network, but also calculations and analysis of electromagnetic interference, that constitute a set of electromagnetic compatibility problems.

### **8.6.2. Electromagnetic interference within enterprise power supply systems**

Power supply system of enterprise is a source of a number of electromagnetic interference. Power lines, distributor gears, buslines, cables as well as automation equipment, control equipment, and protection equipment are among them.

In the first place, initiation of emergency (transient) electromagnetic processes depends on short circuits within power-supply systems and on switching in power circuits. These processes are sources of oscillatory and aperiodic disturbance incidentally arising and having, as a rule, wide frequency spectrum.

Usually steady-state electromagnetic processes accompanied by disturbance in the range of low, medium and high frequency (from several Hz to 100 GHz) are originated by all power installations.

Frequency spectra of pulse and periodical electromagnetic interference in electric installations are represented in Fig. 8.16.

Electrical installations are not just generators of electromagnetic interference but also the objects to which interference is affecting in emergency and in normal conditions.

The main reasons of such affect are:

- Processes of switching on the side of high voltage taking place at the planned switchover and in the case of faults (short circuit, power lines insulation overlapping, switching including operations with power isolating switches);

- Switching on the side of low voltage at switching on and off electric equipment containing inductive circuits, heavy-current devices that produce strong electric and magnetic fields of industrial frequency;

- Availability of powerful high-frequency communication devices, data transfer units, and also of voltage fluctuations with frequency of higher harmonics, breaks in power supply of control current circuits, etc.;

- Electrostatic discharges, direct lightning strokes to objects closely located to power transmission lines or arranged close to them.

Switching on the side of high voltage arising as a result of at the planned switchover and in the case of faults (short circuit, power lines insulation overlapping, switching, etc.) and on the side of low voltage cause transient electromagnetic disturbances, first of all affecting to equipment of automation, control and protection systems. At this damped oscillations with frequency of hundreds kilohertz and overvoltage many times greater than rated voltage may arise in high-voltage power networks. Electromagnetic interference produced in the course of inductive circuit commutation is most intensive and dangerous for technical means of low voltage discrete automation equipment. Under unfavorable conditions at inductive circuits commutation, considerable level of interference is possible such as overvoltage at the place of their occurrence in power networks of voltage up to 10 kV; at steepness of overvoltage curve up to 100 V/ns the pulse rise time in the lines of data transfer is in the range from 1 ns to 1 ms.

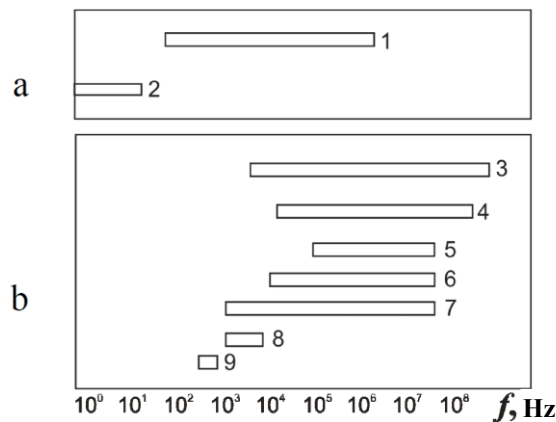


Fig. 8.16. Frequency spectra of pulse electromagnetic processes (a) and periodic processes (b) initiating disturbance in electrical systems and facilities:

1 – switching processes; 2 – load surges; 3 – radio and TV receiving sets; 4 – computer systems; 5 – supply-line switches; 6 – electrical installations; 7 – power consumers; 8 – centralized control systems; 9 – power supply lines

Under certain conditions, as a rule when normal levels are exceeded, electromagnetic interference can cause violation of interference immunity of technical means at power stations and substations, in particular of widely now implemented microprocessor protection devices. Studies of 100 substations performance have shown that in up to 15% of secondary equipment damage or malfunction are caused by noncompliance with conditions of electromagnetic compatibility [....].

Some types and characteristic of electromagnetic interference affecting the technical means of electric power stations and substations are given Table 8.1.

Table 8.1

Some types and characteristic of electromagnetic interference affecting the technical means of electric power stations and substations

Interference of prolonged nature	Interference of transient nature having high probability of appearance	Interference of transient nature having low probability of appearance
Slow voltage variation: - In AC power supply systems - In DC power supply systems	Supply voltage dips with duration not more than 0.02 s: - In AC power supply systems - In DC power supply systems	Supply voltage dips with duration more than 0.02 s: - In AC power supply systems - In DC power supply systems
Harmonics and interharmonics of supply voltage	Fluctuation of supply voltage	Breaks of power supply in AC supply systems
Voltage of industrial frequency	Damped oscillating magnetic field	Microsecond impulse disturbances of high energy
Conductive disturbances in the range of 0 – 150 kHz (excluding disturbances at frequency of 50 Hz)	Electrostatic discharges	Short appearance of voltage of industrial frequency

In normal symmetrical conditions of power supply systems of industrial enterprises there are background interferences which level is in acceptable standard limits. As a rule deviation of voltage varies in the limits of  $\pm 2\%$ ; in the same limits there is imbalance of three-phase voltage system by zero sequence. The background level of nonsinusoidality caused in the first place by asymmetry of transformer cores does not exceed as a rule 2 – 3%.

In practice, non-periodic voltage dips take place that are connected with switching of motors, transformers, capacitors, etc. As a rule these dips depth is not more than several per cent of the rated voltage, and their duration is from 100 ms till several seconds. Voltage dips arising due to damage of insulation under short circuit cause reduction of the voltage till 10% which last from 500 ms till several seconds. There are also periodical reductions of the voltage, caused by operation of controlled electronic power converters, and periodic and non-periodic overvoltage, having duration till several dozens of microseconds. One of reasons of the overvoltage is the lightning discharges.

### 8.6.3. Sources of artificial electromagnetic interference arising in enterprise electric power supply system

At the modern industrial enterprises more than half of electricity is used in modified form (at metallurgical enterprises – more than 90%). Powerful sources of electromagnetic interference are widely used semiconductor valve converters, different types of inverters and home appliances operating as in steady-state as in transient conditions.

Non-linear loads (arc steel-making furnaces and electric welding equipment, wind stations, power transformers and motors) also generate considerable disturbance.

Consider sources of artificial electromagnetic disturbance in enterprise power supply system.

Operation of electric equipment produces voltage deviations at every plant. The voltage fluctuation takes place at operation of electric installations of machine-building enterprises (with powerful welding facilities), steel plants with arc steel furnaces) and non-ferrous enterprises with electrolysis facilities.

Voltage asymmetry arises at operation of powerful one-phase consumers, tracking substations of electrified railway transport, and enterprises with powerful welding facilities. In these cases non-balanced distribution of one-phase consumers and non-simultaneous variation of phase loads are observed. Voltage nonsinusoidality is caused by operation of arc steel furnaces, electrolysis facilities, and traction substations of railway transport.

Valve converters are powerful concentrated sources of harmonic interference. In most frequently used 6-pulse bridge circuits prevail the 5<sup>th</sup>, 7<sup>th</sup>, 11<sup>th</sup>, and 13<sup>th</sup> harmonics called the characteristic harmonics. Their level (relative to 1<sup>st</sup> harmonic) is inversely proportional to number; that is 1/5 1/7, 1/11, 1/13. When 12-pulse circuits are used, 5<sup>th</sup> and 7<sup>th</sup> harmonics are theoretically absent, and 11<sup>th</sup> and 13<sup>th</sup> harmonics prevail. Such converters are applied in circuits of main drives of roll mills, in electrolysis installations, etc.

Voltage dips and fluctuations take place at suddenly applied load such as in rolling mills.

Considerable voltage fluctuations appear in the course of rolling mills operation when fast-operating compensating devices are not available. So in the network of 10 kV supplying a slabbing-mill the flicker dose a unit of voltage fluctuation) equals 10.5, and on busses of 10 kV it is 2.2. In the network of a blooming-mill the flicker dose exceeds 4.

The reactive power surge at capturing the metal with rolls of cold-rolling mill reaches 2,000 Mvar that causes voltage dips with depth up to 10 - 12% depending on the rated voltage and the short-circuit power.

A valve converter operating in a network with imbalanced system of line-to-line voltage is a source of negative-sequence current:

$$I_2 = 0.5K_{2U}I_1$$

Where  $K_{2U}$  is line-to-line voltage unbalance factor by negative sequence;  $I_1$  is fundamental of the network line current of valve converter under symmetrical conditions.

Phase of current  $I_2$  is

$$\arg I_2 = \varphi_2 + \alpha$$

where  $\varphi_2$  is phase displacement between current vectors  $I_2$  and  $I_1$ ;  $\alpha$  is the inverter delay angle.

Lately frequency converters are widely applied for motors frequency control as a part of control system of frequency-regulated drives in metallurgy, machine-building, and light industry. The frequency converters are sources of harmonic electromagnetic disturbance of not only higher harmonics but also so-called interharmonics which frequencies are between frequencies of characteristic harmonics. Consumers that continuously or during short time operate in transient are also sources of interharmonics. These include arc steel furnaces, welding machines, valve converters of rolling mills and other non-linear The sources of steel-making furnaces, welding facilities, valve inverters of rolling mill, and other sharply changing loads.

In Fig. 8.17 structure of frequency converter with direct current link consisting of rectifier Rect, inverter Inv (as a rule, it is the voltage inverter) and LC filter (the direct current link).

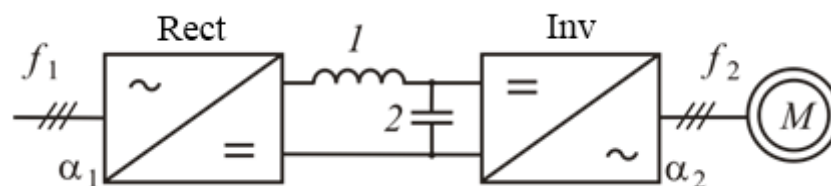


Fig. 8.17. Structure of frequency converter with LC direct current link:

$f_1, f_2$  are rectifier Rect input frequency and inverter Inv output frequency respectively;  $\alpha_1$  and  $\alpha_2$  are respectively delay angles of rectifier and inverter; 1 is reactor; 2 is capacitor; M is alternating-current motor

Rectifier and inverter can be either controllable or uncontrollable. Filer of powerful frequency converters has smoothing reactor reducing the alternating component of the current a pulsations. In low-power frequency converters the direct current link has a capacitor only.

In all cases, frequency converters are directly connected to the network (without use of special transformer).

Two interharmonics (of 4<sup>th</sup>, 6<sup>th</sup>, 8<sup>th</sup>, .. order) correspond to each of characteristic harmonics. Spectral contents of input (network) current is represented as

$$f_{in} = (kp_1 1)f_1 + p_2 f_2 v.$$

As an example the frequency spectrum around 7<sup>th</sup> characteristic harmonic is presented in Fig. 8.18.

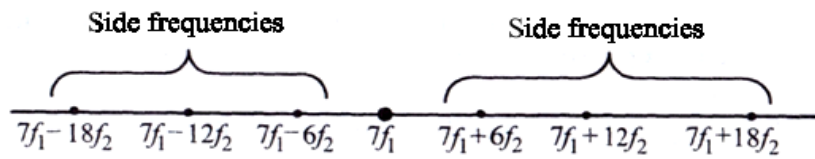


Fig. 8.18. Frequency spectrum of side frequencies around 7<sup>th</sup> harmonic

As a rule,  $f_1 \neq f_2$ . A number of side frequencies – interharmonics – appear around each characteristic harmonic (Fig. 10.17). The inequalities taking place are:  $(7f_1 - 18f_2) \geq 5f_1$  and  $(7f_1 - 18f_2) < 11f_1$ .

Fig. 8.19 represents frequency spectrum of harmonics and interharmonics for frequency converter (see Fig. 8.17) if  $f_1 = 50$  Hz, and  $f_2 = 30$  Hz. As it follows from Fig. 8.19 amplitude of interharmonics of the 2<sup>nd</sup> and 4<sup>th</sup> order is 22% of the fundamental rated current; level of the 5<sup>th</sup> harmonic is 40 %.

Direct frequency converters called the cycloconverters consist of two opposing rectifiers (Fig. 8.20). Depending on type of modulating function formed with the help of phase pulse control system, the levels of interharmonics in the network current exceed the levels of characteristic harmonics.

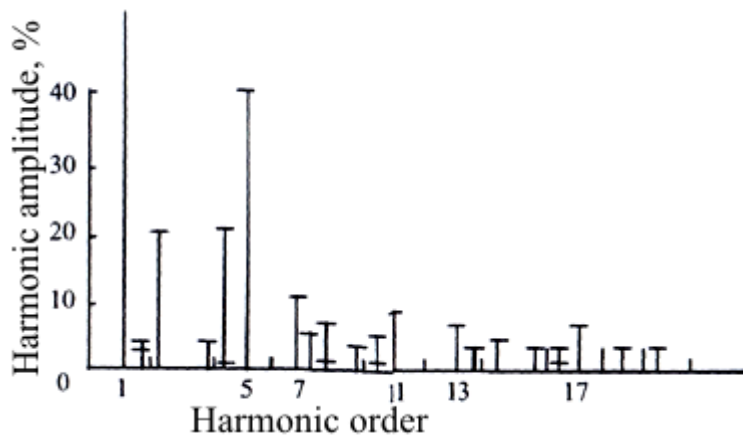


Fig. 8.19. Calculated current harmonics spectrum of frequency converter with direct-current link



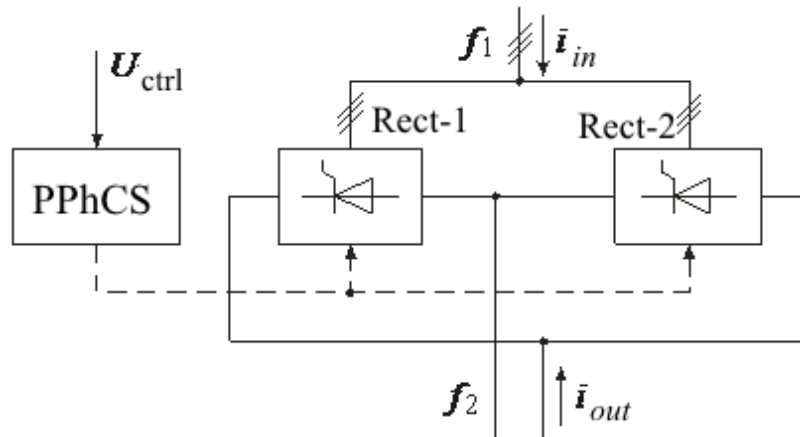


Fig. 8.20. Structure of direct frequency converter with single-phase output:

Rect-1 and Rect-2 are oppositely connected rectifiers;  $f_1$  and  $f_2$  are input and output current frequencies respectively

Levels of harmonic produced by converters (see Figures 8.17 and 8.20) at different control laws are given in Table 8.2.

**Arc steel furnaces** with arc current equal to 25...70 kA are sources of sizeable electromagnetic disturbance such as higher harmonics, voltage oscillations and unbalance.

In most degree the disturbance appear in the stage of metal melting. Averaged values of higher current harmonics generated by arc steel furnace are given below:

Harmonic number	1	2	3	4	5	6	7	8	9
Relative value of current harmonic, %	100	2.7	2.3	0.65	2.3	1.3	1.5	1.0	0.85

Table 8.2

Level of harmonics produced by cycloconverters and converters with direct current link

Parameter	6-pulse bridge cycloconverter					3-ph – 3-ph, 6-pulse frequency converter with direct current link, $f_2 = 30 \text{ Hz}$
	3-ph – 1-ph at $f_2 = 10 \text{ Hz}$ under control law				3-ph – 3-ph at $f_2 = 20 \text{ Hz}$ and sinusoidal control law	
	linear	sinusoidal	triangular	rectangular		
Rms of input current, %	147	208	183	173	128	113
Fundamental component of input current, %	100	100	100	100	100	100
Rms of total harmonic distortion, %	9	23	24	33	56	42
Rms of interharmonic distortion, %	107	181	152	138	56	30
Ratio of rms of input current interharmonic content to rms of input current harmonic content, %	12	8	6	4	1	1.3

Spectrum of a process of arc steel furnace current variation during metal melting is mixed. It consists of discrete and continuous components.

Power of interharmonics produced by steel furnaces of 100t and 200t productivity reaches 20% of total power of the mixed spectrum.

Voltage unbalance at the furnace busses of equals 5...6% for networks of 6...35kV and 3% for networks of 110 kV. For these cases, value of flicker (fluctuation) dose is usually within 1.5...10.

**Electric welding facilities** produce practically all basic electromagnetic interference characterizing the power quality: nonsinusoidality, unbalance, voltage dips and fluctuations.

Currents of  $n^{\text{th}}$  harmonic of contact welding machines are determined by the expression:

$$I_n = \frac{S_r k_{l\ st} \sqrt{DF_{st}}}{n^2 U_r}$$

where  $S_r$  is rating power of electric welding installation;  $k_{l\ st}$  and  $DF_{st}$  are average coefficients of load ratio and actual duty factor, respectively;  $n = 3, 5, 7$  is number of harmonic;  $U_r$  is grid rated voltage.

Limits of current harmonic values of a single-operator welding for  $n = 3$  are 12...30%, for  $n = 5 - 4...15\%$ , if  $n = 5$ , for  $n = 7 - 2...8\%$ .

Harmonics currents of direct-current welding sets and welding rectifiers commuting according to 6-pulse circuit are calculated by similar expression:

$$I_n = \frac{S_r k_{l\ st} \sqrt{DF_{st}}}{\sqrt{3} n^2 U_r}$$

where  $n = 5, 7, 11$ .

Unbalance factor for networks with electric welding sets is within 1...5 %.

Considered electric welding sets are also source of interharmonics. At spot welding, interharmonics appear in the range of frequencies of 35...75 Hz, and their amplitudes reach 20% of welding current fundamental harmonic. For all electric welding plants, power of discrete spectrum is 6...20 % of total energy of the mixed spectrum. Parameters of voltage dips are presented in Table 8.3.

Table 8.3  
Parameters of voltage dips for some types of electric welding installations

Installation type	Power, kVA	Voltage dip depth, %	Dip duration, s
Stationary spot installation	75	1.2	0.18
Butt-welding machine	750	13.0	0.70
Multiple-spot welding machine	63	16.3	0.36
Arc welder	60	1.0	27.00

Gas-discharge lamps (both luminescent and arc mercury ones) are the sources of harmonics of order  $n = 3, 5, 7$ . Generation of harmonics is caused by nonlinearity of volt-ampere characteristics of the electric arc inside the lamps as well as availability of a choke as part of ballasts. Relative value of 3<sup>rd</sup> and 5<sup>th</sup> harmonics is 16...21% and 0.9...3% respectively for luminescent lamps with inductance-capacitance ballast, and 18% and 5.8...7.2% for arc mercury lamps provided with compensation. At this, values of current harmonics are determined by the expression:

$$I_n = \frac{0.2 S_r}{n^2 U_r}$$

where  $S_r$  is rated power of arc mercury lamps.

Wind power installations are intensive sources of electromagnetic interference mainly of higher harmonics and fluctuations. Typical scheme of a wind power installation is shown in Fig. 8.21. The inverter is connected to the network of 10 kV either directly or through a step-up transformer. The long-term dose of flicker  $P_L$  and the distortion factor of voltage curve  $K_U$  were measured on the busses sections of 10 kV.

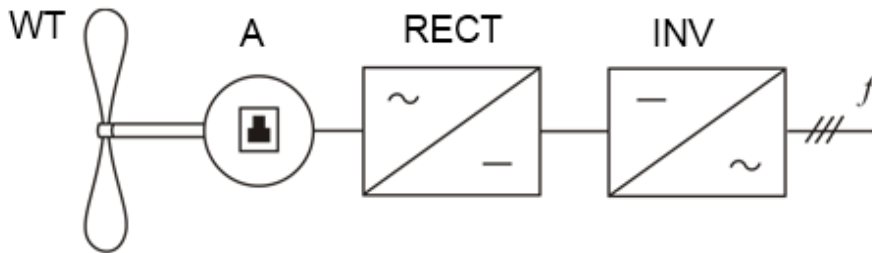


Fig.8.21. Scheme of a wind power installation:  
A is a synchronous generator; RECT - rectifier;  
INV - inverter; WT - wind turbine

Three wind generators with power of 150 kW each are connected directly to section 1 with voltage of 10 kV; another wind generator with 150 kW power is connected through a step-up transformer. To section 2 with voltage of 10 kV, 15 wind results generators with power of 150 kW each are connected directly.

Results of measurement of the long-term flicker dose  $P_L$  and distortion factor  $K_U$  KU at two substation sections are:

	Section I				SectionII		
Phase	A	B	C	Phase	A	B	C
$P_L$ in per unit	3.15	3.9	3.03	$P_L$ in per unit	8.55	6.81	8.00
$K_U, \%$	1.76	1.85	1.82	$K_U, \%$	2.95	3.13	3.28

As it is seen, levels of voltage fluctuation and flicker doses are not equal in different phases and can reach significant values (especially the flicker dose).

Quantitative assessment of interference level is given in Table 8.4 on a logarithmic scale in per units. Such an approach makes it possible to compare values differing by several orders.

The equipment presented above from the point of view of electromagnetic compatibility belongs to non-linear loads. In modern power supply systems, ever increasing application of loads based on semiconductor converting power equipment and electronics is observed. As a rule, rectifiers and/or inverters commutated on 6-pulse or 12-pulse bridge configuration are used in circuits of these devices.

Table 8.4  
Maximum level of electromagnetic interference at industrial enterprise power distribution network of 0.38...35kV

Reason of electromagnetic interference	Disturbance level, Np	Pulse duration, $\mu$ s
Lines switching off/on: with a switch	+1.7	5...20
with a switch under short circuit	-	5...20
with isolating switch	+2.5	500...1500
Lightning strike	+2.5	Till 1000
Corona discharge on feed lines of 110 kV and higher	-5.0	Constant

#### 8.6.4. Calculation of electromagnetic transients with account requirements of electromagnetic compatibility

At power stations and substations of industrial enterprises the current and voltage harmonics is prevailing type of electromagnetic disturbance caused, first of all, by wide application of power converter equipment. The continued availability of harmonics results in electromagnetic loss and extra heating of electric equipment. When level of higher harmonics exceeds background values, heating the electric equipment takes place. As a result, electric strength of insulation reduces, insulation wear becomes more intensive, and the insulation damage (breakdown) probability arises. It is obvious that on this reason both reliability and service life of the insulation and equipment decrease.

It is known that for electric cables the service life  $T_{slx}$  of class A insulation (its relative value) shortens twice at temperature increase by every  $6^\circ$ .

Degree of short circuit current impact is estimated using Joule integral  $B_x$  (thermal pulse)

$$B_{SC} = \int_0^{t_{off}} i_{SC} dt$$

where  $i_{SC}$  is short circuit current at time instant  $t$ , A;

$t_{off}$  is calculated duration of short circuit, s.

It is possible to perform estimation using thermally equivalent current of short circuit  $I_{te}$ :

$$I_{te} = \sqrt{\frac{B_{SC}}{t_{off}}}$$

While calculations it permissible to determine Joule integral by formula:

$$B_{SC} = B_{SCp} + B_{SCnp}$$

where  $B_{SCp}$  and  $B_{SCnp}$  are Joule integral of periodical and non-periodical components of short circuit current respectively.

In the case of arbitrary design circuit at remote short circuit and/or availability synchronous compensating devices, the Joule integral as well as thermally equivalent short circuit source are determined according to the approximate equations:

$$B_{SC} = I_p^2 (t_{off} + T_{npe})$$

$$I_{te} = I_p \sqrt{1 + \frac{T_{npe}}{t_{off}}}$$

Where  $I_p$  is rms of periodic component of equivalent power source (system) current, A;

$T_{npe}$  is equivalent time constant of aperiodic short current component damping, s.

Electric apparatuses and cables meet the requirements of thermal stability if

$$B_{SC} \leq B_{tea}, I_{te} \leq I_{tea}$$

For cables of 6 to 10 kV with paper insulation maximum allowable heating temperature at short circuit equals  $\theta_a = 200^\circ\text{C}$ .

Extra loss due to electromagnetic interference results in extra heating of electric equipment  $\Delta\tau$  and insulation service life decrease (of electric equipment as a whole) by  $\Delta T_{sl} = T_{sl} - T_{sl}^{int}$ , where  $T_{sl}$  and  $T_{sl}^{int}$  is service life of power supply system elements at presence and absence of electromagnetic interference respectively. Insulation service life is estimated by Montzinger's formula:

$$T_{sl} = Ae^{-0.086\tau}$$

where  $A$  is a coefficient depending on insulation type;

0.086 is Montzinger's parameter basing upon assumption that overheating by  $8^\circ$  results in double decrease in insulation service life;

$\tau$  is insulation heating temperature.

Relative decrease of electric equipment insulation service life is

$$\Delta T = \frac{T_{sl} - T_{sl}^{int}}{T_{sl}} = 1 - e^{-0.086\Delta\tau} \cong 0.086\Delta\tau + \frac{(0.086\Delta\tau)^2}{2}$$

Decrease of insulation service life leads to corresponding decrease in electric equipment functioning reliability, increase in probability of its damage and initiation of interphase or three-phase short circuit. Estimation reliability after the time of normal operation  $t_{no}$  is made as a rule with the help of Weibull's law. Reliability function  $R(t)$  by Weibull is

$$R(t) \cong 1 - \lambda t^\alpha$$

where  $\alpha$  is a parameter of distribution function;  $\lambda$  is a parameter of failure flow.

In practice tabulated values of these parameters are used in calculation of electric equipment of different types.

Values of reliability function  $R(t)$  for time  $t$  taking into account decrease of the insulation service life correspond to reliability level, probability of damage and short circuit initiation.

#### Example 8.1

Estimate thermal stability for the case of remote switch of BBTЭ 10 630 20Y3 type installed within cable line circuit at 10 kV substation. Estimated short circuit current duration is  $t_{off} = 0.5$  s. Periodical component of short circuit current  $I_p = 15$  kA. Equivalent time constant of aperiodic component damping is  $T_{np\epsilon} = 0.05$  s. Standardized value of thermal stability current is  $I_{te} = 20$  kA.

Time of thermal stability current flow is  $T_{t\text{ norm}} = 3$  s.

*Solution*

1. Determine value of Joule integral  $B_{SC}$ . According to the condition  $t_{off} > T_{np\epsilon}$ . Calculate  $B_{SC}$ :

$$B_{SC} = I_p^2 (t_{off} + T_{np\epsilon}) = 15000^2 (0.5 + 0.05) = 123.75 \cdot 10^6 \text{ A}^2 \cdot \text{s}$$

2. Allowable value of Joule integral  $B_{t\text{ allow}}$ , taking into account that  $t_{off} < T_{t\text{ norm}}$ , is:

$$B_{t\text{ allow}} = I_{te}^2 T_{t\text{ norm}} = 20000^2 \cdot 3 = 1200 \cdot 10^6 \text{ A}^2 \cdot \text{s}$$

Since it was turned out that  $B_{SC} < B_{t\text{ allow}}$ , thermal stability of the switch is provided.

3. In accordance with requirements of thermal stability, minimum cable with aluminum conductors cross-section area is

$$S_{t\text{ min}} = \frac{\sqrt{B_{SC}}}{c_\tau}$$

Coefficient  $c_\tau$  may be taken equal to  $90 \text{ s}^{1/2} \text{ mm}^2$ .

$$S_{t\text{ min}} = \frac{\sqrt{123.75 \cdot 10^6}}{90} = 123.6 \text{ mm}^2$$

It is necessary to use standard cable with the next larger cross-section area of  $150 \text{ mm}^2$  for which admissible continuous current is  $I_{adm} = 210$  A.

**Example 8.2**

Determine decrease of insulation service life of a cable of 10 kV loaded with 6-pulse rectifier of 3 MV·A. The rectifier is loaded by  $K = 95\%$ .

*Solution*

1. Calculate additional heating of cables  $\Delta\tau_v$  caused by using the formula:

$$\Delta\tau_v = \Theta_{vn} \sum_{v=2}^n K^2 I_{vx}^2 (0.187 + 0.532\sqrt{v})$$

where  $\Theta_{vn}$  is the cable temperature rise caused by losses at normal operation,  $\Theta_{vn} = 60^\circ\text{C}$ ;  $K$  is the cable load factor:  $K = 0.95$ ,  $K^2 = 0.9$ ;  $I_{vx} = \frac{1}{v}$  is relative value of a harmonic;  $v = 5, 7, 11, 13$  the harmonics numbers.

$$\Delta\tau_v = 2.15^\circ\text{C}$$

2. Relative decrease of the cable insulation service life

$$\Delta T = 0.086 \cdot 2.15 + \frac{(0.086 \cdot 2.15)^2}{2} = 0.2$$

At continues normal operation the cable service life is  $T_{sl} = 20$  years. Then absolute value of the cable service life decrease equals

$$\Delta T_{sl} = 20 \cdot 0.2 = 4 \text{ years}$$

**Example 8.3**

Estimate probability of the cable fault (or short circuit) emergence using data of previous example.

Value of  $\lambda(t)$  after 10-year service of a cable having class A insulation is 0.13 – 0.17. Assume  $\lambda(t) = 0.15$ .

*Solution*

The cable service life is  $T_{sl} = 20 - 4 = 16$  years.

Value of reliability function:

$$R(t) = 1 - 0.15 \cdot \sqrt{16} = 0.4$$

i. e. probability of the cable line fault (short circuit emergence) after 16-year operation is 40%.

Presented in examples 8.1 – 8.3 calculations of electromagnetic transients show a close relationship of problems considered while studying the processes of short-circuit (stability of parallel operation of power stations and networks) and electromagnetic compatibility. So, correct estimation of short circuit emergence probability and its thermal effect is possible only at taking into account the electromagnetic disturbance such as harmonics, unbalance, etc. Mutual influence of electric power sources should be also taken into account in calculation the short circuit in complicated electric networks. It means that at calculation of short circuit currents corrections taking into account electromagnetic compatibility should be made.

The examples permit to come to a conclusion that despite variety of types, forms, and features of behavior (short circuit currents, pulse disturbance, overvoltage, etc.) electromagnetic transients in power supply systems of enterprises are theoretical and practical components of electromagnetic compatibility which is a modern science branch devoted to analysis of transients in complex physical systems.

**Test questions**

1. What factors and conditions do determine the short circuit current and power levels in a system of electric power supply?
2. What methods of short circuit current and power limitation can be used while designing electric power supply system?
3. What technical means are used to limit short circuit currents?
4. How are current-limiting reactors (of single and double circuit construction) are connected to the system of electric power supply?
5. What is the essence of the problem of short circuit power level optimization in the system of electric power supply?
6. What is the essence of the problem of short circuit current level coordination at the stages of designing and operation of the system of electric power supply? What technical means and methods of short circuit current limitation are used?
7. How do short circuit currents and power levels affect the technical and economic performance of electric power supply system elements and electric power quality?

**Topics for essay**

1. Variation of the short circuit power level on the buses of secondary voltage of a main step-down substation depending on the step-down transformers power.
2. Subdivision of electric power supply system substations fed by the power system.
3. Sequence of short circuit power and currents limiting in the system of electric power supply.





## **Part 2: Electromechanical Transients**



## **CHAPTER 9 ANALYSIS TASKS AND ELECTROMECHANICAL PROCESSES CHARACTERIZATION**

- 9.1. Background of the problem
- 9.2. Consequences of short-time failures in power supply
- 9.3. Conditions of power supply system operation mode maintenance
- 9.4. Mathematical models of power supply system components
- 9.5. Modeling of industrial enterprise electric load

Test questions

Topics for essay

### 9.1. Background of the problem

The task of power supply system operation condition maintenance consists in its parameters on the side of a power system and in the nodes of a power supply system change limitation in the boundaries at which its stability may be provided. Stability is determined by power system ability for recovering of initial or close to them operating conditions after occurrence of abnormal operation caused by various disturbances.

To change power supply system operating conditions in the specified direction it is necessary to be able to foresee the transient pattern, to choose means for its control and emergency automatics devices.

Power supply system stability is determined by analysis and calculation of electromechanical transients with account of normal and post-fault steady operation. Here substantial operating condition changes, nature and level of disturbances influencing the power supply system stability have to be considered.

Unity of power production, distribution and consumption processes stipulate the necessity to consider the fault causes, and its possible consequences in analysis and calculations, taking into account the power and supply systems.

The problems of an electric power system stability are to be solved taking into account occurrence of transients in the system that may be a source of unsafe disturbances for its normal operation.

To disturbances originating in power supply systems belong short circuits in feed lines of 110–220 kV, in distribution networks and equipment, sudden loads-off caused by major consumers cut off, and reactive loads-on after destabilization of electric motors. In all these cases considerable and sharp changes of the consumed power occur. They may cause the power system instability.

The disturbances can be divided into small disturbances at which deviations of operation parameters of their rated values are insignificant and large disturbances causing essential electric power system parameters change.

Electro-mechanical transients are classified into 3 groups according to the following features:

- small variations of the power and the rotational speed;
- large power variations and small changes of the rotational speed;
- large variations of the power and the rotational speed.

The first group includes normal transients that occur in the working conditions of a system resulting from insignificant load variations and from response of regulating and compensating devices. These transients occur when generators, transformers, transmission lines, loads and other components of a power supply system are turned on or off. They cause small perturbations and do not result in progressively increasing parameter variations with respect to normal operation conditions. Recovering the initial operating condition or a condition close to it under small perturbations defines static stability of a power supply system.

Investigations and calculations of transients referring to the second group make possible to define capability of condition stability maintenance at large perturbations being the result of sharp and significant changes in a power supply system causing considerable variation of its parameters from their rated values.

The studied cases are:

- short circuits in an electric system and their subsequent switching off;
- changes in the PSS elements connection due to switching off units and transmission lines carrying large load;
- normal switching on the transmission line possessing large charging capacity;
- switching on generators under action of self-synchronization devices.

Restoration of the initial operating conditions or an operation mode admissible with respect to operation parameters after a large perturbation exposure assures retention of the PSS operation dynamic stability.

Investigations of transients belonging to the third group permit to determine possibility of restoration the system stable operation after violation of synchronous operation of some elements or the whole system dropout.

Ability of a system to restore the initial operation mode after synchronous operation disturbance caused by large perturbation and occurrence of short-term asynchronous operation which are admissible by conditions of this operation mode makes possible to assure resulting PSS operation stability.

In electromechanical transient simplified analysis, some assumptions causing the error within 10% and making calculations easier are made. These assumptions are the following:

- the synchronous machines rotational speed varies during the transient less than for 2..3% of their synchronous speed;
- the generator voltage, its armature and field currents change in stepwise manner;
- the PSS operation change can be accounted by entering new values of the generators EMF, power, own and mutual impedances to the circuit diagram;
- unsymmetrical operation modes could be reduced to symmetrical ones assuming that the generator rotors movement change are caused only by the torques stipulated by the positive current sequence;
- generators and transformers impedance stipulated by their magnetic cores saturation could not be taken into consideration or be accounted approximately reducing corresponding reactance of the equivalent circuit till the value of  $(0,6 \dots 0,8) x'_d$ .

## 9.2. Consequences of short-time failures in power supply

In the course a power supply system operation short-term disturbances in consumer power supply occur influencing significantly on production technological process.

As the result, electromechanical transients occur. They initialize violation of continuous technological processes and cause economic losses. The major consequence of the transients is significant reduction or temporary disappearance of the voltage at consumers, which gives rise to a number of negative effects in the power supply system.

### ***Self-cutting-off electrical installations***

The reason of self-cutting-off is the self-reset of magnetic starter contactors of a standard version, that cannot be held in the position “switch on” when a short-time voltage decrease by 20–40% occurs.

In some cases electric installations are switched off by the action of minimal voltage protections that are not tuned away from short-time failures of power supply when short circuits, automatic circuit re-closure and automatic load transfer actions occur. Self-cutting-off may also appear as the result of technological protections action due to inadmissible change of a technological process variable. Unnecessary disconnections of electric installations when short-time failures of power supply occur may be initiated by automatic control systems if reserve power sources are not available.

### ***Impossibility of induction motors self-running***

Simultaneous motor self-running at a large enterprise required on electrical and technological reasons cannot be performed because of a drastic bucking and current overloads in the circuit. Sometimes self-running of motors cannot be allowed under the technological conditions or due to danger of main equipment breakages (hydraulic impacts unsafe for pipe-lines, rotational frequency variation inadmissible by technology, etc)

### ***Violation of synchronous motors stability***

Synchronous motors are more vulnerable than induction ones concerning stability in transients caused by short-time power failures.

The maximum permissible break of power supply of induction motors is determined by possible self-running at the greatest slip. The maximum permissible break of power supply for

synchronous motors is significantly less and is determined by conditions of dynamic stability. Synchronous motors operate at the power factor close to unit therefore their stability margin is significantly reduced due to critical voltage increasing and reduction of the power supply failure duration when motors remain in synchronism.

For other power installations, synchronous motors instability is stronger disturbance than initial short-time power supply failure. In this case the terminal voltage of some consumers is practically reduced close to zero. It results in their operation failures.

To prevent the negative consequences stipulated by short-time power supply failures it is necessary to analyze transients with consequent calculation of load centers stability and development of anti-damage measures for the power system and the enterprise power supply system.

### **9.3. Conditions of power supply system operation mode maintenance**

Large disturbances in a power supply system are connected with sequent operating irregularities of major industrial enterprises. Due to generating sources cutting off, sharp changes of active and reactive power balances occur. That leads to multitudinous instability of electric motors operation. To maintain the operating conditions of a power supply system, it is necessary to evaluate transients. For that, the following peculiarities should be disclosed. That are the places of imposition of main disturbances that are the most unsafe for consumers and have the maximum influence on system operating condition changes taking into consideration place and type of a short circuit, value and structure of loads, self-cutting-off load capacity, the peculiarities of a motor self-running and other factors that influence a power supply system stability. The main anti-damage measures in the internal and external power supply systems of an enterprise should be substantiated. On the basis of analysis and calculation of enterprise load stability it is possible to estimate technical and economical effectiveness of anti-damage measures and losses caused by each trouble of technological process. To assess effect of power supply failures on the enterprise load centers stability, calculation of all possible disturbances is required. In such cases short circuits should be considered as the strongest disturbances at different voltage levels. Analysis of transients shows that when short circuits occur on high-voltage lines, the power supply failures are very short-time and do not result in the complete break of consumer power supply. But the disturbances influence the work of many enterprises. When short circuits occur in the distribution mains of medium voltage, the disturbance influence on a fewer number of consumers but the transient duration is significantly longer taking into account power breaks caused by automatic re-closing and automatic load transfer.

Ensuring motor self-starting with account of its duration limitation is of great significance for preserving operation modes of a power supply system. To do that, in the course of analysis of power supply system stability, the volume of the load that would not be switched off, and at which motors self-starting would be provided should be determined. Here some over-design is needed taking into account possible errors in load assessment and superposition of different transients in generators and in load circuits.

Supply voltage is the main parameter that defines transients in a power supply system load. Due to calculations errors, the obtained voltage value can be less than allowable. Therefore it is advisable to have some voltage reserve under steady conditions and transients. Voltage reserve under transients should be 5%, but at significant errors in load defining it should not be less than 10%. It should be noticed that 5% voltage reserve corresponds to the power reserve of induction motors that are self-started (about 10%).

One of the methods of transient calculation accounting voltage reserve consists in assigning the initial mode with reduction of the voltage on buses of power stations of the electric power system and substations of the considered power supply system by a safety factor. In this case transient voltage evaluations are reduced respectively.

#### 9.4. Mathematical models of power supply system components

To analyze and calculate electromechanical transient mathematical description of a power supply system is used. The possibility to express system stability in the form of mathematical criteria depends on order of differential equations and intensity of disturbance action.

The problem of process description rigor and of calculation results accuracy should be considered taking into account correspondence between the results and actual physical nature of the process studied. The concept of the rigor follows from the goals of investigation and is determined only by this problem formulation and necessity to achieve certain results needed for practice. In this case levels of assumption should be formulated. They are specified by the problem put.

To simplify the calculation algorithm used for assessment of power supply system stability, linearization of differential equation sets and their order depression are applied. In electric power systems that have essential non-linearity the operation stability is analyzed by the method of small oscillations.

To analyze stability at large disturbances the theorems and methods of motion stability analysis by A.N. Lyapunov, and the methods of numerical integration are used.

Equation system depression can be done in different ways basing on simplification the transients described:

- dividing processes into fast and slow ones and analyzing them separately;
- replacing the group of sources or motors by one equivalent;
- equivalent account of the load using their generalized characteristics;
- selection the most significant factors and neglecting secondary ones, negligible parameters and their variations;
- linearization the system elements characteristics;
- division a complex system into simple subsystems and analyzing them separately.

The structure of equations of power supply system transients depends on mathematical models of its main elements and interrelation between its major operation parameters.

In the process of designing, **synchronous machine** models are chosen with account of a number of calculated variables that contain both in the machine and in the electric network equations. A synchronous machine is represented in the equations of electric network by some EMF  $\dot{E}$  that is connected behind a constant resistance which in turn is present in the electric network resistance matrix. In this case modeling the synchronous machine consists in determining the needed reactance  $x$  and forming the algorithm of the EMF  $\dot{E}$  module and phase calculation.

It may be assumed that the module of this EMF is  $E = const$ , or  $E = var$  and depends on inner electromagnetic transients of the machine that corresponds to actual conditions.

When the EMF is constant it is assumed that  $x = x'_d$ ;  $E = E'$  and the EMF  $\dot{E}'$  angle changes in regard to synchronous axis are determined from solution of following differential equation describing the generator motion:

$$T_J d^2 \delta / dt^2 = M_{tb} - M \quad (9.1)$$

where  $d^2 \delta / dt^2$  is angular acceleration;  $T_J$  is mechanical inertia constant of the power unit;  $M_{tb}$  is the turbine moment;  $M$  is electromagnetic torque produced by a generator.

The drawback of the model with  $E' = const$  is the fact that it is impossible to consider field adjustment and asynchronous torque facilitating damping of generator synchronous swinging. This defect can be removed by artificial introduction into the equation (9.1) of the summand proportional to slip, with damping factor  $P_d$ , i.e. representing the equation as

$$T_J d^2 \delta / dt^2 + p_d d\delta / dt = M_{tb} - M. \quad (9.2)$$

To determine the value of  $P_d$ , knowledge of the transient expected flow is necessary.

Use of the model with  $E' = const$  is advisable in the following two cases:

1) when a long series of calculation are made for disturbances of the same type and at similar initial modes. For that it is necessary to make some preliminary calculations with use of the generator models as precise as possible. Then these models are replaced by the models with  $E' = const$  and such  $P_d$  values are selected that results of the simplified and the exact calculations coincide. After that other simplified calculations are executed.

2) for synchronous machines remote from the disturbance place, when it is known in advance that they remain in synchronism with the power system and have slight effect on the major calculation results.

Models of synchronous machines with variable EMF are chosen depending on the goal of calculation and corresponding required calculation accuracy, pattern of the problems solved, computer performance and memory capacity along with information availability. In this case the following assumptions are made:

- synchronous machine equations are recorded in a rotating coordinates system  $d, q$  rigidly bound with the rotor, where the direct axis coincides with the axis of magnetic flux of the field winding;
- transients in stator circuits are neglected enabling excluding differential form of line, transformer and reactors equations without appreciable error;
  - the current and voltage values at asymmetric short circuits and other operation modes are taken into account only by direct sequence components;
  - damping circuits of synchronous machines are either approximately described or neglected;
  - nonlinearity of magnetization curves, stipulated by cores saturation, is neglected;
  - active stator winding resistance is neglected;
  - synchronous machine is introduced into the electric network equations by its superconducting EMF  $E''$  and reactance values  $x_d'' = x_q'' = x''$ ;
  - frequency variations are taken into account in simplified form or neglected.

**Generators and synchronous compensators excitation system models** in transient calculations should account the upper and lower excitation limits, speed of excitation change and action of automatic on the electromechanical oscillations damping.

Excitation systems and automatic control in transient in power supply systems calculation is commonly described with two differential equations to account the character of control and time lag in the automatic excitation control device and the exciter.

**Turbine models** in transient calculations are intended to investigate the frequency control system effect and the rotational velocity significant changes on the turbine capacity  $P_{tb}$ .

The equation of the turbine torque can be drawn up on the basis of static characteristics  $M_{tb} = \psi(\omega, \mu)$  where  $\mu$  is grade of opening the regulating control valve of the turbine energy carrier flow. When the rotational velocity and valve opening change slowly, nonlinearly of dropping characteristics  $M_{tb}$  are approximately represented by family of straight lines (Fig.9.1.). In this case

$$M_{tb} = \mu[M_{tb0} - \eta_{tb}(\omega - \omega_0)] \quad (9.3)$$



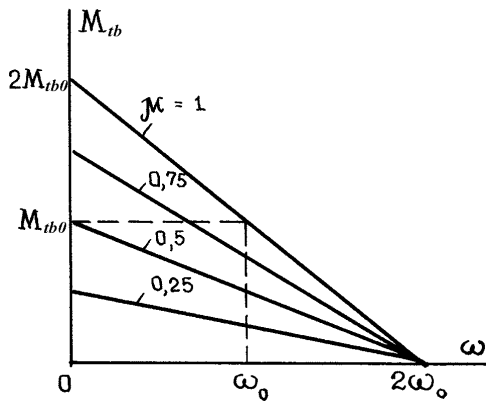


Fig. 9.1. Idealized characteristics of turbine torque

where  $M_{tb0}$ ,  $\omega_0$  are the turbine torque and rotational velocity at normal mode,  $\eta_{tb}$  is the turbine self-regulation factor.

**The load models** at small or slow voltage in the node point changes are described by natural static characteristics. These characteristics show consumers reaction to voltage and frequency deviation. They are defined by dependences  $P(U, \omega)$  and  $Q(U, \omega)$  for the most typical power units, including induction motors and static consumers with  $P(U)$  dependences close to quadratic (resistance furnaces, incandescence lamps, a number of domestic electrical installations).

The static characteristics of active node load against the voltage  $P(U)$  are given as the sum of corresponding characteristics of all the node consumers with account of the distribution network losses. They may be represented by one standard characteristic.

Averaged dependence of  $Q(U)$  can't be represented by the standard characteristic as total static characteristic for reactive load is mainly stipulated by induction motors, and significantly depends on compensating devices availability in the node.

When large disturbances occur in a power supply system the dynamic load model is to be used. Induction and synchronous motors as well as static consumers (domestic load, electric furnaces etc) are introduced into the model. The static load is assigned by constant admittance.

**Model of the induction motor** is described by four equations without account of electromagnetic processes that do not practically influence on the calculation accuracy:

- The motor motion equation is

$$T_J ds_R / dt = M_{WM} - P_{IM} / \omega, \quad (9.4)$$

where  $s_R = (\omega_{rated} - \omega_R) / \omega_{rated}$  is the rotor slip;  $M_{WM}$  is anti-torque moment of a working mechanism;  $P_{IM}$  is active motor power;  $\omega_R$  is the rotor speed.

- Active power equation is

$$P_{IM} = U^2 r_2 s / \left[ (\omega / \omega_{rated})^2 x_s^2 s^2 + r_2^2 \right], \quad (9.5)$$

where  $U$  is voltage on the motor terminals;  $\omega$ ,  $\omega_{rated}$  are actual and rated speed respectively;  $s = (\omega - \omega_R) / \omega$  is rotor slip in regard to voltage vector;  $x_s$  is the rotor and stator leakage reactance;  $r_2$  is rotor active resistance.

Resistances  $x_s$  and  $r_2$  depend on rotor current frequency in a complicated manner (Fig. 9.2, a). Determining  $P_{IM}$  they may be assumed constant at  $s \leq s_{cr}$ , but when  $s > s_{cr}$  significant errors may appear. It is advisable to apply dependences  $x_s(s)$  and  $r_2(s)$ , given in the Fig. 9.2, c where they are defined according to piecewise-linear approximation. Reference slip values (Fig. 9.2, b) used for approximation are  $s_1 \approx s_{cr}$ ;  $s_2 = 0,5 \dots 0,9$ .

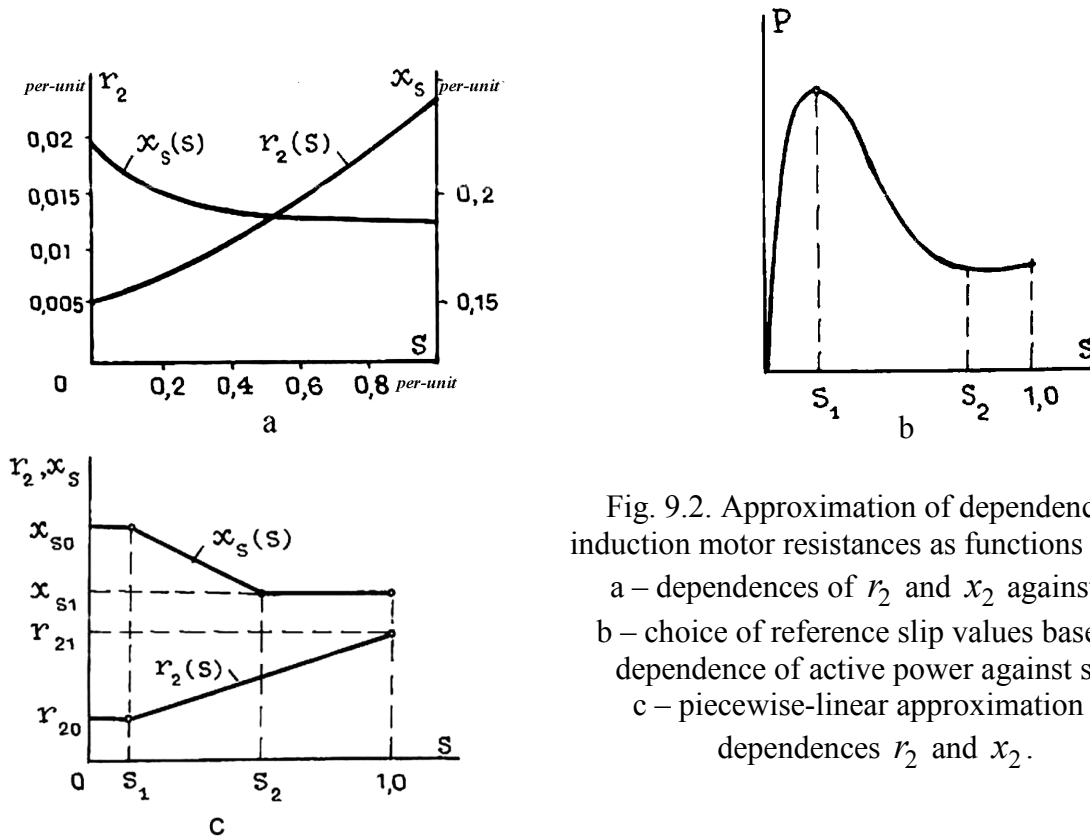


Fig. 9.2. Approximation of dependence of induction motor resistances as functions of slip: a – dependences of  $r_2$  and  $x_2$  against  $s$ ; b – choice of reference slip values based on dependence of active power against slip; c – piecewise-linear approximation of dependences  $r_2$  and  $x_2$ .

Rotor resistance, proportional to the torque on motor shaft, is determined for equal numbers of rotor and stator winding turns:

$$r_2 = \begin{cases} r_{20}, & s \leq s_1; \\ [r_{20}(1-s) + r_{21}(s-s_1)]/(1-s_1), & s > s_1. \end{cases}$$

Leakage reactance is

$$x_2 = \begin{cases} x_{s0}, & s \leq s_1; \\ [x_{s0}(s_2-s_1) + x_{s1}(s-s_1)]/(s_2-s_1), & s_1 < s < s_2; \\ x_{s1}, & s \geq s_2. \end{cases}$$

Coordinates of “reference” points in piecewise-linear approximation are determined by catalogue data with the expressions:

$$s_{cr} = s_{rated} (m_{max} + \sqrt{m_{max}^2 - 1})$$

$$x_{s0} = 1/(2m_{max} \cos \varphi_{rated})$$

$$x_{s1} = x_{\mu} / (I_{start} x_{\mu} - 1);$$

$$r_{20} = s_{rated} (1 + \sqrt{1 - 1/m_{max}^2}) / (2 \cos \varphi_{rated}); \quad (9.6)$$

$$r_{21} = (1 - \sqrt{1 - 4x_{s1}^2 m_{start}^2 \cos^2 \varphi_{rated}}) / (2m_{start} \cos \varphi_{rated}),$$

where  $I_{start}$ ,  $m_{start} = M_{start} / M_{rated}$  are ratios of starting current and starting moment respectively;  $x_{\mu} = 1 / [\sin \varphi_{rated} - (1 - \sqrt{1 - 4x_{s0}^2 \cos^2 \varphi_{rated}}) / (2x_{s0})]$  is magnetization resistance at the rated voltage.

- Equation of reactive motor power is

$$Q_{IM} = P_{IM} (\omega / \omega_{rates}) x_s s / r_2 + Q_{\mu} \omega_{rates} / \omega \quad (9.7)$$

Here  $Q_{\mu}$  is reactive magnetization power. Due to non-linearity of no-load characteristic the reactive power  $Q_{\mu}$  is a function of  $U_M$   
Approximately:

$$Q_{\mu} = (U_M^2 / x_{\mu}) f(U_M)$$

Function  $f(U_M)$  reflects non-linearity of the no-load characteristic. It may be expressed as

$$f(U_M) = 1 - \alpha - \alpha (U_M / U_{M,rated})^4$$

or

$$f(U_M) = \left( U_M / U_{M,rated} \right)^{k_{\mu}^{-2}}$$

If non-linearity is not taken into account ( $\alpha = 0$ ,  $k_{\mu} = 2$ ), the function adopts the value

$$f(U_M) = 1$$

- Equation of the drag torque on a motor shaft is

$$M_{WM} = M_{WM,static} + (M_{WM,rated} - M_{WM,static}) \times \left[ (1 - s_R) / (1 - s_{R,rated}) \right]^P, \quad (9.8)$$

where  $M_{WM,rated}$  is rated torque of the working mechanism;  $P$  is exponent depending on its type;  $M_{WM,static}$  is the static drag torque at  $\omega = 0$ ;  $s_{R,rated}$  is the motor slip under rated conditions.

The following parameters are adopted from the catalogue data with account of the induction motor equivalent circuit:  $P_{rated}$ ;  $U_{rated}$ ;  $\cos \varphi_{rated}$ ;  $s_2$ ;  $k_{\mu}$  (or  $\alpha$ );  $m_{static}$ ;  $T_J$ ;  $m_{max} = M_{max} / M_{rated}$ ;  $m_{start}$ ;  $I_{start}$ ;  $s_{rated}$ ;  $x_{\mu}$ ,  $x_{s0}$ ,  $x_{s1}$ ,  $r_{20}$ ,  $r_{21}$ .

As asynchronous loads have common properties it is possible to use the generalized induction motor for their simulation. The following parameters are assumed for that:

$\cos \varphi_{rated} = 0,8$ ;  $m_{start} = 0,73$ ;  $I_{*start} = 4,1$ ;  $s_{rated} = 2\%$ ;  $s_2 = 70\%$ ;  $k_\mu = 4$ ; ( $\alpha = 0,5$ );  
 $m = 0,7$ ;  $m_{WM,static} = M_{WM,static} / M_{WM,rated} = 0,5$ ;  $T_J = 0,8$  s at short perturbations and  
 $T_J = 0,6$  s at deep slowdown. The following values of the motor parameters are obtained from its  
 equivalent circuit:  $x_\mu = 2,95$ ;  $x_{s0} = 0,368$ ;  $x_{s1} = 0,266$ ;  $r_{20} = 0,0226$ ;  $r_{21} = 0,0424$ .

Active power of the generalized induction motor in the initial regime is prescribed by its share  
 of active power consumed in the node:  $P_{IM,stand} = K_{IM} P_{stand}$ . The slip in the initial mode  
 $s_{stand} = s_{R,stand}$  under known voltage  $U$ , is determined according to expression (9.5). The  
 induction motor reactive power  $Q_{IM,stand}$  is found according to expression (9.7). The value of static  
 load can be determined by the found values of the induction motor active and reactive power:

$$P_{static,stand} = P_{stand} - P_{IM,stand}, \quad Q_{static,stand} = Q_{stand} - Q_{IM,stand}$$

These values are used in transient calculation in the following form:

$$P_{static} = P_{static,stand} F_1(U, \omega); \quad Q_{static} = Q_{static,stand} F_2(U, \omega),$$

where functions  $F_1(U, \omega)$  and  $F_2(U, \omega)$  correspond to characteristics pre-assigned for the static  
 load.

### 9.5. Modeling of industrial enterprise electric load

**Asynchronous load of a production plant** is modeled on the basis of initial induction motor  
 models for each node of a power supply system design circuit with use of averaged parameters and,  
 if necessary, making some refinements.

Using main parameters of an asynchronous motor, it is possible to determine its equivalent  
 circuit parameters in per-unit by means of expressions (9.6).

Induction motor averaged parameters ( $\cos \varphi$ ,  $m_{max}$ ,  $m_{start}$ ,  $I_{*start}$ ) are determined  
 according to the table 9.1. The rated slip is taken equal to  $s_{rated} = 0,02$ .

Table 9.1  
 Averaged parameters of induction motors

Rated voltage of the motor, $U_{rated}$ , kV	Rated voltage of the load node (in the design circuit)	$\cos \varphi$	$m_{max}$	$m_{start}$	$I_{*start}$
0.38	0.38	0.85	2	1.7	6
0.38	6–10	0.84	1.9	1.4	5.5
6	6–10	0.84	2.2	1.4	5.4
0.38 i 6 (one motor)	6–10	0.84	2	1.4	5.5

Average load factor of a modeled motor group is

$$m = P_\Sigma / P_{\Sigma rated},$$

where  $P_\Sigma$  is total power consumed by the motors;  $P_{\Sigma rated}$  is total rated power of all the motors  
 connected to the network.

Static braking torque of the enterprise asynchronous load as a whole is defined as weighted average value for three most typical consumer groups that differ by dependences of the drag torque on the shaft against speed.

For the first motor group having total rated power  $P_{1\Sigma rated}$  that includes mechanisms of a ventilator type it is assumed that  $m_{static1} \approx 0$ . For the second group with  $P_{2\Sigma rated}$  that includes mechanisms of a pump type it is assumed that  $m_{static2} \approx 0,5$ . For the third group having  $P_{3\Sigma rated}$  that includes mechanisms with compressor characteristics the static braking torque is assumed equal  $m_{static3} \approx 1$ .

Here

$$P_{\Sigma rated} = P_{1\Sigma rated} + P_{2\Sigma rated} + P_{3\Sigma rated}$$

$$m_{static} = \sum_{i=1}^3 m_{static,i} (P_{i\Sigma rated} / P_{\Sigma rated}) \approx$$

$$\approx (0,5P_{2\Sigma rated} + P_{3\Sigma rated}) / P_{\Sigma rated}$$

When the number of motors is great, the averaged value of the mechanical inertia constant can be determined by empirical formula

$$T_{JM} = 0,12(P_{\Sigma rated} / N)^{0,45}, \quad (9.9)$$

where  $N$  is the number of motors.

The mechanical inertia constant of the unit as a whole can be defined by the expression

$$T_J = K_J T_{JM}$$

where  $K_J$  is the factor characterizing a mechanism type.

For the assumed motor groups  $K_{J1} \approx 5$ ;  $K_{J2} \approx 1,2$ ;  $K_{J3} \approx 2$ , and the averaged value of the inertia constant for the equivalent motor is

$$T_J = \sum_{i=1}^3 K_{Ji} T_{JMi} (P_{i,rated} / P_{\Sigma rated}) \approx$$

$$\approx \left( \begin{array}{l} 5T_{JM1}P_{1\Sigma rated} + 1,2T_{JM2}P_{2\Sigma rated} + \\ + 2T_{JM3}P_{3\Sigma rated} \end{array} \right) / P_{\Sigma rated},$$

where  $T_{JM,i}$ ,  $i = 1, 2, 3$  are averaged values of  $T_{JM}$  for each of the three motor groups according to (9.9).

To provide sufficient accuracy of transient calculation especially at self-starting of a large motor group of variety dissimilar types, the load can be divided into more than three parts, consisting of homogeneous components. These groups should be homogeneous by criterion basing  $m_{static}$  and  $T_J$  values.

**Synchronous motors of an enterprise** are modeled in steady state mode with the use of the static characteristic of reactive power that is described with equation

$$Q_{SM} = -[E_q U_M \cos \delta / x_d -$$

$$- U_M^2 ((x_d + x_q) - (x_d - x_q) \cos 2\delta) / (2x_d x_q)] / \omega. \quad (9.10)$$

The interior angle  $\delta$  in (9.10) is determined with the help of expression

$$P_{SM} = [E_q U_M \sin \delta / x_d + U_M^2 \cdot (x_d - x_q) \sin 2\delta / (2x_d x_q)] / \omega. \quad (9.11)$$

From the expressions (9.10) and (9.11), it follows that when  $x_d = x_q$

$$Q_{SM} = U_M^2 / (\omega x_d) - \sqrt{(E_q U_d / (\omega x_d))^2 - P_{SM}^2}. \quad (9.12)$$

View of the static characteristic of active power doesn't depend on the motor type.

**At transients calculation basing on dynamic stability of synchronous motors** operation it is necessary to consider influence of automatic excitation control in the process of starting and self-starting of the motors, and other influences on enterprise complex load by automatic devices of the excitation system.

Electric network elements of an enterprise power supply system (nodes, arms of a network, transformers, etc.) which parameters enter in the motion equations are described on the base of dependences known from the theory of electric system and networks.

**For a node of the network** having  $n$ -arms the balance of active and reactive power is true:

$$\sum_{i=1}^n P_i = 0; \quad \sum_{i=1}^n Q_i = 0. \quad (9.13)$$

Elements of the network arms are given in the equivalent circuits as series-parallel connected resistances and reactances (Fig 9.3). They are described with the following equations:

- at the beginning of a section

$$\left. \begin{aligned} P_1 &= U_1^2 y_{11} \sin \alpha_{11} + U_1 U_2 y_{12} \sin(\delta_{12} - \alpha_{12}); \\ Q_1 &= U_1^2 y_{11} \cos \alpha_{11} - U_1 U_2 y_{12} \cos(\delta_{12} - \alpha_{12}); \end{aligned} \right\} \quad (9.14, a)$$

- at the end of a section

$$\left. \begin{aligned} P_2 &= -U_2^2 y_{22} \sin \alpha_{22} + U_1 U_2 y_{12} \sin(\delta_{12} + \alpha_{12}); \\ Q_2 &= -U_2^2 y_{22} \cos \alpha_{22} + U_1 U_2 y_{12} \cos(\delta_{12} + \alpha_{12}); \end{aligned} \right\} \quad (9.14, b)$$

Self- and mutual admittances in (9.14) are defined by expressions:

$$Y_{11} = 1/[Z_1 + Z_2 Z_3 / (Z_2 + Z_3)] = y_{11} \exp \varphi_{11};$$

$$Y_{22} = 1/[Z_2 + Z_1 Z_3 / (Z_1 + Z_3)] = y_{22} \exp \varphi_{22};$$

$$Y_{12} = 1/[Z_1 + Z_2 + Z_1 Z_2 / Z_3] = y_{33} \exp \varphi_{12},$$

and complementary angles are

$$\alpha_{11} = 90^\circ - \varphi_{11}; \quad \alpha_{22} = 90^\circ - \varphi_{22}; \quad \alpha_{12} = 90^\circ - \varphi_{12}.$$

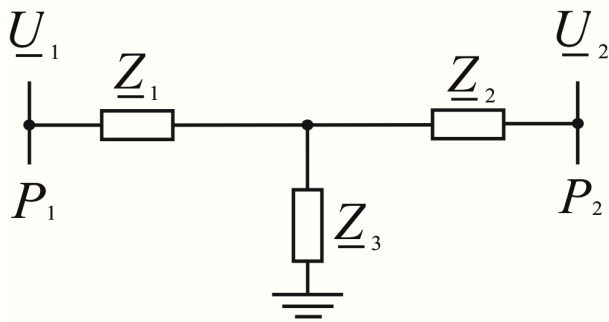


Fig.9.3 Equivalent circuit of electric network section

**Design model of an enterprise load node** is defined by composition of its consumers and the distribution network. In general case it is multi-component and includes mathematical formulation of equivalent induction and synchronous motor parameters and static load (lighting, rectifiers, inverters, heating appliances, electric furnaces, etc.) as well.

Besides the static load characteristics  $P(U, \omega)$ ;  $Q(U, \omega)$ , the load dynamic characteristics reflecting fast changes of the load operation mode are used:

$$\left. \begin{aligned} &P(U, \omega, t, dU / dt, d\omega / dt, \dots); \\ &Q(U, \omega, t, dU / dt, d\omega / dt, \dots). \end{aligned} \right\} \quad (9.15)$$

The choice of a load model type depends on the required accuracy of power supply system stability calculation. In approximate calculations of electromechanical transients, the load model in the form of complex resistance is often used.

Taking into account the simplifying assumptions, mathematical models of power supply systems are divided into conservative positional and dissipative ones. Description of the system by means of **conservative positional model** corresponds to: assumption of absence of power losses depending of the machine rotational speed (any operation disturbances result in continuous oscillations); independence of all exerting torques on machine speed variation, except machines with great inertia of rotor.

Therefore at positional model idealization, the torques in per-unit entering to the equations of motion can be replaced by powers. Turbines power, EMF of machines, frequency and resistances of the network passive elements are assumed constant and the developed electromagnetic power depends only on position of synchronous machine rotors.

**In dissipative system model** the following factors are taken into account. These are dependence of the machine torque on rotor position, and dependence of their rate of change on regulating and compensating devices parameters, as well on electromagnetic and electromechanical processes in the network elements.

To analyze stability of an enterprise power supply system operation, it is acceptable to use simplification basing on to the calculation of electromechanical transients using the positional model. It could be explained by the fact that in centralized power supply of enterprises, a source of unlimited power (precondition of the voltage and frequency stability) is practically always available. The problem of stability maintenance is solved for short period of time after disturbance occurrence (0.2...0.5 s). Moreover, the synchronizing torques but not inertial ones depending on rotational speed and being insignificant are vital for the PSS operation mode stability.

### Test questions

1. What are the purposes of electromechanical transient calculation?
2. What kinds of disturbances in a power supply system result in transients?
3. What assumptions are taken to simplify transient calculations?
4. What are consequences caused in a power supply system by short- time disturbance of consumer power supply?
5. What are the necessary conditions to provide operation stability of a power supply system?
6. What is the role of mathematical models of power supply system elements in transient calculations?
7. What equations are used to describe synchronous machine models?
8. What is the function of turbine models at transient calculations?
9. What are the equations of an induction motor model?
10. How is modeling of total enterprise asynchronous load carried out?
11. Describe the process of enterprise synchronous load modeling.
12. How are electric network elements considered in modeling of an enterprise power supply system?

### Topics for essay

1. Electromechanical transients influence on power supply system operation.
2. Mathematical modeling of enterprise load in transient calculations.

## **CHAPTER 10: METHODS OF STATIC STABILITY CALCULATION**

10.1. Static stability criteria

10.2. Application of practical static stability criteria

10.3. Analysis of static stability by method of small oscillations

10.4. Taking into account automatic excitation control

Test questions

Topics for essay



**10.1. Static stability criteria**

Examples of simplest mechanical systems (Fig.10.1) show that there are certain states of system parts equilibrium to which these parts tend to return after random disturbance (that is, they restore initial condition). Besides, there are such equilibrium states which the system leaves at any disturbance. In the first case the system equilibrium state is stable, and in the second case it is unstable.

Physical estimation of stability disturbance mechanism can be performed on the basis of force or energy approaches. *Under the energy approach* the conditions of stability are determined according to the Dirichlet's theorem. By the theorem a conservative system is stable if its potential energy in equilibrium state is minimal. *Under the force approach*, stability of a system is determined by the condition according to which the sum of forces applied to a body in equilibrium state must be equal to zero. When the body moves aside of the equilibrium state, the force tending to return it to the equilibrium should arise. If a body being in equilibrium state moves at some velocity, the forces arising should be directed against the velocity vector.

Give energy interpretation to a steady state of a power supply system (PSS).

There is balance of the source energy  $W_G$  entering the system and the energy  $W$  consumed in the load, i.e.  $W_G = W$ . At some small disturbance that appears as the operation parameter  $V$  change by  $\Delta V$  the balance is upset. As  $W_G = f_1(V)$ ; and  $W = f_2(V)$  the source varies energy production by  $\Delta W_G$ , and energy consumption in the system varies by  $\Delta W$ .

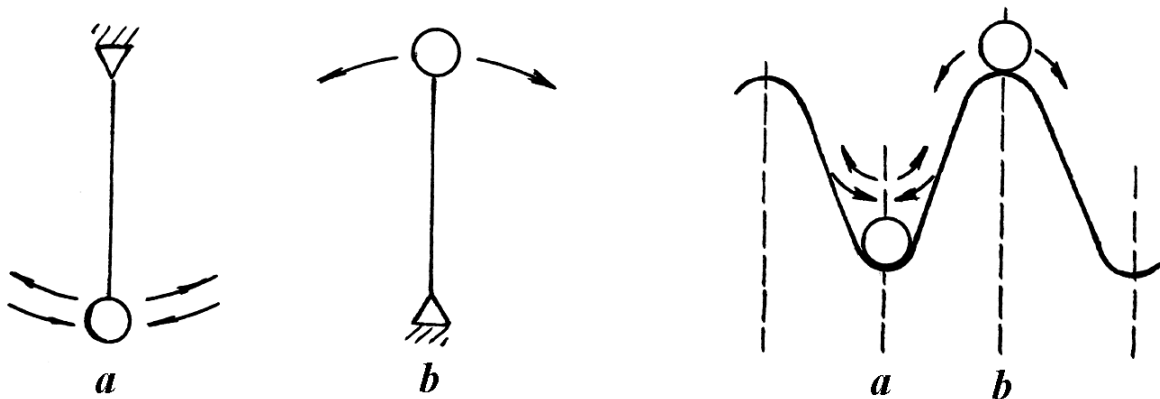


Fig. 10.1. Equilibrium states of mechanical system: a – stable; b – unstable

If energy consumption of a system after disturbance becomes more intensive than its arrival from an external source, a new operation mode taking place after the disturbance can not be supplied with energy. On that reason the previous or close to it system mode should be restored. Such a system is stable.

It follows from the stability definition given above that the criterion of the system stability conservation (stability criterion) is expressed as  $\Delta W / \Delta V > \Delta W_G / \Delta V$ . The view of the criterion in differential form is  $d(W_G - W) / dV < 0$ .

The difference  $W_G - W = \Delta W_\Sigma$  is called *the excess energy*. With it stability criterion will be

$$d(\Delta W_\Sigma) / dV < 0 \tag{10.1}$$

This means that the operation is stable if derivative of excess energy with respect to the diagnostic parameter  $V$  is negative.

Approximate estimations of stability based on stability criterion (10.1) are used in engineering analysis. They are called practical stability criteria.

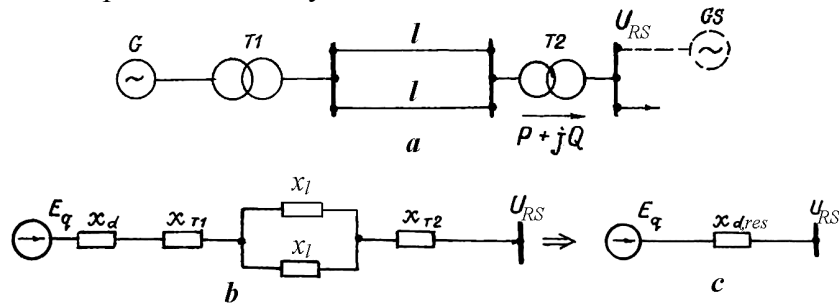


Fig. 10.2. Power supply system: a – design circuit; b, c – equivalent circuit and its conversion

Consider the simplest PSS circuit in which a generator is connected through a transformer and power line to buses of invariable voltage. The buses belong to a receiving system which power is so great that voltage on its buses can be recognized as invariable in amplitude and phase under any operation conditions (Fig. 10.2, a).

For analysis, it is convenient to use the power-angle curve  $P = f(\delta)$ , where  $P$  is electromagnetic power of the generator;  $\delta$  is the angle of phase displacement between EMF  $\dot{E}_q$  of the generator and voltage on the buses of the receiving system  $\dot{U}_{RS}$ .

Assuming  $r = 0$  we get from equivalent circuits (Fig.10.2, b, c) that resulting resistance is

$$x_{d,res} = x_d + x_{T1} + x_l / 2 + x_{T2}$$

Vector diagram for normal operation is given in Fig.10.3, from which it is seen that  $bc = E_q \sin \delta$  or  $bc = I x_{d,res} \cos \varphi = I_{act} x_{d,res}$ . Here  $E_q \sin \delta = I_{act} x_{d,res}$ . Multiplying the both sides of the equality by  $U_{RS} / x_{d,res}$  we find that active power transferred to the receiving system is determined as

$$P = E_q U_{RS} \sin \delta / x_{d,res} \tag{10.2}$$

where  $E_q = \sqrt{(U_{RS} + I_{react} x_{d,res})^2 + (I_{act} x_{d,res})^2}$   
or

$$E_q = \sqrt{(U_{RS} + Q x_{d,res} / U_{RS})^2 + (P x_{d,res} / U_{RS})^2} \tag{10.3}$$

It follows from expression (10.2) that under constant values of the generator EMF  $E_q$  and voltage  $U_{RS}$  on receiving system buses variation of the transferring power  $P$  depends only on the angle  $\delta$  variation.

Power delivered to the network by the generator can also be changed by action onto actuator valves of turbine. In the initial operation mode the turbine torque is balanced by the generator

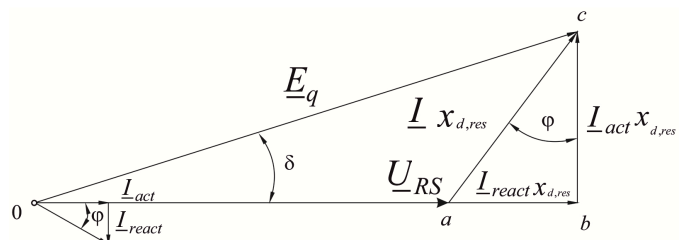


Fig. 10.3. Vector diagram for normal operation

torque performing with constant rotational speed. As the actuator valves open, the turbine torque increases, and balance of the rotary and braking torques upsets, resulting in acceleration of the generator rotor. When the rotor accelerates, the vector  $\dot{E}_q$  moves relatively to the vector of the receiving system voltage  $\dot{U}_{RS}$  rotating with invariable speed. The angle  $\delta$  increase caused by the described process will result in corresponding generator power increase till it will have balanced the increased turbine power again. As dependence  $P = f(\delta)$  is sinusoidal, the power initially increases when the angle increase takes place, and then starts to reduce after achievement its peak value.

Under given values of the generator EMF  $\dot{E}_q$  and the voltage  $\dot{U}_{RS}$ , maximum value of the transmitted power is called *the ideal power limit*, and

$$P_{max} = E_q U_{RS} / x_{d,res} \tag{10.4}$$

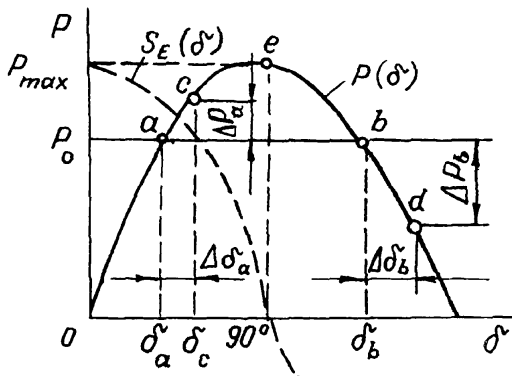


Fig. 10.4. Generator active power and synchronizing power against power angle characteristics

Under stable operation of the PSS, the power developed by the turbine equals to the power developed by the generator. At that two points of equilibrium on power-angle curve, and hence two values of angle ( $\delta_a$  and  $\delta_b$ ) correspond to the turbine power  $P_0$  (Fig. 10.4). But stable operation is provided only in the point  $a$  that can be easily shown by assessment the character of generator rotor movement under insignificant deflection from the equilibrium point (Fig. 10.5).

Assume that due to insignificant disturbance the power angle increases its value by  $+\Delta\delta_a$ . This causes the operating point on the

power-angle curve displacement from  $a$  to  $c$ , and power delivered by the generator increases by  $+\Delta P_a$  (positive increment in power angle corresponds with positive increment in power).

As the result of the generator power increase at invariable power supplied by the turbine, the balance of torque developed by the turbine and braking torque of the generator is upset, and the resulting braking torque appears on the set shaft. The generator rotor begins decelerate being exposed to it, and this causes the

EMF vector  $\dot{E}_q$  displacement in the direction of the angle  $\delta$  reduction. The angle  $\delta$  being reduced, the initial operation mode at the point  $a$  is restored. So the considered mode is stable. The same conclusion can be obtained in the case of negative increment of the power angle  $-\Delta\delta_a$  as well.

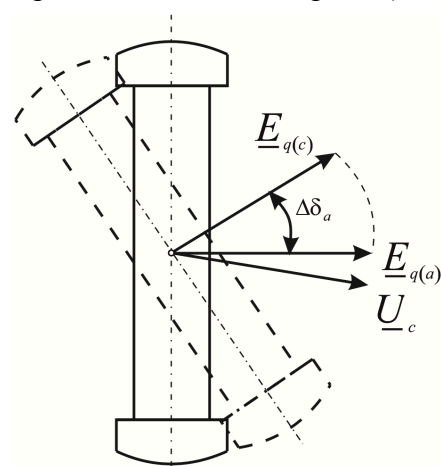


Fig. 10.5. Change of generator rotor position under perturbation

In point  $b$  (Fig. 10.4), negative increment in the generator power  $-\Delta P_b$  corresponds to positive increment in the angle  $+\Delta\delta_b$ . Reduction of the generator power causes the accelerating torque applied to the rotor, under which influence angle  $\delta$  does not reduce but increases. The angle  $\delta$  being increased, the delivered by the generator power continues to reduce. This causes further angle  $\delta$  increase, and so on. The process is progressive, and the generator pulls out of step. Operation in point  $b$  is unstable.

Thus, the system state corresponding to point  $a$  as well as any other point within the rising part of the torque-angle curve is stable, and the state corresponding to each point of falling part of characteristic is unstable. Hence the following criterion of static stability within the portion of the curve is true:

$$\Delta P / \Delta \delta > 0$$

and in a point:

$$dP / d\delta > 0 \quad (10.5)$$

It follows from (10.5) that static stability takes place if increments of the torque angle and power delivered by generator have the same sign.

The derivative  $\Delta P / \Delta \delta = S_E$  is called the synchronizing power. Its sign can serve as a criterion of static stability. Synchronizing power is determined with the help of the expression (automatic load transfer isn't available):

$$S_E = dP / d\delta \Big|_{E=const} = E_q U_{RS} \cos \delta / x_{d,res} \quad (10.6)$$

Synchronizing power is positive if  $\delta < 90^\circ$ , and operation of the system is stable. Numerically the static stability is described with help of the power assurance factor  $k_{paf} = (P_{max} - P_0) / P_0$ .

## 10.2. Application of practical static stability criteria

Analysis of static stability in increasing order of its complexity can be divided into the following stages. Firstly, the fact of stability or instability of steady operation is determined, and then its character (non-periodic or oscillatory) is determined by type of the transient curve.

Non-periodic instability arises if balance of torques on the generator shaft is upset, and the turbine torque exceeds the peak electromagnetic torque of the generator available under given conditions. Analysis of stability includes determination of the generator limiting stable state on the basis of drift or flow of normal operation indicators with use of the steady operation equations.

Within the following stage the conditions of oscillating instability for the determined area of stable operation are identified. The oscillating instability can take place due to:

- availability of parametric self-swing, or feedback at automatic load transfer;
- self-excitation when a synchronous generator is loaded with capacitance or a line with distributed parameters;
- instability of the load center.

Analysis of oscillating stability requires the account of the system components dynamics using the system model described with differential equations. In this case, the problem of determination of the control devices elements parameters and structure, needed for the system stable operation, is stated and solved.

Stability or instability of stable operation under relatively insignificant running variations of its parameters ("drift" of the steady operation indicators) can be determined with help of practical criteria based on physical representations of stability mechanism disturbance. They are used when

self-swing conditions are not present, and there is no necessity to study the character of transient, and to determine the type of stability disturbance (non-periodical or oscillatory). Stability estimation on practical criteria is a rough one, and overestimation on stability reserve takes place. It determines only the fact of the given operation stability.

The use of PSS static stability practical criteria is based on estimation of its properties by the power criterion (10.1) which establishes relation between intensity of external influences to the system, and its response for it. It stipulates the excess power determination in the system as a whole.

Analysis of static stability by practical criteria helps to determine limiting operation state as well as stability limit under chosen method to influence PSS. The latter is called the method of operation charging. It can be chosen only under the condition if the electric power supply circuit as well as structure of the using equipment is given. The power supply circuit is transformed into the design circuit.

Design circuits are brought to forms being in accordance with the equivalent PSS relatively its electric power distribution nodes:

- equivalent generator – power line – buses of invariable voltage;
- bilateral feeding of load given as a fixed resistance;
- equivalent power source – nodal point of the network;
- feeding of induction motors by a powerful power system;
- equivalent source energizing a complex load of comparable power.

Consider typical examples of practical criteria use to study static stability of the indicated equivalent circuits of PSS.

**Design circuit "Equivalent generator – electric power transmission line - buses of invariable voltage"**

See the equivalent circuit “equivalent generator - power line - buses of invariable voltage”. The circuit with parameters of its elements is in Fig.10.6. Under steady condition, the torque delivered by the turbine is balanced with electromagnetic torque by the generator, that is

$$M_G - M = 0 \tag{10.7}$$

where electromagnetic torque is equal to

$$M = E'U \sin \delta / \left[ \omega_0 (x'_d + x_{ext}) \right] \tag{10.8}$$

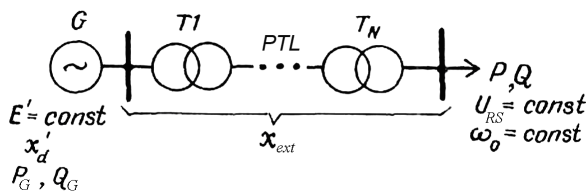


Fig. 10.6. Equivalent circuit “Equivalent generator - power line - buses of invariable voltage”

Equation (10.7) has the only essential variable as other parameters ( $E', U, \omega_0$ ) are invariable on the condition, and the turbine torque doesn't depend on the variable  $\delta$  as it follows from (9.3).

Expressing the system and operation parameters in per units, it can be assumed

that  $M_{tb} \equiv P_{tb}$ , and equation (10.7) can be written down as:

$$P_{tb} - P = 0 \tag{10.9}$$

There are two points of the system operation balance depending on variable  $\delta$  (Fig.10.4). Violation of the powers equality (10.9) is an indicator of its power balance change and availability of excess power in the system. In this case the power criterion (10.1) can be expressed as:

$$d(P_{tb} - P) / d\delta < 0 \tag{10.10}$$

As  $P_{tb}(\delta) = const$ , inequality (10.10) takes the form of (10.5).

**Design circuit with bilateral load energizing**

As for this circuit (Fig.10.7), power criterion can be indirectly characterized with the help of such an operation variable as the active power. If branches containing generators have the same load ( $P_1 = P_2 = P$ ) the steady condition will be described by means of following system of equations:

$$\left. \begin{aligned} P_{tb} - P &= 0; \\ P &= E_{q1}^2 \sin \alpha_{11} / |\underline{Z}_{11}| + E_{q1} E_{q2} \sin(\delta_{12} - \alpha_{12}) / |\underline{Z}_{12}|; \\ P_1 + P_2 - U_{ld}^2 / R_{ld} &= 0, \end{aligned} \right\} \quad (10.11)$$

where

$$\underline{Z}_{11} = jx_1 + jx_2 \underline{Z}_{ld} / (jx_2 + \underline{Z}_{ld}) = |\underline{Z}_{11}| \exp(\pi/2 - \alpha_{11})$$

$$\underline{Z}_{12} = jx_1 + jx_2 + jx_2 jx_1 / \underline{Z}_{ld} = |\underline{Z}_{12}| \exp(\pi/2 - \alpha_{12}) = R_{12} + jx_{12}$$

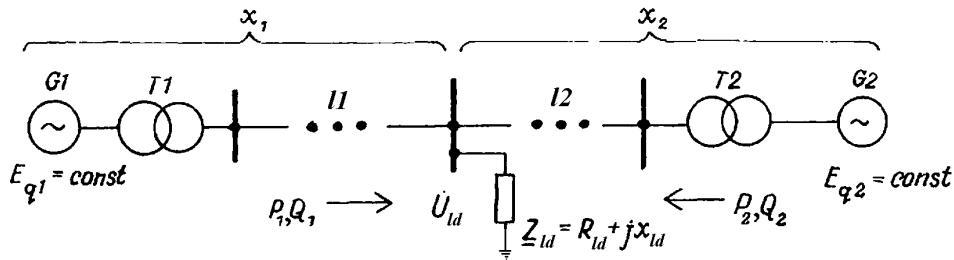


Fig. 10.7. Design circuit with bilateral load node energizing

In equations (12.11) the angle  $\delta_{12}$  is an unrestricted essential variable on which the operation mode parameters ( $P, U_{ld}$ ) depend. That's why the derivative  $dP / d\delta_{12}$  can be used as the practical criterion of static stability on the analogy of expression (10.5):

$$dP / d\delta_{12} > 0 \quad (10.12)$$

The utmost state at which stability is kept corresponds with the condition:

$$dP / d\delta_{12} = E_{q1} E_{q2} \cos(\delta_{12} - \alpha_{12}) / |\underline{Z}_{12}| = 0, \quad (10.13)$$

whence

$$\delta_{12,cr} = \pi/2 + \text{arctg}(x_{12} / R_{12}) \quad (10.14)$$

Substituting (10.14) into equation of the station power-angle curve (10.11) we receive the operation variable critical value:

$$P_{max} = E_{q1}^2 \sin \alpha_{11} / |\underline{Z}_{11}| + E_{q1} E_{q2} / |\underline{Z}_{12}| \quad (10.15)$$

Coefficient of static stability reserve, i.e. power assurance factor, is determined by the expression:  $k_{pqf} = (P_{max} - P_{ld}) / P_{ld}$

**Design circuit “equivalent source – node point of network”**

In complex PSS, loads connected to a node point can replace parts of the power system (Fig.10.8), and are given as their static or dynamic characteristics. The nodes are specified by the voltage vectors which play the role of equivalent EMF of implicit sources and loads, and they differ from explicit sources as they have not the inertia of generators and motor loads.

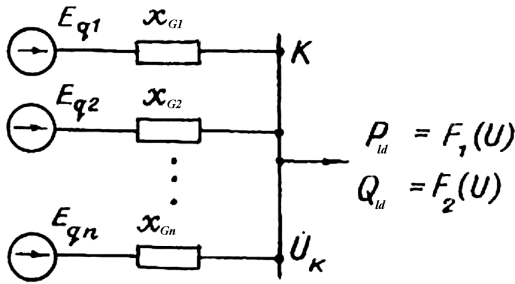


Fig. 10.8. Equivalent circuit of electric system relative node point

The practical criterion of operation static stability is chosen under the conditions of equivalent circuit (invariable frequency, balance of active power delivered by the generator branches ( $E; P = const$ )).

Analysis of the operation parameters set  $\{E, P, Q, U_K, \delta\}$  shows that the voltage in the node point  $U_K = \text{var}$  is essential independent variable which determines the condition of all elements of the system (variable  $\delta$  does not determine load condition in this case). Excess power in the system being the result of disturbance appears as change of the node reactive power balance (9.13):

$$\Delta Q = Q_{G,\Sigma} - Q_{ld}, \quad (10.16)$$

where  $Q_{G,\Sigma} = \sum_{i=1}^n Q_{G,i}$  is total reactive power generated in the node.

Relation of these essential variables under steady condition is determined with equation

$$E_{q,equiv} = \sqrt{(U_K^2 + Q_{G,\Sigma} x_{G,res})^2 + (P x_{G,res})^2} / U_K \quad (10.17)$$

where  $x_{G,res} = 1 / \sum_{i=1}^n 1 / x_{G,i}$ ;  $E_{q,equiv} = \sum_{i=1}^n E_{q,i} x_{G,i} / x_{G,res}$ ;  $P = \sum_{i=1}^n P_{G,i}$ .

The practical stability criterion for such an equivalent circuit can be, in accordance with (10.1), expressed as

$$d(Q_{G,\Sigma} - Q_{ld}) / dU_K < 0. \quad (10.18)$$

It characterizes reaction of the system for the perturbation causing change of the voltage. The perturbation appears as the node reactive power imbalance.

Analysis by this criterion consists in examining the steady state equations by the variable  $U_K$ :

$$\left. \begin{aligned} Q_{G,\Sigma} &= \left( -U_K^2 + \sqrt{E_{q,equiv}^2 U_K^2 - P^2 x_{G,res}^2} \right) / x_{G,res}; \\ Q_{ld} &= F_2(U_K) \end{aligned} \right\} \quad (10.19)$$

The equation of generated reactive power is received as the result of equation (10.17) transformation. The system of equations (10.19) is solved either analytically or graphically depending on the way of the load static characteristic representation. The solution is in accordance

with critical values of the essential parameters of operation  $U_{\kappa,cr}$  and  $d\Delta Q / dU_{\kappa} = 0$  (Fig.10.9). The derivative (10.18) sign is checked beginning from a certainly steady condition with its step-by-step charging on  $U_{\kappa}$  value. Curve of reactive power  $\Delta Q(U_{\kappa})$  imbalance is plotted on the basis of calculations by equations (10.16) and (10.19) to find the extreme point being in accordance with the critical voltage.

The factor of static stability assurance is determined through the indicators of steady and limiting operation:  $k_{vaf} = (U_{\kappa} - U_{\kappa,cr}) / U_{\kappa}$ .

**Design circuit of feeding the asynchronous load by a powerful system.** It is supposed, that the latter has unlimited power, and there is a node point with invariable voltage, or a point energized by equivalent source with invariable EMF. The PSS equivalent circuit is shown in Fig. 10.10, a where load is represented as the equivalent induction motor.

Let us analyze static stability of the asynchronous load feeding condition for the cases when the consumed active power is variable and constant.

If active load power is changed ( $P = var$ ), the equilibrium condition (the steady condition) is described by equation:

$$P_{WM} - P = 0 \tag{10.20}$$

or in expanded form:

$$\omega M_{WM} - U_{RS}^2 r_2 s / \left[ r_2^2 + (x_{equiv} + x_s)^2 s^2 \right] = 0. \tag{10.21}$$

The subtrahend in the left side of the equation is the active power consumed by induction motor (9.5), and the minuend is defined by braking torque of the working mechanism (9.8). Equation (10.21) has the only essential variable – slip  $s$ , and other parameters and values are constant under steady operation. Disturbances impact only on the active power balance in the point of load connection. Its violation can be revealed using the criterion

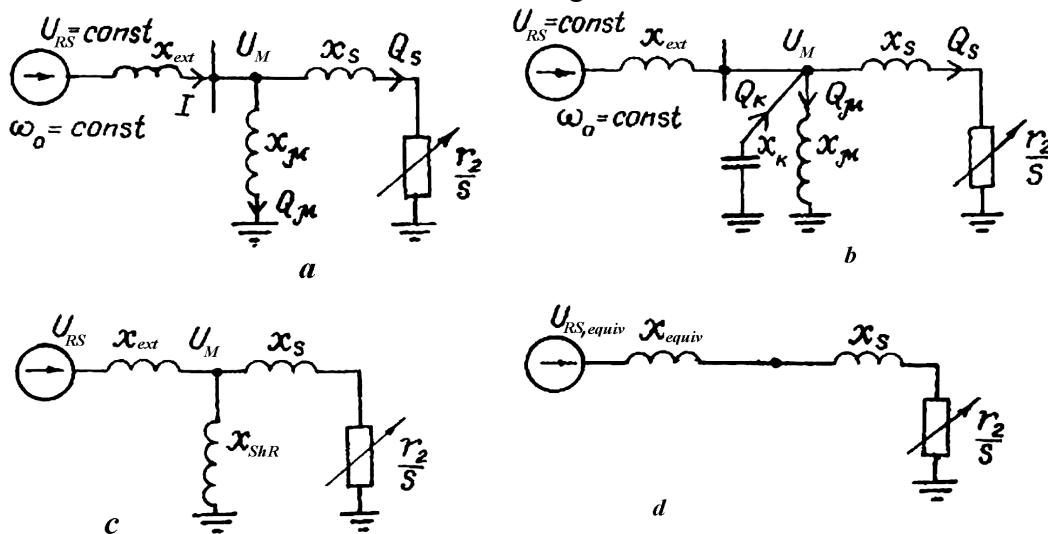


Fig. 10.10. Equivalent circuits for PSS with asynchronous load: a – without reactive power compensation; b – with reactive power compensation; c, d – transformations of equivalent circuits

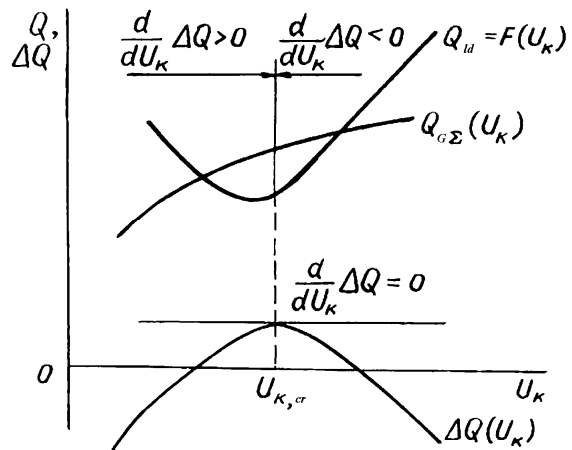


Fig. 10.9. Graphical solution of equations (12.16) and (12.19)



$$d(P_{WM} - P) / ds < 0 \quad (10.22)$$

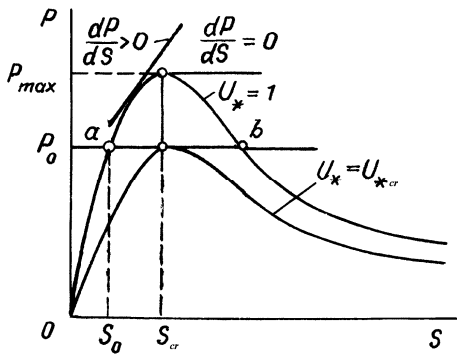


Fig. 10.11. Power of equivalent induction motor as function of slip

From here, under  $P_{WM}(s) = const$ , we have:

$$dP / ds > 0 \quad (10.23)$$

The limit of stability takes place under condition  $dP / ds = 0$  (Fig.10.11). Physically, the variable  $dP / ds$  shows the PSS reaction for the disturbance which appears as the slip of asynchronous load increase. If the derivative is positive, increase of the slip causes the electromagnetic power rise. It is taken from the network, and increases more rapidly in comparison with the working mechanism static breaking power. With it the power excess results in acceleration of the motor rotors, and working point returns to the slip value which corresponds to the initial condition.

From equilibrium equation (10.21) and the operation charging on the variable  $s$  it is possible, according to criterion (10.23), to determine critical values of changing parameters  $P, s$  for the condition that is limiting for stability (Fig. 10.11). When  $dP / ds = 0$  we have

$$s_{cr} = r_2 / (x_s + x_{equiv}) \quad (10.24)$$

Substituting  $s_{cr}$  into equation (9.5) we receive the greatest value of active power consumed by the load

$$P_{max} = U_{RS}^2 / [2(x_s + x_{equiv})] \quad (10.25)$$

which corresponds to the overturning torque of the equivalent induction motor. As the torque is proportional to the squared voltage on the terminals, it reduces when voltage drops. Voltage under which overturning torque becomes equal to the motor load is called critical voltage (Fig. 10.11). It is determined with the help of the expression

$$U_{RS,cr} = \sqrt{2mP_{rated}(x_s + x_{equiv})} \quad (10.26)$$

where  $P_{rated}$  is rated power of the equivalent motor; and  $m$  is its load factor.

Motors of the load nodes slow down if the voltage is less than critical one. The critical voltage value together with  $s_{cr}$  and  $P_{cr}$  characterize the limiting operation state by condition of induction motors stability. The more the critical voltage is, the less reserve of stability of motors is obtained.

Static stability reserve depends on the motors load  $m$ , their electrical remoteness of the invariable voltage busses, and of the reactive power fraction compensated in the point of load connection. The equivalent resistance  $x_{equiv}$  is determined by conditions of the given point with the buses of invariable voltage coupling. The reactance  $x_{equiv} = x_{ext}$  when  $x_{ext} \leq (0,1...0,15)x_s$ . If the external resistances have great values it is necessary to use the equivalent circuit (Fig.10.10, d), where

$$U_{RS,equiv} = U_{RS}x_\mu / (x_{ext} + x_\mu); \quad x_{equiv} = x_\mu x_{ext} / (x_\mu + x_{ext}) \quad (10.27)$$

Under connection of static compensating capacitors to the load node which power is  $Q_\kappa$  and resistance is  $x_\kappa = U^2 / Q_\kappa$  (Fig. 10.10, b), at first the resistance of equivalent shunt (Fig. 10.10, c) is determined by the formula

$$x_{ShR} = -x_\mu x_\kappa / (x_\mu - x_\kappa),$$

and then transition to equivalent circuit (Fig. 10.10, d) is made by (10.27) if  $x_{ShR} \equiv x_\mu$ .

Voltage in the load connection point is not independent variable as it is determined by the load node operation conditions:

$$U_M = I \sqrt{x_s^2 + (r_2/s)^2}, \quad I = U_{RS} / \sqrt{(x_{equiv} + x_s)^2 + (r_2/s)^2}$$

whence

$$U_M = U_{RS} \sqrt{(x_s^2 s + r_2^2) / [(x_{equiv} + x_s)^2 s^2 + r_2^2]} \quad (10.28)$$

In this case the values of the limiting operation indicators can not be calculated with use of the voltage found from (10.28). They can be determined only using the voltage  $U_{RS}$  which does not depend on operation mode changes by definition.

When the consumed active power is constant ( $P = const$ ) the excess energy can be estimated using balance of the reactive power:

$$Q = Q_\mu + Q_s \quad (10.29)$$

The components of (10.29) are described by equation (9.7), where system voltage  $U_{RS}$  is an independent operation variable:

$$Q_\mu = U_{RS}^2 / x_\mu \quad \text{under } \alpha = 0 \quad (10.30)$$

$$Q_s = U_{RS}^2 (x_{equiv} + x_s) / [(x_{equiv} + x_s)^2 + r_2^2 / s^2] \quad (10.31)$$

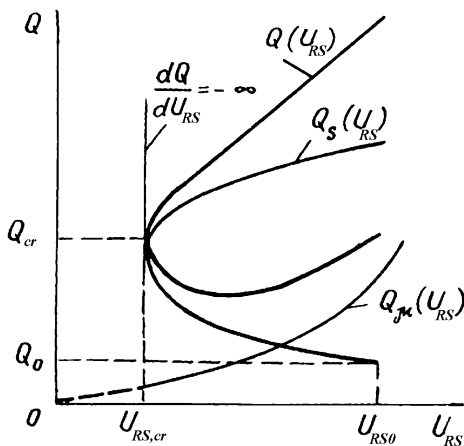


Fig. 10.12. Dependences of reactive power against the system voltage

Critical parameters of the system are determined on the basis of expression (10.29) analysis which is done in the following order:

the current values corresponding to invariable load  $P_0 = const$  are determined with formula

$$I_i = \sqrt{P_0 / (r_2/s_i)},$$

using the obtained current values, the series of the voltage values is determined as

$$U_{RS,i} = I_i \cdot \sqrt{(x_{equiv} + x_s)^2 + (r_2/s)^2},$$

the components of  $Q$  (10.29) are determined using the found voltage values.

Graphs of the reactive power  $Q(U_{RS})$  and its components  $Q_\mu$  and  $Q_s$  (Fig.10.12) show that marginal operation with critical parameters  $U_{RS,cr}$ ,  $Q_{cr}$  corresponds to the criterion  $dQ/dU_{RS} = -\infty$ .

**Circuit with an equivalent source energizing complex load of commensurable power (Fig.10.13, a)**

In the point of load connection, the equilibrium condition is characterized with changing values of operation parameters  $U$ ;  $P_{ld} = F_1(U)$ ;  $Q_{ld} = F_2(U)$ . In this circuit, the voltage in the load connection point is an essential free variable. It indicates the state of all system elements. The operation indices  $E_{equiv}$  and  $Q_{equiv}$  indicate indirectly to availability of the excess energy in the system being the result of disturbance.

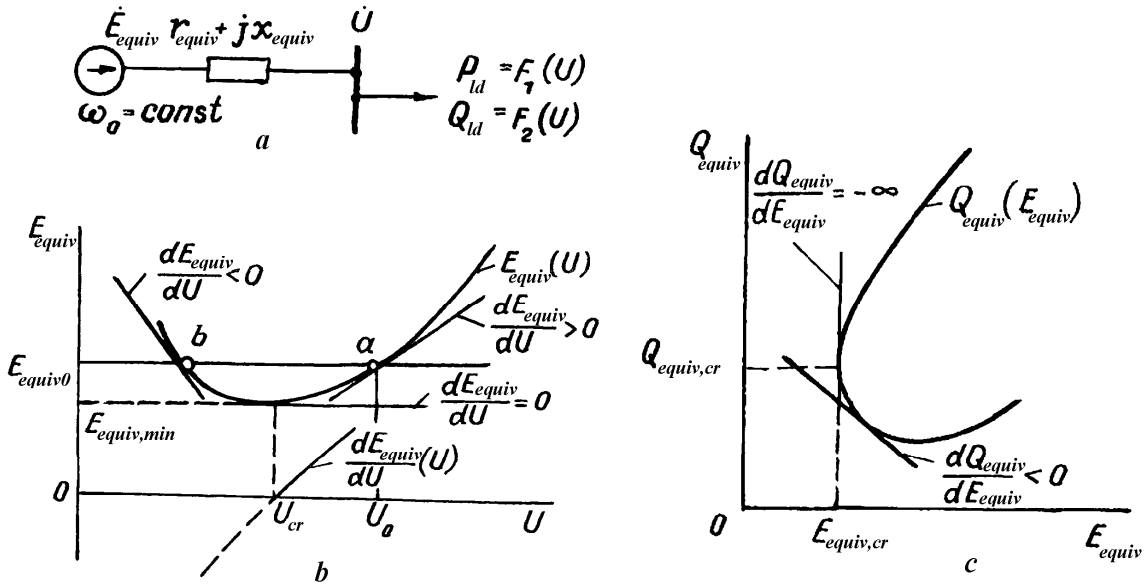


Fig. 10.13. EPS circuit with complex load: a – equivalent circuit; b,c – dependences of essential variables

Graphical analysis of dependence

$$E_{equiv} = \sqrt{\left( U^2 + P_{ld}r_{equiv} + Q_{ld}x_{equiv} \right)^2 + \left( P_{ld}x_{equiv} - Q_{ld}r_{equiv} \right)^2} / U \quad (10.32)$$

is made with operation charging on variable  $U$ . The analysis shows that there is minimum at  $dE_{equiv}/dU = 0$  with coordinates  $E_{equiv,cr}$ ,  $U_{cr}$  corresponding to marginal condition (Fig. 10.13, b, c).

Parallel between physical conditions for the complex and asynchronous load stability described with Fig.10.11 can be drawn. Under  $E_{equiv} = E_{equiv,0}$  the steady condition equilibrium is possible in the points  $a$  and  $b$  (Fig.10.13, b) similarly to the operation equilibrium points in Fig.10.11. In the point  $a$ , inequality  $dE_{equiv}/dU > 0$  corresponds to stable equilibrium in the power-angle curve. With charging on variable  $U$ , a growing slip value corresponds to every point of the curve  $E_{equiv}(U)$  (Fig.10.11). The critical voltage takes place at the critical slip value. The point  $b$ , in which the derivative is  $dE_{equiv}/dU < 0$ , corresponds to unstable steady operation equilibrium in accordance with the power-angle curve.

Thus, sign of the derivative  $dE_{equiv} / dU$  makes possible to judge of the system stability. The criterion of stability

$$dE / dU > 0 \quad (10.33)$$

was first introduced by Prof. P.S. Zhdanov, and is called Zdanov's stability criterion.

If we evaluate coordinates of the marginal operation state when stability is not yet broken using (10.32) and dependence

$$Q_{equiv} = Q_{ld} + (P_{ld}^2 + Q_{ld}^2) x_{equiv} / U_{ld}^2 \quad (10.34)$$

they correspond to extreme point  $(E_{equiv,cr}, Q_{equiv})$  (Fig.10.12,c) for which

$$dQ_{equiv} / dE_{equiv} = -\infty. \quad (10.35)$$

When stability is checked on (10.35), the operation charging on variable  $E_{equiv}$  is performed.

The curve  $Q_{equiv}(E_{equiv})$  region, where the condition

$$dQ_{equiv} / dE_{equiv} < 0. \quad (10.36)$$

is fulfilled, corresponds to the states in which stability is preserved.

Factors of static stability assurance are determined by the expressions

$$k_{vaf} = (U_0 - U_{cr}) / U_0, \quad k_{EMFaf} = (E_{equiv} - E_{equiv,cr}) / E_{equiv,0}$$

Thus, static stability of PSS steady operation can be evaluated by practical criteria only for a specific electric power supply system, under assumptions indicated above. The analysis is performed in the following steps:

- the PSS equivalent circuit is composed;
- the array of the varying operation parameters is determined, and main assumptions are specified;
- essential independent variable, which determines the PSS elements operation mode is specified in the variables array;
- essential variables that permit to assess indirectly the excess power under disturbance in the system are specified;
- interrelation of the essential variables for steady operation is determined;
- using the practical criteria, coordinates of the limiting operation at which static stability is preserved are determined (choose of essential variables defines the trajectory of charging);

The stability margin is evaluated and compared with the critical value.

### 10.3. Analysis of static stability by method of small oscillations

Under steady operation of PSS, values of its operation parameters are influenced by different factors (load mainly) change around the equilibrium state. When operation parameters get increments which are incommensurable small in comparison with these parameters steady values, such effects are considered as small perturbations.

If PSS operation is described with low order equations, evaluation of static stability can be made by the method of small oscillations. In contrast to static stability evaluation on practical criteria, the technique is based on study of the system motion equations written as equations for minor deviations.

Assume that PSS operation at any instant is described by a non-linear differential equation of perturbed motion that is given by

$$Y\left(t, y_i, \frac{d^k y_i}{dt^k}\right) = F\left(f_i, \frac{d^l f_i}{dt^l}\right); \quad i = \overline{1, J}; \quad k = \overline{1, K}; \quad l = \overline{1, L}, \quad (10.37)$$

where  $y_i$  is a set of PSS operation parameters which are functions of time;  $f_i$  are external actions which variation can be arbitrary. Under equilibrium, the PSS operation is characterized with the help of steady values ( $y_{i,0}$ ) of these parameters:

$$Y_0(t, y_{i,0}, 0) = F_0(f_{i,0}, 0) \quad (10.38)$$

If  $F_0(f_{i,0}, 0) = 0$ , perturbations are temporary (suppose they terminate under  $t = t_0$ ). Then, equation (10.38) describes unperturbed motion. Left side of equation (10.37) can be expressed by small deviations  $x_i = y_i - y_{i,0}$  of operation parameters  $y_i$  relative to their steady values  $y_{i,0}$  as

$$Y\left(t, y_{i,0} + x_i, \frac{d^k (y_i + x_i)}{dt^k}\right) = F\left(f_i, \frac{d^l f_i}{dt^l}\right); \quad i = \overline{1, J}; \quad k = \overline{1, K}; \quad l = \overline{1, L}. \quad (10.39)$$

Components containing  $x_i$  characterize the transient  $X(t)$  caused by the perturbations. Initial values for transient under  $t = t_0$  are

$$x_i(t_0) = x_{i,0} = y_i(t_0) - y_{i,0}(t_0) \quad (10.40)$$

If motion is unperturbed then  $x_i \equiv 0$ . Stability evaluation on the basis of system of equations in the form of (10.39) is substantiated with help of Lyapunov's theorems. Unperturbed motion is stable relatively indices  $y_i(t)$  if for any positive number  $\varepsilon$ , despite its minority, another positive number  $\eta(\varepsilon)$  can be chosen so that for any initial conditions  $x_i(t_0)$ , satisfying the inequalities  $|x_i(t_0)| \leq \eta$ , the inequalities  $|x_i(t)| < \varepsilon$  are true when  $t > t_0$ .

If unperturbed motion is stable and the condition  $\lim_{t \rightarrow \infty} |x_i(t)| = 0$  is additionally satisfied, it is referred as asymptotically stable.

When engineering problems are solved, it is possible to use physical interpretation of stability definitions given above regarding nature of the transient  $X(t)$  by its peak value. In the case of dead-beat stability the transient is damping in its peak value. If oscillation process takes place, the stability will be available under invariable peak value of oscillations, and negative stability will take place if peak value of oscillations increases.

Character of transient can also be found with the help of analysis of non-linear differential equations set (10.39). It is linearized by the first A.M. Lyapunov's approximation method, and then the characteristic equation roots of the linearized equations set are analyzed. Linearization is based

on assumption of such change of the variables during the transient under which their deviations from steady values stay to be rather minor all the time.

Linearization of the system equations (10.39) is performed by means of the equations left-side expansion into power series (Taylor's or Moclairin's) in powers of minor deviations  $x_i$  (all derivatives of  $x_i$  are considered as independent variables):

$$Y_0(t, y_{i,0}) + (\partial Y / \partial y_i)_{y_{i,0}} x_i + \sum_{s=1}^{s=n} \left[ \partial Y / \partial (d^s y_i / dt^s) \right]_{y_{i,0}} \times \\ \times (d^s x_i / dt^s) + s_{x_i} = F(f_i, d^l f_i / dt^l). \quad (10.41)$$

Here

$$s_{x_i} = \sum_{k=2}^{k=K} \sum_{s=1}^{s=n} \left[ \partial Y / \partial (d^s y_i / dt^s) \right]_{y_{i,0}} \times \\ \times (d^s y_i / dt^s) + \sum_{k=2}^{k=K} (\partial^k Y / \partial y_i^k)_{y_{i,0}} \quad (10.42)$$

is sum of the terms consisting of products of function  $Y$  partial derivatives (of the second and higher order derivatives), and minor deviations as well as their derivatives  $(\partial Y / \partial y_i)_{y_{i,0}} = a_{i,n}$ ;  $\left[ \partial Y / \partial (d^s y_i / dt^s) \right]_{y_{i,0}} = a_{i,s}$  are coefficients at variables  $x_i$  and  $d^s y_i / dt^s$  (their values are computed using expressions of the partial derivatives of function  $Y$  with respect to  $y_i$  in the points  $y_{i,0}$ ).

If equilibrium equations (10.38) are subtracted from equations (10.41), and the sum of terms of power series  $s_{x_i}$  is neglected, the linearized equations of perturbed motion (first approximation equations) are

$$\sum_{s=1}^{s=n} \left[ \partial Y / \partial (d^s y_i / dt^s) \right]_{y_{i,0}} \cdot (d^s x_i / dt^s) + (\partial Y / \partial y_i)_{y_{i,0}} x_i = \\ = F(f_i, d^l f_i / dt^l) - F_0(f_i, 0).$$

Reduce them to more compact form with the use of the assumed notation of the coefficients  $a_{i,s}$ , and of the differentiation operator  $p = d / dt$ :

$$\sum_{s=0}^{s=n} a_{i,s} p^{n-s} x_i(t) = F(f_i, d^l f_i / dt^l) - F_0(f_i, 0). \quad (10.43)$$

Stability of non-linear system (10.41) is estimated by transient damping which availability is determined according to form of the system characteristic equation roots (10.43)

$$D(p) = a_0 p^n + a_1 p^{n-1} + a_2 p^{n-2} + \dots + a_{n-1} p + a_n = 0. \quad (10.44)$$

It is necessary and sufficient for stability if characteristic equation roots have negative real parts. Under null real part of roots, extra study of the suppressed terms of equation (10.42) should be made. Change of sign of a root real part, or a zero root, or a pair of purely imaginary roots availability as well as an infinite root, when other roots have negative real part, defines the stability limit.

The root negative real part availability can be revealed by direct solution of equation (10.44) if it is not higher than of the fourth power. Mathematical stability criteria which do not require evaluation of the characteristic equation roots are used to study equations of higher order.

The applied stability criteria are divided into algebraic (by Hurwitz, by Ljenare-Shepare, and by Routh), and frequency ones (by Mikhailov, by Nyquist, and logarithmic). Method of D-division is used as well. With it, interrelation between negative sign of characteristic equation roots real part

and its coefficients ( $a_s, s = \overline{0, n}$ ) is analyzed. The necessary but not sufficient indicator of unperturbed movement stability is availability of the same sign of the characteristic equation coefficients (usually, positive sign of coefficients is specified as when negative sign is available it can be easily changed by means of multiplication by minus one). This necessary stability index is too sufficient one for equations of the first and the second order.

If not every coefficient of characteristic equation has the same sign, unperturbed motion is not stable, and there is no necessity to perform extra study of stability.

Hurwitz criterion determines the system stability by characteristic equation (10.44).

For that purpose, a determinant consisting of  $n$  rows and columns is formed:

$$\Delta_n = \begin{vmatrix} a_1 & a_3 & a_5 & \dots & 0 & 0 \\ a_0 & a_2 & a_4 & \dots & 0 & 0 \\ 0 & a_1 & a_3 & \dots & 0 & 0 \\ 0 & a_0 & a_2 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & a_{n-1} & 0 \\ 0 & 0 & 0 & \dots & a_{n-2} & a_n \end{vmatrix}, \quad (10.45)$$

where the coefficients  $a_1$  up to  $a_n$  are recorded on the main diagonal in increasing numbering of indices, and the rows are completed with coefficients so as the rows with odd and even (including  $a_0$ ) numbers alternate and the coefficient number in each the row increases from left to right. The missing coefficients having index numbers less than zero and more than  $n$  are replaced by zeros.

For the stable initial state of equilibrium by Hurwitz, the following conditions should be met:

$$a_0 > 0; \quad \Delta_s > 0, \quad s = \overline{1, n}. \quad (10.46)$$

Here the diagonal minors are extracted from the principal Hurwitz determinant as

$$\left. \begin{aligned} \Delta_1 = a_1 > 0; \quad \Delta_2 = \begin{vmatrix} a_1 & a_3 \\ a_0 & a_2 \end{vmatrix} > 0; \\ \Delta_3 = \begin{vmatrix} a_1 & a_3 & a_5 \\ a_0 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{vmatrix} > 0; \quad \dots \end{aligned} \right\} \quad (10.47)$$

Stability limit of the system is determined with the help of equation:

$$\Delta_n = 0 \quad \Delta_s > 0 \quad s = 1, 2, \dots, n-1 \quad (10.48)$$

As the last column of the determinant (10.45) has only one coefficient which is not equal to zero then

$$\Delta_n = a_n \Delta_{n-1} \quad (10.49)$$

With it, the equation (10.48) resolves into two equalities:  $a_n = 0$  and  $\Delta_{n-1} = 0$ .

The first equality determines the first type stability limit – the non-periodic stability limit, and the second one - the second type stability limit – the oscillating stability limit.

The analytical stability conditions (10.47) are clumsy for equations being higher than the fourth order, and it stipulates practical complexity of their use for analysis. That complicates definition the system movement indicators contribution in the operation charging and stability limit formation.

For the stable system, in accordance with the Ljénare-Shepare criterion, it is required that the following two conditions should be met:

1) coefficients of characteristic equation (10.44) should be positive, that is  $a_s > 0$ , where  $s = 1, 2, \dots, n-1$ ,

2) diagonal minors  $(n-1)$  and  $(n-3)$  of determinant (10.45) should be positive, that is

$$\Delta_{n-1} > 0; \quad \Delta_{n-3} > 0 \quad (10.50)$$

The Routh's criterion for the system stability also stipulates the two conditions:

1) coefficients of characteristic equation (10.44) should be positive;

2) coefficients of the Routh's table first column (Table 10.1) which includes  $(n+1)$  row should be positive.

To the first row of Table 10.1 the even-numbered coefficients of the characteristic equation (including  $a_0$ ) are recorded in increasing order and to the second row – the odd-numbered coefficients. The coefficients recorded to the following rows of the table are calculated on the formula

$$c_{k,i} = c_{k+1,i-2} - \lambda_{I-2} \cdot c_{k+1,i-1}, \quad i \geq 3, \quad (10.51)$$

where  $\lambda_{I-2} = c_{1,i-2}/c_{1,i-1}$ .

The coefficients in Table 10.1 have been obtained by the Hurwitz determinant (10.45) transformation using determinants property according to which the value of determinant will not change if coefficients from either row multiplied by the same number will be added to coefficients of its another row.



Table 10.1  
 Routh's table

Row number ( $i$ )	Column number ( $k$ )					Coefficient $\lambda_{i-2}$
	1	2	3	4	...	
1	$c_{11} = a_0$	$c_{21} = a_2$	$c_{31} = a_3$	$c_{41} = a_6$	...	-
2	$c_{12} = a_1$	$c_{22} = a_3$	$c_{32} = a_5$	$c_{42} = a_7$	...	-
3	$c_{13} = a_2 - \lambda_1 a_3$	$c_{23} = a_4 - \lambda_1 a_5$	$c_{33} = a_6 - \lambda_1 a_7$	$c_{43} = a_8 - \lambda_1 a_9$	...	$\lambda_1 = a_0/a_0$
4	$c_{14} = a_3 - \lambda_2 c_{23}$	$c_{24} = a_5 - \lambda_2 c_{33}$	$c_{34} = a_7 - \lambda_2 c_{43}$	$c_{44} = a_9 - \lambda_2 c_{53}$	...	$\lambda_2 = a_1/c_{13}$
...	...	...	...	...	...	...

Algorithmic form of Routh's criterion is convenient for study of systems stability with the help of computers.

Mikhailov's criterion, being used to study the system stability conditions, gives their evident geometric interpretation. Using the roots  $p_1, p_2, \dots, p_n$  of characteristic equation (10.44) the latter can be presented as

$$D(p) = a_0 (p - p_1)(p - p_2) \dots (p - p_n) \quad (10.52)$$

When the operator  $P$  is replaced with imaginary unit multiplied by angular velocity of oscillations  $j\omega$ , equation (10.52) can be written down as the product of complex factors

$$D(p) = a_0 (j\omega - p_1)(j\omega - p_2) \dots (j\omega - p_n) \quad (10.53)$$

or

$$D(j\omega) = |\operatorname{Re} D(j\omega) + j \operatorname{Im} D(j\omega)| \cdot \exp(j\varphi(\omega)) \quad (10.54)$$

Expression (10.54) under fixed angular velocity describes characteristic radius-vector in complex plane by its polar coordinates with the modulus  $|D(j\omega)|$  and the argument  $\varphi(\omega)$ . Coordinates of the radius-vector end in the real axis is

$$\operatorname{Re} D(j\omega) = a_n - a_{n-2}\omega^2 + a_{n-4}\omega^4 - \dots \quad (10.55)$$

and in imaginary axis is

$$\operatorname{Im} D(j\omega) = a_{n-1}\omega - a_{n-3}\omega^3 + a_{n-5}\omega^5 - \dots \quad (10.56)$$

If we prescribe a series of growing angular velocity values, the the radius-vector end movement (10.54) describe a curve called the Mikhailov's curve.

Graphical analysis of equation (10.54) in the complex plane with respect to the independent variables  $|D(j\omega)|$  and  $\varphi(\omega)$  shows that characteristic radius-vector at the angular velocity, changing within  $0$  to  $+\infty$ , varies in module and in direction. Change of the radius-vector direction is connected with the sign of the roots real part  $p_1, p_2, \dots, p_n$  as follows from the characteristic equation representation as product of complex factors (10.53) which arguments (the angles of

rotation  $\varphi_1, \varphi_2, \dots, \varphi_n$ ) are added in algebraic way. Every of the cofactors contains characteristic equation root, and when angular velocity changes from zero to plus infinity it corresponds (if the root real part is negative) to turn of the radius-vector by the angle of  $+\pi/2$ , and if its real part is positive the vector rotates by  $-\pi/2$ .

For the  $n$  order characteristic equation, its  $m$  roots having positive real part comply the radius-vector turn by the angle of  $-m\pi/2$ , and  $n-m$  roots having negative real part –turn by the angle of  $(n-m)\pi/2$ , and the resulting change of the radius-vector direction will be

$$-m\pi/2 + (n-m)\pi/2 = n\pi/2 - m\pi \tag{10.57}$$

Expression (10.57) illustrates the stability criterion of  $n$ -order linear system equations formulated by A.V. Mikhailov: if the characteristic radius-vector has resulting turn angle  $+\pi n/2$  under subsequent change of the angular velocity within  $0$  to  $+\infty$ , the equations system solution is stable.

The statements following from the graphic analysis of the system stability conditions according to the Mikhailoff's criterion (Fig.10.14, a) are:

- the initial point of Mikhailov's curve is located on the real axis as at  $\omega=0$  the equality  $\text{Re}D(0) + j\text{Im}D(0) = \text{Re}D(0)$ ; takes place;
- the Mikhailoff's curve passes subsequently counter-clockwise all quadrants of complex plane, and tends to infinity in quadrant corresponding to order of equations system (curve transit through origin of coordinates needs extra study for stability);
- zeroes of expressions

$$\text{Re}D(j\omega) = 0; \text{Im}D(j\omega) = 0. \tag{10.58, a}$$

should alternate.

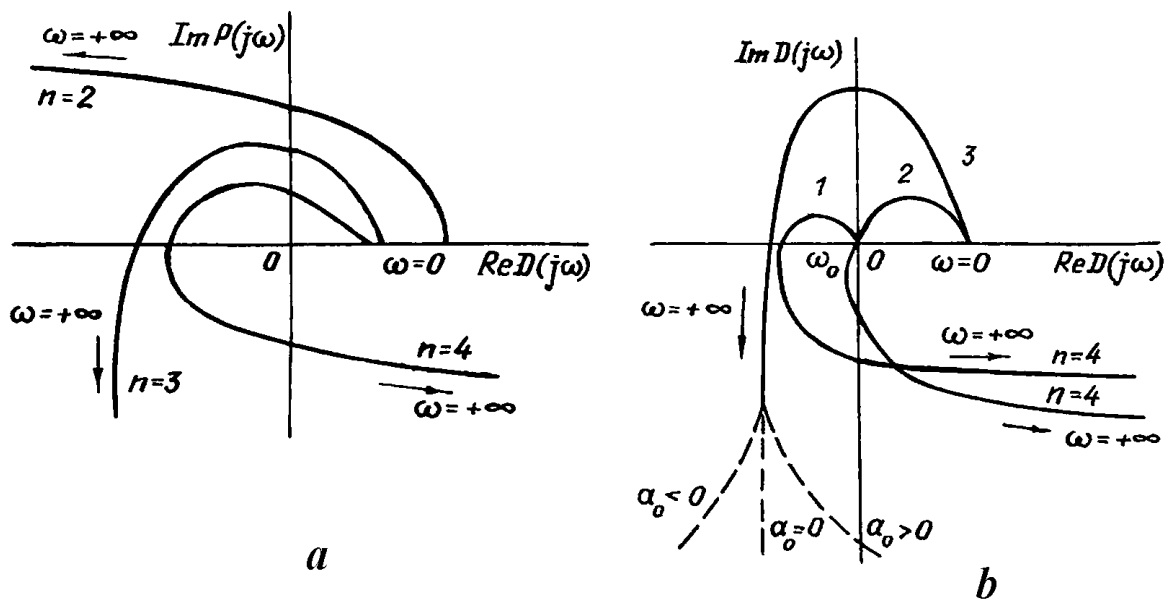


Fig. 10.14. The Mikhailov's curves: a – for stable solutions of  $n$ -order equations set; b – trace of the curve for stability limits of the first (1), the second (2) and the third (3) type

From the above statements, other formulation of the Mikhailov's stability criterion follows: to have stable solution of the equations set it is necessary that of zeroes of the characteristic radius-vector real and imaginary parts alternated (10.58, a), be real, and under  $\omega = 0$

$$\operatorname{Re} D(j\omega) > 0; \quad d(\operatorname{Im} D(j\omega))/d\omega > 0. \quad (10.58, b)$$

Proceeding from the Mikhailoff's curve view, the limits of all three types of stability can be determined. Under  $a_n = 0$ , that corresponds to non-periodic stability limit, the Mikhailov's curve starts from the origin of coordinates. That means that zero root is available (Fig.10.14, b, curve 1).

The characteristic equation for the limit of oscillatory stability has the appearance

$$D(j\omega_0) = \operatorname{Re} D(j\omega_0) + j \operatorname{Im} D(j\omega_0) = 0.$$

This implies the equations:  $\operatorname{Re} D(j\omega_0) = 0; \operatorname{Im} D(j\omega_0) = 0$ , and under angular velocity of continuous oscillations the Mikhailov's curve passes through the origin of coordinates (Fig. 10.14,6, curve 2).

Availability of an infinite root of the characteristic equation corresponds to stability limit of the third type. With it, the Mikhailov's curve changes its trajectory depending on change of sign of the coefficient  $a_0$  from plus to minus as it is seen in Fig. 10.14,6 (curve 3).

Method of  $D$ -partition helps to determine stability area in the plane of essential variables of the system which are functionally connected with coefficients of characteristic equation (10.44). In the study, the latter are considered as variables to evaluate their influence on the stability region formation.

The method is based on the fact that roots of the characteristic equation drift along trajectories on the complex plane of roots (in general the roots are represented as complex numbers) when the equation coefficients vary. Sign change of a root real part means that its trajectory intersects the imaginary axis of the roots plane. Coordinates of intersection points form so called  $D$ -partition limit of the characteristic equation coefficients (technical system parameters) space. In these points characteristic equation has roots on imaginary axis of complex plane. A closed limit of  $D$ -partition separates areas with different number of roots having real part. Among these areas, the stability area is determined using the previously described criteria by means of check for arbitrarily chosen points of the area (in it all the roots have negative real part).

To determine the stability area limits, the three attributes of all their existing types are used:  $a_n = 0$  for the first type;  $\Delta_{n-1} = 0$  from the Hurwitz criterion, or  $D(j\omega_0) = 0$  from the Mikhailov's criterion for the second type; and  $a_0 = 0$  for the third type. The limits of  $D$ -partition separate areas with different number of roots having negative real part. It is indicated with the help of proper cross-hatching. Within area to which cross-hatches are directed, the number of roots having negative real part is more in comparison with contiguous area as intersection of the area limit corresponds to the root trajectory across the imaginary axis of the roots plane transition.

For the common case of the stability area limit in the plane of two indices  $A$  and  $B$  determination, such a sequence is recommended:

- 1) Characteristic equation (10.44) is presented in the form of (10.54)
- 2) Equating the real and imaginary parts of equation (10.54) to zero, it is obtained

$$\operatorname{Re} D(j\omega, A, B) = 0; \quad (10.59)$$

$$\operatorname{Im} D(j\omega, A, B) = 0 \quad (10.60)$$

3) After simultaneous solution of equations (10.59) and (10.60) the parametric equations are found:

$$A(\omega) = \Delta_A(\omega) / \Delta(\omega); \quad (10.61)$$

$$B(\omega) = \Delta_B(\omega) / \Delta(\omega) \quad (10.62)$$

The latter equations define coordinates of the points  $A$  and  $B$ , and the limits of D-partition for values of  $\omega$  within  $-\infty$  to  $+\infty$ . In (10.62) and (10.62)  $\Delta$ ,  $\Delta_A$ ,  $\Delta_B$  are accordingly the main and auxiliary determinants of the system equations (10.59) and (10.60).

4) At  $\Delta(\omega_k) = 0$ , the D-partition limit are special straight lines, which equations are obtained for the straight lines matching  $\omega = +\infty$  - from equality  $a_0(A, B) = 0$ , and for the straight line matching  $\omega = 0$  - from equality  $a_n(A, B) = 0$  for  $\Delta(\omega_k) = \Delta_A(\omega_k) = \Delta_B(\omega_k) = 0$  by substitution of  $\omega_k$  into equations(10.59) and (10.60).

5) D-partition limit is cross-hatched in the direction of  $\omega$  increase from the left if  $\Delta > 0$ , and from the right if  $\Delta < 0$ .

6) Special straight lines are cross-hatched in a way that makes them directed against each other in the point of intersection (when tangency the region limit takes place) only with cross-hatched or uncross-hatched sides.

7) Stability area is selected by its checking for stability in an arbitrary point  $(A_0, B_0)$  with the help of any criterion using the characteristic equation  $D(j\omega, A_0, B_0) = 0$ . If the given point corresponds to the stable condition, the studied area is the area of stability.

#### 10.4. Taking into account of automatic excitation control

The generator power against power-angle curve without automatic excitation control under  $r = 0$ , and constant EMF  $E_{q,0}$  is defined by the expression

$$P = E_{q,0} U \sin \delta / (x_d + x_{ext}). \quad (10.63)$$

When power delivered to the network increases comparably with the initial mode ( $I_1 > I_0$ ), the angle  $\delta$  increases simultaneously. Then the vector of EMF  $\dot{E}_{q,0}$  changes its direction when  $|\underline{E}_{q,1}| = |\underline{E}_{q,0}|$  (Fig.10.15,a). At the same time, the voltage on the generator terminals decreases ( $|\underline{U}_{G,1}| < |\underline{U}_{G,0}|$ ). The power limit in (10.63) decreases at  $\delta = \pi/2$  (Fig.10.15, b). If generator has an automatic excitation control device, increase in power and concerned with it increase of the angle  $\delta$  cause change of the generator EMF magnitude. It corresponds to transition from the power-angle curve drawn up for constant EMF modulo to the curve that corresponds to the EMF  $\underline{E}_{q,1} > \underline{E}_{q,0}$  (Fig.10.15, b). With increase of the angle  $\delta$ , the generator EMF rises. As it is seen from the power-angle curve, the powers limit increases and shifts in the range of  $\delta > \pi/2$ . Thus, automatic excitation control influences the limit of power delivered to the network, and hence the stability of PSS operation.

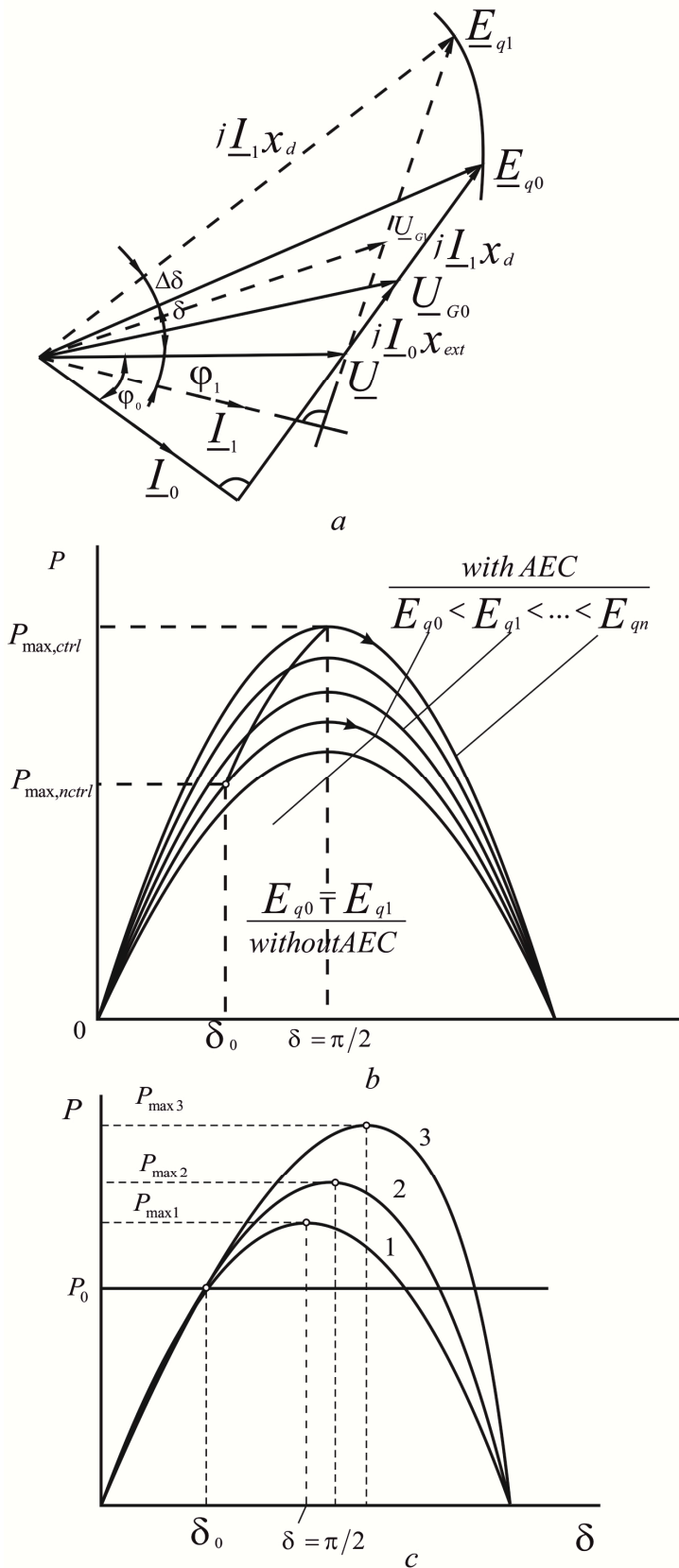


Fig. 10.15. To consideration of ALT influence

$$(T_J / \omega_0) \cdot d^2 \delta / dt^2 = P_0 - E_q U \sin \delta / x_d, \tag{10.64}$$

It is necessary to take into account the type of automatic excitation control devices while calculating stability. There are automatic devices of proportional type, and controllers of intense action. The devices of the first type respond to deviation of one or several operation parameters as for values under control, and the latter respond to rate of velocity and acceleration of parameter change. Introduction of derivatives of operation parameters variation into control action not only stabilizes the control system but considerably increases the power limit (stability limit). Automatic control devices of proportional type guarantee operation stability over lesser range of the angle change and the power transmitted (curve 2 in Fig.10.15, c) to compare with the automatic controllers of intense action (curve 3); curve 1 characterizes the power-angle curve without an automatic excitation control device.

Consider the way of taking into account the generator automatic excitation control when PSS stability is calculated by means of the refined method and the practical criteria. Evaluation of PSS stability limits for the cases of the generator with and without automatic excitation control can be deduced to evaluation of the limiting power delivered to the network by generator.

Analyze static stability of operation of the simplest electric system with a generator having automatic controller of proportional type responding to the voltage deviation, and working for the buses with constant voltage (Fig.10.16, a). Transient is described by means of the set of equations that includes the equation of relative motion of the generator rotor

the equation of transient within the generator field winding circuit with independent excitation

$$T_{d0} dE'_q / dt + E_q = E_{q,e}, \tag{10.65}$$

and the equation of transient within the exciter

$$T_e \cdot d E_{q,e} / dt + E_{q,e} = U_{ROV}. \tag{10.66}$$

Quantities entering in the equations (10.64) – (10.66) mean:  $T_{d,0}$  – time constant of the generator field winding when the stator winding is open;  $E'_q$  and  $E_q$  – transient and synchronous direct-axis EMF;  $E_{q,e}$  – EMF under steady operation (it is equal to voltage of exciter in per units);  $T_e$  – the time constant of the exciter field winding;  $U_{ROV}$  – the steady value of the exciter field winding voltage (it equals in per units to the regulator output voltage).

The equation of an ideal proportional type automatic voltage regulator which instantly changes voltage on the exciter field winding proportionally to the voltage deviation on the generator terminals:

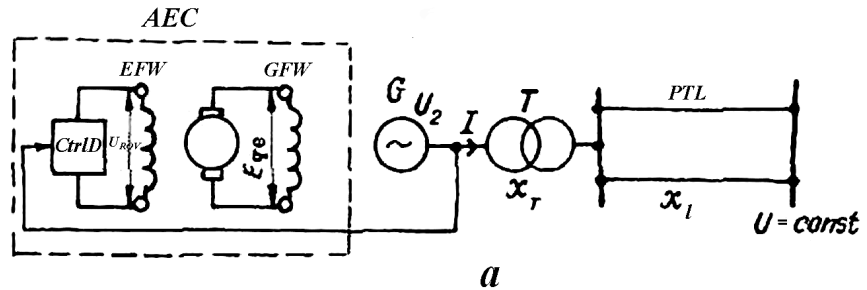


Fig. 10.16a Analysis of operation stability for generator with proportional type automatic excitation control

$$U_{ROV} - U_{ROV,0} = -K_U (U_G - U_{G,0}), \tag{10.67}$$

where  $K_U$  is the regulator gain factor.

Set of equations (10.64)-(10.67) contains six variables:  $\delta, E_q, E'_q, E_{q,e}, U_{ROV}, U_G$ . Extra equations connecting them can be obtained from the vector diagram of generator (Fig.10.16, 6):

$$E_q = U \cos \delta + I_d X_d, \tag{10.68}$$

$$E'_q = U \cos \delta + I_d X'_d, \tag{10.69, a}$$

$$U_G \approx U_{G,q} = U \cos \delta + I_d X_{ext}, \tag{10.70, a}$$

where  $X_{ext} = x_T + 0,5x_L$ ;  $X_d = x_d + X_{ext}$ ;  $X'_d = x'_d + X_{ext}$ .

Determining the current  $I_d$  from (10.68), and substituting it into equations (10.69, a) and (10.70, a) write down the missing equations:

$$E'_q = U \cos \delta \cdot (X_d - X'_d) / X_d + E_q X'_d / X_d; \quad (10.69, b)$$

$$U_G = U \cos \delta \cdot (X_d - X_{ext}) / X_d + E_q X_{ext} / X_d. \quad (10.70, b)$$

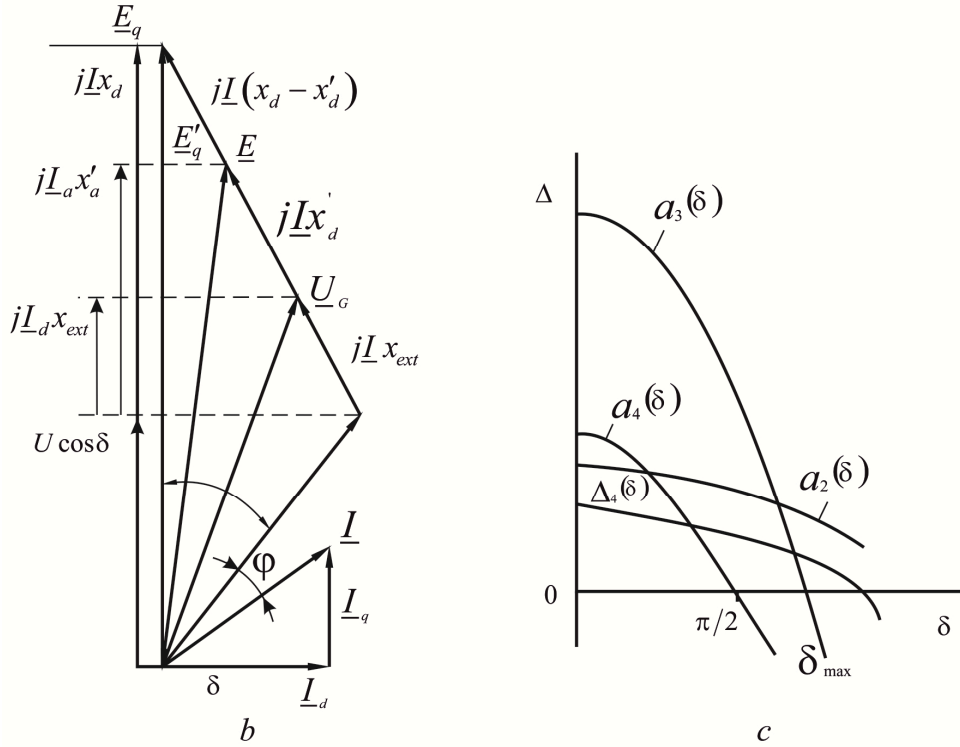


Fig. 10.16(b, c) Analysis of operation stability for generator with proportional type automatic excitation control

Present the set of equations (10.64)-(10.67), (10.69, b) and (10.70, b) by means of corresponding linearized equations based on the minor increments. In operator form they look as:

$$\Delta \delta \cdot (T_J / \omega_0) p^2 = \Delta \delta \cdot \partial P / \partial \delta - \Delta E_q \cdot \partial P / \partial E_q; \quad (10.71)$$

$$\Delta E'_q T_{d,0} \cdot p + \Delta E_q = \Delta E_{q,e}; \quad (10.72)$$

$$\Delta E_{q,e} (1 + T_e \cdot p) = \Delta U_{ROV}; \quad (10.73)$$

$$\Delta U_{ROV} = -K_U \cdot \Delta U_G; \quad (10.74)$$

$$\Delta E'_q = \Delta \delta \cdot \partial E'_q / \partial \delta + \Delta E_q \cdot \partial E'_q / \partial E_q; \quad (10.75)$$

$$\Delta U_G = \Delta \delta \cdot \partial U_G / \partial \delta + \Delta E_q \cdot \partial U_G / \partial E_q; \quad (10.76)$$

where the variables partial derivatives are

$$\begin{aligned} \partial P / \partial \delta &= S_{E_q} = E_q U \cos \delta / X_d; \quad \partial P / \partial E = U \sin \delta / X_d; \\ \partial E'_q / \partial \delta &= U \sin \delta \cdot (X'_d - X_d) / X_d; \quad \partial E'_q / \partial E_q = X'_d / X_d. \end{aligned} \quad (10.77)$$

To obtain the characteristic equation of equations set (10.71)-(10.76) it is necessary to perform the following transformations: substitute (10.76) into (10.74), and the obtained expression – into (10.73), the value obtained from this equation together with (10.75) substitute to (10.72). As the result, the characteristic equation takes the form of

$$a_0 p^4 + a_1 p^3 + (a_2 + K_U \cdot \Delta_2) p^2 + a_3 p + a_4 + K_U \cdot \Delta_4 = 0 \quad (10.78)$$

where

$$a_0 = \frac{T_J}{\omega_0} T_{d,0} T_e \frac{\partial E'_q}{\partial E_q}; \quad a_1 = \frac{T_J}{\omega_0} \left( T_e + T_{d,0} \frac{\partial E'_q}{\partial E_q} \right);$$

$$a_2 = \frac{T_J}{\omega_0} + T_{d,0} T_e \left[ \frac{\partial E'_q}{\partial E_q} \cdot \frac{\partial P}{\partial \delta} - \frac{\partial E'_q}{\partial \delta} \cdot \frac{\partial P}{\partial E_q} \right];$$

$$a_3 = T_{d,0} \left[ \frac{\partial E'_q}{\partial E_q} \cdot \frac{\partial P}{\partial \delta} - \frac{\partial E'_q}{\partial \delta} \cdot \frac{\partial P}{\partial E_q} \right]; \quad a_4 = \frac{\partial P}{\partial \delta};$$

$$\Delta_2 = \frac{T_J}{\omega_0} \frac{\partial U_G}{\partial E_q}; \quad \Delta_4 = \frac{\partial U_G}{\partial E_q} \cdot \frac{\partial P}{\partial \delta} - \frac{\partial U_G}{\partial \delta} \cdot \frac{\partial P}{\partial E_q}.$$

After substitution the partial derivatives (10.77) into these expressions, we have

$$\left. \begin{aligned} a_0 &= \frac{T_J}{\omega_0} T'_d T_e; \quad a_1 = \frac{T_J}{\omega_0} (T'_d + T_e); \\ a_2 &= \frac{T_J}{\omega_0} + T'_d T_e \left( \frac{E_q U}{X_d} \cos \delta + U^2 \frac{X_d - X'_d}{X_d X'_d} \sin^2 \delta \right); \\ a_3 &= T'_d \left( \frac{E_q U}{X_d} \cos \delta + U^2 \frac{X_d - X'_d}{X_d X'_d} \sin^2 \delta \right) + T_e \frac{E_q U}{X_d} \cos \delta; \\ a_4 &= \frac{E_q U}{X_d} \cos \delta; \quad \Delta_2 = \frac{T_J}{\omega_0} \cdot \frac{X_{ext}}{X_d}; \\ \Delta_4 &= \left( \frac{E_q U}{X_d} \cos \delta + U^2 \frac{X_d - X_{ext}}{X_d X_{ext}} \sin^2 \delta \right) \frac{X_{ext}}{X_d}, \end{aligned} \right\} \quad (10.79)$$

where  $T'_d = T_{d,0} X'_d / X_d$ .

In the transformed characteristic equation (10.78)

$$a_0 p^4 + a_1 p^3 + a_2 p^2 + a_3 p + a_4 + K_U \cdot (\Delta_4 + \Delta_2 p^2) = 0$$



the part with coefficients  $a_0, \dots, a_4$ , which do not depend on excitation control, determines stability of uncontrolled electric power system. The remaining equation part reflects influence of automatic load transfer ( $\Delta_2$  and  $\Delta_4$  are unit additions of the characteristic equation corresponding coefficients being proportional to the regulator gain factor).

Basing on (12.79) the following conclusions can be made:

- the coefficients  $a_0$  and  $a_1$  are always positive, and do not depend on operating conditions and parameters of the network the generator works for;
- the coefficients  $a_2, a_3, a_4, \Delta_4$  depend on the operation mode and the network parameters; with the angle  $\delta$  increase some of them become negative (Fig.1016, c), that indicates to violation of the necessary condition of the system stability which corresponds to positive values of all coefficients of the characteristic equation.

To determine conditions of the system stability, the Hurwitz criterion may be used:

$$a_2 + K_U \Delta_2 > 0; \quad (10.80)$$

$$a_3 > 0; \quad (10.81)$$

$$a_4 + K_U \Delta_4 > 0; \quad (10.82)$$

$$a_1 a_3 (a_2 + K_U \Delta_2) - a_1^2 (a_4 + K_U \Delta_4) - a_0 a_3^2 > 0. \quad (10.83)$$

To simplify the analysis of stability conditions it is firstly assumed that the exciter time constant  $T_e = 0$ . It permits to obtain a characteristic equation of lower (up to the third) order as  $a_0 = 0$  (see (10.78) and (10.79)). Stability of generator without automatic excitation control ( $K_U = 0$ ) is disturbed when the power and angle  $\delta$  increase and the coefficient  $a_4$  sign changes for negative. That takes place when criterion (12.82) is violated. Under  $T_e = 0$  and  $K_U = 0$  criterion (10.81) looks as

$$a_3 = T_d' \left( a_4 + U^2 \frac{X_d - X_d'}{X_d X_d'} \sin^2 \delta \right)$$

When  $a_4$  becomes negative it cannot serve as a limit condition of stability. Under  $T_e = 0$  and  $K_U = 0$ , criterion (10.80) is always positive. Under  $T_e = 0$  and  $K_U = 0$ , criterion (10.83) appears as

$$a_2 a_3 - a_1 a_3 = T_d' U^2 \frac{T_J}{\omega_0} \cdot \frac{X_d - X_d'}{X_d X_d'} \sin^2 \delta$$

and is always positive under any value of the angle  $\delta$ .

Thus, if only electromagnetic transients in the field winding are taken into consideration, stability of the generator without excitation control is determined by the limit condition i.e. by positive value of the synchronized power  $S_{E_q} = a_4$  being determined under EMF  $E_q$  constancy.

Applying the error-closing control on the voltage ( $K_U = 0$ ), inequality (10.82) free term may be given a positive value at angles being more than  $\pi/2$ . Besides, it will help to remove the reason of disturbance of the generator without excitation control stability.

The condition at which the coefficient has positive sign defines the limit of the least gain of the voltage displacement values. That follows from (10.82):

$$K_{U,min} = -a_4 / \Delta_4 \quad (10.84)$$

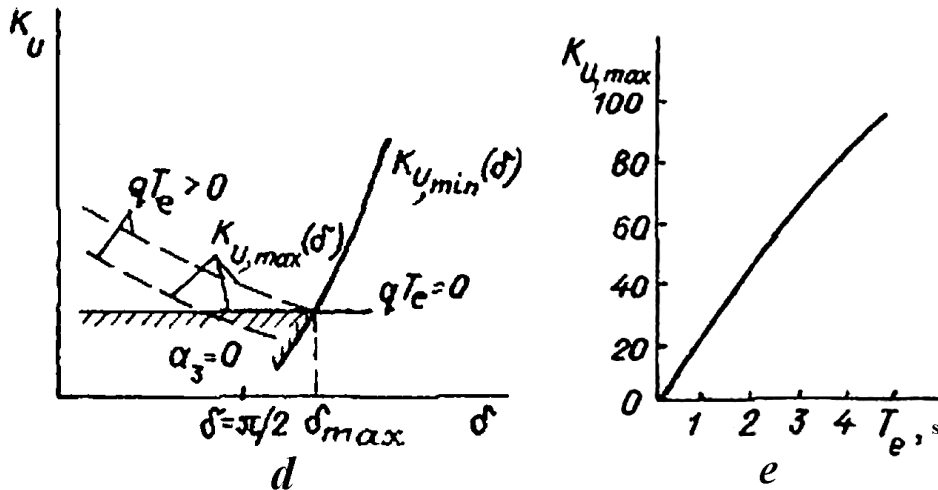


Fig. 10.16(d,e) Analysis of operation stability for generator with proportional type automatic excitation control

Character of the dependence  $K_{U,min}(\delta)$  change is shown in Fig.10.16, d. If coefficient  $a_4$  is positive, the stability can be disturbed when the coefficient  $a_3$  sign (voltage error-closing control does not influence it), or the sign of inequality (10.83) changes. It can be shown that the coefficient  $a_3$  depends on the synchronizing power sign which is determined under the transitional EMF  $E'_q$  constancy.

If EMF in (10.63) defined by (10.69) is substituted for  $E_{q0}$  then

$$P = U \cdot E'_q \sin \delta / X'_d - U^2 \cdot \frac{X_d - X'_d}{2X_d X'_d} \sin(2\delta) \quad (10.85)$$

Under  $E'_q = const$ , the partial derivative

$$\frac{\partial P}{\partial \delta} = S_{E'_q} = U \cdot E'_q \cos \delta / X'_d - U^2 \cdot \frac{X_d - X'_d}{X_d X'_d} \cos(2\delta)$$

Taking into account (10.69, b), and the equality  $\cos 2\delta = \cos^2 \delta - \sin^2 \delta$  we have

$$S_{E'_q} = U \cdot E'_q \cos \delta / X'_d + U^2 \cdot \frac{X_d - X'_d}{X_d X'_d} \sin^2 \delta$$

Taking into consideration (10.77) we obtain

$$S_{E'_q} = S_{E_q} + U^2 \cdot \frac{X_d - X'_d}{X_d X'_d} \sin^2 \delta \quad (10.86)$$

Comparison of (10.86) with expression of  $a_3$  (10.79) shows that  $a_3 = T'_d S_{E'_q}$  under the condition  $T_e = 0$ . Thus, the sign of coefficient  $a_3$  is defined by the synchronizing power  $S_{E'_q}$  sign.

Analyzing (10.83) and taking into consideration (10.79), find that maximum value of the regulator gain factor:

$$K_{U,max} = X_d / \left[ X_{ext} - x_d X_d / (x_d - x'_d) \right]$$

The coefficient value range is:

$$-a_4 / \Delta_4 < K_U < (a_1 a_4 - a_2 a_3) (a_1 \Delta_4 - a_3 \Delta_2) \quad (10.87)$$

Fig.10.16, e shows that operation stability is disturbed under the angles  $\delta = \delta_{max}$  being out of the allocated area.

For  $\delta = \delta_{lim}$  after transformation by (10.87), we obtain

$$a_3 (a_4 \Delta_2 - a_2 \Delta_4) = 0$$

and with account of (10.79) at  $T_e = 0$ , the latter equality can be written as:

$$\frac{T'_d T_J}{\omega_0} S_{E'_q} U^2 X_{ext} \sin^2 \delta \cdot \frac{X_d - X_{ext}}{X_d^2} = 0$$

Consequently, the synchronizing power is still positive if  $K_U \neq 0$  and  $\delta = \delta_{lim}$ . It permits to draw a conclusion that the equality  $S_{E'_q} = 0$  is limit condition of a system with automatic regulator of proportional type on the voltage deviation stability loss. Besides, such a regulator introduction can not increase the limit of the system stability determined for the condition of the synchronizing power  $S_{E'_q}$  positive value and constancy of  $E'_q$  as this condition does not depend on the regulator gain factor.

When the time constant of exciter  $T_e > 0$ , evaluation of the system stability consists in the following. The limiting condition is determined by the Hurwitz criterion (10.80)-(10.83). Inequality (10.80) does not define the operation stability because it is always disturbed later than inequality (10.81) as it is seen from (10.79). Comparison of inequalities (10.81) and (10.82) shows that inequality (10.81) is the first to be disturbed: under  $T_e = 0$  and  $\delta = \delta_{lim}$  the equalities are disturbed simultaneously, and under  $T_e > 0$  values  $K_U$  increase, and it guarantees inequality (10.82) satisfaction under the angle  $\delta$  large values.

Thus, limit condition of the system stability is determined with the help of inequalities (10.81) and (10.83). It follows from inequality (10.83) that maximum value of the regulator gain factor is determined by expression

$$K_{U,max} = (a_0 a_3^2 / a_1 - a_1 a_4 - a_2 a_3)(a_1 \Delta_4 - a_3 \Delta_2) \quad (10.88)$$

When time constant of the exciter increases, the value of  $K_{U,max}$  increases too (Fig.10.16,e). With the angle increase the values of  $K_{U,max}$  decrease as the result of the coefficient  $a_3$  reduction (10.79). Under  $a_3 = 0$  the gain factor  $K_{U,max} = a_4 / \Delta_4$ , and it is less than its value under  $T_e = 0$  (10.87). That is why when the angle  $\delta$  increases and  $T_e > 0$ , the stability condition (10.83), which is limiting one for determination the power limit on static stability of the system with automatic regulator of proportional type, is disturbed first.

Under the automatic generator excitation control by voltage deviation with given value of the regulator gain, the limit of the system operation static stability is calculated in the following order:

1. The EMF value under steady operation is determined as

$$E_q = E_{q,0} + K_U (U_{G,0} - U_G) \quad (10.89)$$

where  $E_{q,0}$  and  $U_{G,0}$  are the EMF determined with use of the synchronous inductive reactance, and voltage on terminals of the generator under initial regime respectively.

2. Substituting the value of (10.70, b) into equation (10.89) the value of synchronous EMF is determined on the formula

$$E_q = \left[ E_{q,0} + K_U \left( U_{G,0} - U \cos \delta \frac{X_d - X_{ext}}{X_d} \right) \right] / \left( 1 + K_U \frac{X_{ext}}{X_d} \right) \quad (10.90)$$

3. The value of the transient EMF  $E'_q$  is calculated from formula (10.69, b).

4. Transforming equation (10.63) with account of (10.90) the power-angle characteristic is determined:

$$P = \left[ (E_{q,0} + K_U U_{G,0}) U \sin \delta - U^2 K_U \sin 2\delta \frac{X_d - X_{ext}}{2X_d} \right] / (X_d + K_U X_{ext}) \quad (10.91)$$

With sufficient for practical consideration accuracy, the power-angle characteristic can also be obtained by formula (10.85) using synchronous EMF value calculated from (10.69, b).

5. The limit of transmitting power as well as the limiting value of the displacement angle of the generator rotor is determined. With the use of the power-angle characteristic (10.91) the limit of transmitting power is determined using the limiting value of the angle  $\delta_{lim}$  obtained from equation (10.91) solution. The latter is equated to the given gain factor value, and the coefficients values and the synchronous EMF are determined from (10.79 and (10.90):

$$c_1 \cos^2 \delta_{lim} + c_2 \cos \delta_{lim} + c_3 = 0 \quad \text{and} \quad \delta_{lim} = \arccos \delta_{lim} \quad (10.92)$$

where

$$c_1 = U^2 \left[ \frac{K_U (T'_d + T_e) (X_d - X_{ext})}{X_d (X_d - K_U X_{ext})} + T_d \frac{X_d - X'_d}{X_d X'_d} \right];$$

$$c_2 = -U(T_d' + T_e)(E_{q,0} + K_U U_{G,0}) / (X_d + K_U X_{ext});$$

$$c_3 = -U^2 T_d' \frac{X_d - X_d'}{X_d X_d'} + T_J (T_d' + T_e) / \left\{ T_e^2 \omega \left[ \left( \frac{X_d' - X_{ext}}{X_d - X_{ext}} + \frac{T_e X_d'}{T_d' X_d} \right) K_U - 1 \right] \right\}.$$

6. Using the power-angle characteristic (10.85) the limiting value of the angle  $\delta_{lim}$  can be found from equation (10.86) assuming  $S_{E_q'} = 0$ .

Study of the system operation stability for the case, when generators have automatic excitation regulator of proportional type, shows that performance of automatic excitation control device provides widening the area of the system static stability operation beyond  $\delta = \pi / 2$  with stability limit being within  $S_{E_q'} = 0$  and  $S_{E_q} = 0$ .

### Test questions

1. Which are simplified mathematic equations describing processes in the main EPS components such as synchronous machines, induction motors, operating mechanisms, an electric power network, and load nodes?
2. What is the basis of practical EPS stability criteria use?
3. What is evaluation of EPS operation static stability with use of practical criteria? What assumptions are made?
4. Why can not practical criteria of EPS stability be considered as universal ones?
5. To what characteristic calculation circuits the EPS circuit can be reduced to?
6. What is the nature of "synchronizing power" concept for evaluation of static operation stability?
7. What indicators are used to characterize the limit operation state by static stability?
8. What practical criteria of static operation stability are used for analysis the typical EPS circuits?
9. What is the linearization of nonlinear equations on the first approximation, and why is it used?
10. What is the necessary and sufficient condition of stability, and what mathematic criteria are used to evaluate the conditions of an EPS static stability?
11. Why are criteria by Hurwitz, Gauss, and Mikhailov, and curves of D-division used to evaluate the EPS static stability?

### Topics for essay

1. Adjustment of EPS circuits to standard calculation ones depending on the aim of static operation stability study.
2. Determination of conditions and study of limit operation states on the basis of static stability for real EPS of industrial enterprises.
3. Consideration of different types of automatic generator excitation control while static operation stability analyzing.

## **CHAPTER 11: STABILITY OF ELECTRIC POWER SUPPLY SYSTEM OPERATION UNDER LARGE DISTURBANCES**

11.1. Dynamic stability of operation

11.2. Simplified methods of dynamic operation stability evaluation

11.3. Estimation of dynamic operation stability of complex electric power system

11.4. Estimation of dynamic stability of operation with account of generators excitation control

11.5. Resulting operation stability

11.6. Estimation of resulting operation stability

11.7. Computation facilities application to study operation stability

Test questions

Topics for essay

**11.1. Dynamic stability of operation**

To ascertain principles of dynamic stability analysis consider phenomena taking place when one of two parallel circuits of power transmission line (PTL) are disconnected (Fig. 11.1, a)

Equivalent circuits of PSS under normal operation while working with two connected power transmission circuits, and under forced operation with one disconnected circuit are in Fig. 11.1 (b, c). Resulting resistance under normal operation is determined by expression

$$x_{res,I} = x'_d + x_{T1} + x_L / 2 + x_{T2},$$

and after one of the circuits disconnection by expression

$$x_{res,III} = x'_d + x_{T1} + x_L + x_{T2}.$$

As  $x_{res,III} > x_{res,I}$  then

$$P_{max,III} = E'U_{RS} / x_{res,III} < P_{max,I} = E'U_{RS} / x_{res,I}. \tag{11.1}$$

It follows from (11.1) that under invariable values of  $E', U_{RS}$ , and  $x_{res}$ , maximum value of the transmitted power diminishes.

If one of the power transmission lines (PTL) circuits is disconnected, the rotor of generator can not change the angle ( $\delta$ ) due to inertia, and it will remain the same as in the point *a* which corresponds to normal PTL operation. In Fig.11.2 power  $P_I(\delta_a) \equiv P_0(\delta_a)$ . That's why the new operation will be characterized by point *b* on the power- angle curve of the generator. When its power drops the excess accelerating torque appears, the angle  $\delta$  increases, and the generator output capacity increases in accordance with the dependence  $P_{II}(\delta)$ .

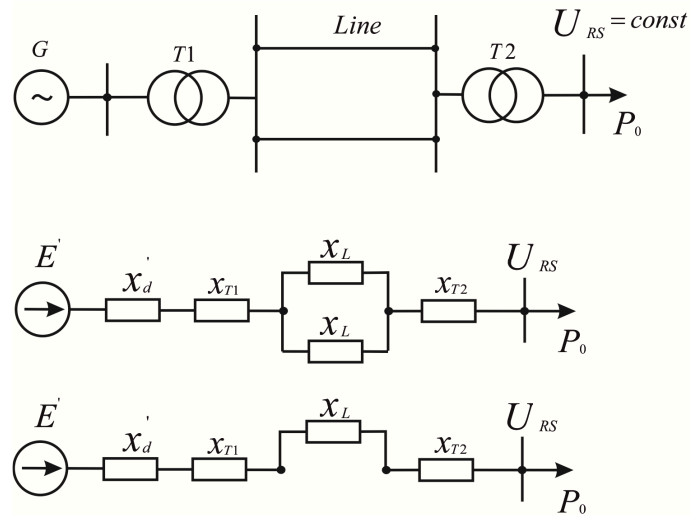


Fig. 11.1. Circuits for study of dynamic stability of PSS operation: a – design circuit; b – equivalent circuit for normal operation; c – equivalent circuit for emergency operation

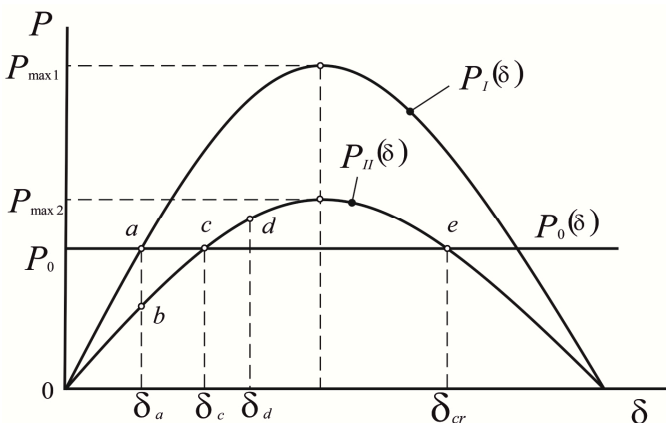


Fig. 11.2 Power-angle curves of generator while one of power transmission lines is disconnected

During the generator rotor acceleration, the point *c* is passed mechanically and after that the generator torque becomes greater than the turbine torque. Rotor begins to retard under impact of the prime mover turbine, and its angular velocity decreases beginning from some point *d*. Damped vibrations originate around the new steady operation mode that corresponds to the point *c*. If angular velocity of rotor increases up to value corresponding to point *e* or other points following it within descending part of the curve  $P_{II}(\delta)$ , the generator falls out of step.

Hence, the angle  $\delta$  variation in time can show the new system operation mode stability.

Examples of the angle  $\delta$  variation in time that are shown in Fig.11.3, correspond to the system stable (curve 1), and unstable (curve 2) operation.

Consider transient under short circuit at the beginning of one of the PTL circuits with its subsequent disconnection (Fig.11.1 a). The equivalent PSS circuits for normal and post-emergency operations correspond to those in Fig. 11.1b, c.

Power-angle curves of the generator for normal operation is determined with the help of expression

$$P_I = E'U_{RS} \sin \delta / X_I, \tag{11.2}$$

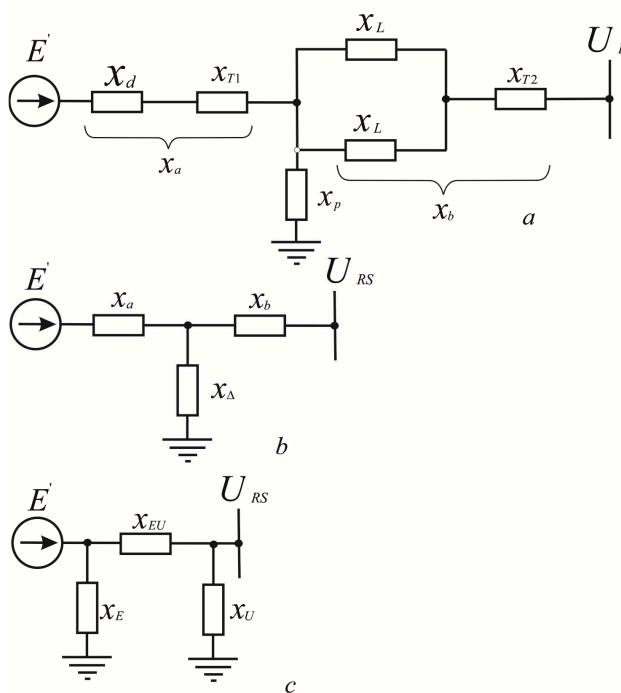


Fig. 11.4. PSS equivalent circuit for emergency operation (a) and its transformation to star (b) and to delta (c)

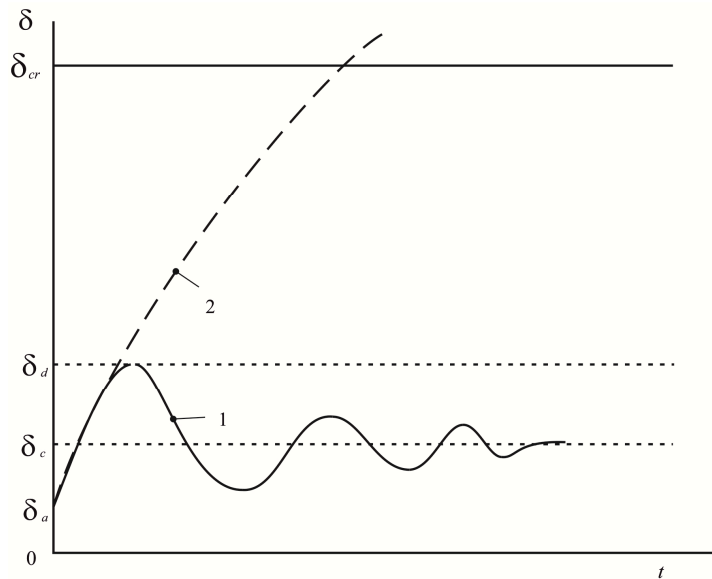


Fig. 11.3. Evaluation of dynamic operation stability by variation the angle  $\delta$  in time. Curve 1 is trajectory of stable and curve 2 of unstable operation

where  $X_I \equiv x_{res,I}$ , and for post-emergency operation – with the help of expression

$$P_{III} = E'U_{RS} \sin \delta / X_{III},$$

where  $X_{III} \equiv x_{res,III}$ .

Equivalent circuit for emergency operation can be obtained if shunting reactance is connected to the point of short circuit (Fig.11.4, a). Its value depends on the type of short circuit:  $x_\Delta^{(3)} = 0$  under three-phase SC;  $x_\Delta^{(2)} = x_2$  under two-phase SC;  $x_\Delta^{(1)} = x_2 + x_0$  under single-phase SC, and  $x_\Delta^{(1,1)} = x_2 x_0 / (x_2 + x_0)$  under two-phase earth, where  $x_0$  and  $x_2$  are resulting resistances of equivalent circuits of zero and negative sequences relatively to the SC point.

Equivalent circuit in Fig. 11.4, a can be transformed in consecutive order from star (b) to delta (c) in which

$$x_E = x_a + x_\Delta + x_a x_\Delta / x_b;$$

$$x_U = x_\Delta + x_b + x_\Delta x_b / x_a;$$

$$x_{EU} = x_a + x_b + x_a x_b / x_\Delta.$$



Reactance  $x_E$  and  $x_U$  have not great influence on the generator active power transmission at emergency operation and can be not taken into consideration. Active power of generator is sent through reactance  $x_{EU} \equiv x_{II}$ , and the power-angle curve of generator is

$$P_{II} = E'U_{RS} \sin \delta / x_{II} \quad (11.4)$$

When resistance  $x_{\Delta}$  decreases, resistance  $x_{EU}$  increases, and it results in the power-angle amplitude decrease. The hardest emergency operation will take place under three-phase SC at the beginning of PTL when  $x_{EU} \rightarrow \infty$ , and amplitude of the power-angle curve is equal to zero. The lightest emergency operation corresponds to single-phase SC under which reactance  $x_{\Delta}$  will be the greatest.

The power-angle curves of the generator for normal operation  $P_I(\delta)$ , emergency operation  $P_{II}(\delta)$ , and post-emergency operation  $P_{III}(\delta)$  are shown in Fig.11.5. Under normal condition the generator output power, and the angle between the EMF  $E'$  and voltage  $U_{RS}$  are noted as  $P_0$  and  $\delta_0$ . At the initial time of SC the angle cannot vary immediately due to inertia of rotor of the generator ( $\delta = \delta_0$ ). Quick power decrease takes place from the value at the point  $a$  on  $P_I(\delta)$  characteristic to the value at the point  $b$  on  $P_{II}(\delta)$  characteristic. As the result some excess accelerating torque arises on generator shaft stipulated by difference of the prime motor power and the power produced by generator. Under its influence the vector of EMF  $E'$  starts moving as for voltage vector of the receiving system (the angle  $\delta$  increases). The movement to point  $c$  is in accordance with the generator power increase on  $P_{II}(\delta)$  characteristic.

If under some value of  $\delta_{discon} = \delta_d$  the PTL faulted circuit is disconnected, at the very instant generator power will vary from the value determined by the point  $c$  on  $P_{II}(\delta)$  curve to the value matching the point  $d$  on  $P_{III}(\delta)$  curve. The prime motor power will stay invariable and equal to  $P_0$  because of inertia of the control system of the turbine rotational speed. After the SC disconnection, electromagnetic generator power will be more than turbine power, and retarding torque will occur on its shaft. The generator rotor continues accelerating within some time, and the vector of EMF  $E'$  moves towards angle  $\delta$  magnification. The generator covers the excess of its output electromagnetic power at the expense of kinetic energy accumulated by its rotor while accelerating from  $\delta_0$  to  $\delta_{discon}$ .

If all kinetic energy is spent before angle  $\delta_{cr}$  will have been attained (the point  $f$  on the  $P_{III}(\delta)$  characteristic), the rotor will start retarding under the effect of excessive value of retarding torque. The angle  $\delta$  variation in the reverse direction will take place in accordance with  $P_{III}(\delta)$  characteristic, and after some oscillations the new steady condition at the point  $e$  with the angle  $\delta_e$

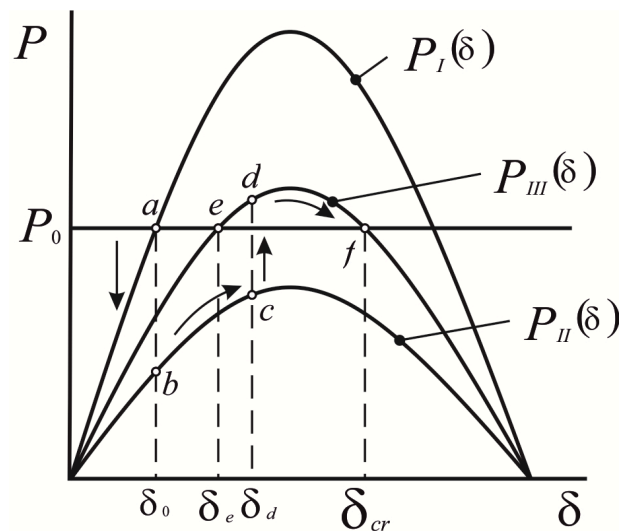


Fig. 11.5. Power-angle curves of generator for normal, emergency and post-emergency condition

will occur. If the angle  $\delta_{cr}$  is passed, the excessive resulting torque will become accelerating again. With  $\delta > \delta_{cr}$  angle magnification, the accelerating torque effecting the rotor will increase progressively, and the generator will fall out of synchronism. Thus, the system is dynamically stable in the first case, and is unstable one in the second case.

Under transient conditions and dynamic stability conservation the varying values of the system parameters as well as condition parameters after the disturbances disappearance can differ from their initial values but stay to be within admissible limits proceeding from the conditions of operation realizability.

## 11.2. Simplified methods of dynamic operation stability evaluation

Large disturbances result in heavy variations of PSS system performance. They occur due to variation of electric power network components composition, and their interconnection under SC, and imbalance of powers being generated and consumed within the PPS nodal points. The most dangerous disturbances are short circuits.

Tasks of PSS dynamic stability study are to evaluate transient character under large disturbances, and calculate important operation parameters in conversion from one operation condition to another, and determine critical values of operation parameters. To solve the problems, approximate methods are used as accurate evaluation of dynamic stability with taking into account every type of transients and variations within PSS caused by the disturbances is rather complicated.

Approximate methods being used to study dynamic stability of PSS operation are based upon a number of assumptions:

- Separation the electromagnetic and electromechanical transients on the basis of their rate assessment taking into account instantaneous change of electric power value under change of operation modes;
- Insignificance of the generator rotational speed deviations relatively the synchronous one;
- Invariability of prime motors torque values and inertia constants within transient;
- Substitution of a generators group of by an equivalent generator;
- Consideration of transient within bounded time period;
- Maintaining of the three-phase sources system symmetry under its violation within the network;
- Consideration just the main non-linear characteristics of the circuit components, etc.

The assumptions made must match the final target of the problem of dynamic stability study being solved. Approximate study methods can be divided into simplified methods and refined ones which are different owing to the level of assumptions, and the problem solution accuracy achieved.

The approximate methods make possible to perform preliminary study of the simplest electric power systems fast and easily but they can be used for rough estimation only.

The refined methods are aimed to consider a number of factors (not taken into account as for the simplified methods) which influence transient essentially:

Automatic control of excitation which effects the generators EMF and hence their electromagnetic torque;

Automatic control of prime motors rotational speed of as well as their torque;

Extra retarding torques occurring during SC caused by periodic current component of stator, and by currents induced in the damper windings;

Dynamic characteristics of load nodes.

The major simplified methods of PPS dynamic stability study are: the method of areas used for determination the limiting values of angle and SC disconnection time; the step-by-step method used for evaluation the transient performance via the angle  $\delta$  in the course of time variation.

**The limiting angle of the SC switch off** can be found without determination of operation mode changes in the course of transient. For this purpose the method of areas is used which permits to evaluate relation of power change within different phases of the PPS operation mode change. As an example, give energetic estimation of the simplest power supply system transition from the

normal operating condition to emergency one, and after that to post-emergency operation. The system consists of the generator connected to the buses of a loading system of infinite power through the transformer and double-circuit PTL. (Fig. 11.1)

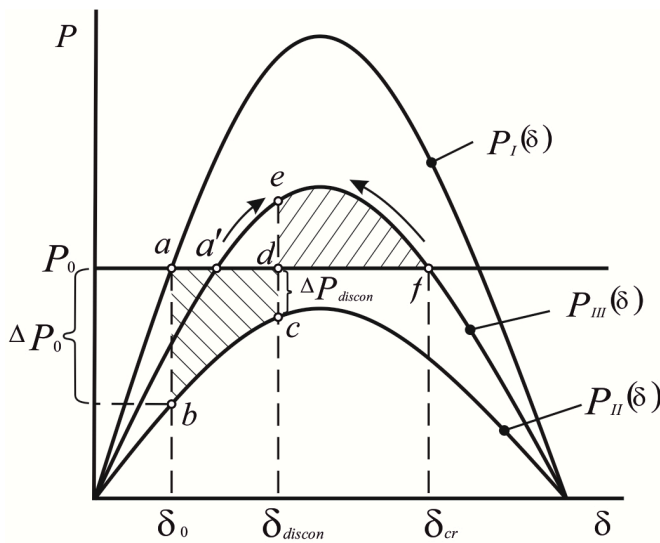


Fig. 11.6. Analysis of power-angle curves for normal, emergency, and post-emergency conditions with use of method of areas

If the angle  $\delta_{discon}$  corresponds to the time of the fault circuit disconnection then while accelerating the generator rotor will accumulate kinetic energy

$$A_{acceler} = \int_{\delta_0}^{\delta_{discon}} \Delta P_{II}(\delta) d\delta \equiv F_{abcd} \quad (11.5)$$

Graphically it corresponds to the area  $F_{abcd}$  shaded in Fig. 11.6. It is called the area of acceleration.

Disconnection of the faulty power transmission circuit results in of increase power supplied to the network from  $P_{II}(\delta_{discon})$  up to  $P_{III}(\delta_{discon})$ . As  $P_{III}(\delta_{discon}) > P_0$ , the torque that for the generator rotor is retarding one occurs. It corresponds to the power  $\Delta P_{III}(\delta_{discon}) = P_{III}(\delta_{discon}) - P_0$ . But the angle growth continues till the kinetic energy of the generator rotor accumulated while acceleration is consumed.

During the time of retardation the power that can be spent has limiting value for the interval of angle variation  $[\delta_{discon}; \delta_{cr}]$  determined by the expression:

$$A_{ret} = \int_{\delta_{discon}}^{\delta_{cr}} \Delta P_{III}(\delta) d\delta \equiv F_{def} \quad (11.6)$$

The area  $F_{def}$  of the figure  $def$  shaded in Fig. 11.6 and called the retarding area corresponds to kinetic energy to be returned by the generator rotor while retarding. The following condition corresponds to dynamic stability retention and to return the working point into position  $a'$ :

$$F_{acceler} \leq F_{ret} \quad (11.7)$$

Change of the system operating conditions (Fig. 11.6) is shown with use of the power-angle curves. The working point under normal conditions has the coordinates  $(P_0, \delta_0)$  that correspond to equality of the prime motor power output and the power conveyed to the network by the generator (see the angle  $\delta_0$  between the EMF  $E'$  and voltage  $U_C$ ).

Under SC shedding the power supplied by generator from  $P_I(\delta_0)$  down to  $P_{II}(\delta_0)$  takes place that results in the power excess  $\Delta P_{II}(\delta_0) = P_0 - P_{II}(\delta_0)$ , and acceleration of generator rotor occurs. With it the working point moves along the curve  $P_{II}(\delta)$  towards the angle  $\delta (\delta > \delta_0)$  magnification along the arc  $bc$ .

Condition (11.7) being expressed by means of power-angle characteristics for the operation modes looks like

$$\int_{\delta_0}^{\delta_{discon}} (P_0 - P_{II}) d\delta - \int_{\delta_{discon}}^{\delta_{cr}} (P_{III} - P_0) d\delta \leq 0.$$

Hence limiting value of the angle of the faulty PTL circuit disconnection can be found as:

$$\delta_{discon,lim} = \arccos \left\{ \left[ P_0 (\delta_{cr} - \delta_0) + P_{imax} \cos \delta_{cr} - P_{imax} \cos \delta_0 \right] / (P_{imax} - P_0) \right\}. \quad (11.8)$$

The limiting time of SC disconnection  $t_{discon,lim}$  corresponds to the limiting angle of disconnection found using (11.8). For any instant of time these values interrelation is expressed by means of motion equation (9.1). Its analytical solution is possible only for special case of complete break of the generator and the loading system buses connection when  $P_{II}(\delta) \equiv 0$  (it takes place under three-phase SC within one of the PTL circuits). In this case, equation (9.1) is simplified:

$$T_J d^2 \delta / dt = P_0. \quad (11.9)$$

This equation is solved by means of step-by-step integration under integration constants  $c_1 = (d\delta / dt)_{t=0}$  and  $c_2 = \delta_0$ :

$$\delta = P_0 / (2T_J t^2) + \delta_0. \quad (11.10)$$

The limiting time value of three-phase SC can be found as:

$$t_{discon,lim} = \sqrt{2T_J (\delta_{discon,lim} - \delta_0) / P_0}. \quad (11.11)$$

If the angle  $\delta$  is expressed in degrees, and the time constant in seconds, the formula (11.11) will be:

$$t_{discon,lim} = \sqrt{T_J (\delta_{discon,lim} - \delta_0) / (9000 P_0)}. \quad (11.12)$$

When the fault is short-term (0.1 – 0.2 c) expression formula (11.12) is used to determine limiting time of disconnection of asymmetric SC when  $P_{II}(\delta) > 0$  (Fig.11.6). In this case power  $P_0$  in equation (11.11) is replaced by the quantity

$$\Delta P = (\Delta P_0 + \Delta P_{discon}) / 2 = P_0 - P_{imax} (\sin \delta_0 - \sin \delta_{discon}) / 2. \quad (11.13)$$

Character of transient from the viewpoint of operation modes change is determined with the help of dependence  $\delta = f(t)$  which is found by means of numerical solution of equation (9.1) by the method of successive intervals. With it, influence of control actions that appear due to the excitation control, time of the fault element disconnection variation, the automatic re-closure, etc. can be taken into consideration. Transient is divided into a series of equal time intervals which are considered successively. As for practical calculations, this time interval is taken within 0.02...0.1s depending upon SC duration, as well as of system automatic equipment characteristics. Power excess (right side of the equation) within every interval is considered as invariable, and under this assumption an increment in  $\Delta \delta$  is calculated.

Power excess  $\Delta P_0$  arises at the instant of SC  $t=0$  (Fig 11.7). The angle increment within the first time interval  $\Delta t$  is found under initial conditions  $(d\delta / dt)_{t=0}$  and  $\delta = \delta_0$  by means of successive integration of equation (9.1):

$$\Delta\delta_1 = \Delta t^2 \Delta P_0 / (2T_J). \tag{11.14}$$

At the end of the first time interval  $\delta_1 = \delta_0 + \Delta\delta_1$ . Within the second interval the generator movement is stipulated by the power excess  $\Delta P_1 = P_0 - P_{II \max} \sin\delta_1$  and some initial velocity obtained in the first interval:

$$(d\delta / dt)_1 = \Delta t(\Delta P_0 + \Delta P_1) / (2T_J). \tag{11.15}$$

Having solved equation (9.1) as for the angle increment within the second interval we obtain:

$$\Delta\delta_2 = \Delta t^2 \Delta P_1 / (2T_J) + \Delta t(d\delta / dt)_1. \tag{11.16}$$

After transformation of expression (11.16), and taking into account (11.14) and (11.15) it is received:

$$\Delta\delta_2 = \Delta\delta_1 + \Delta t^2 \Delta P_1 / T_J. \tag{11.17}$$

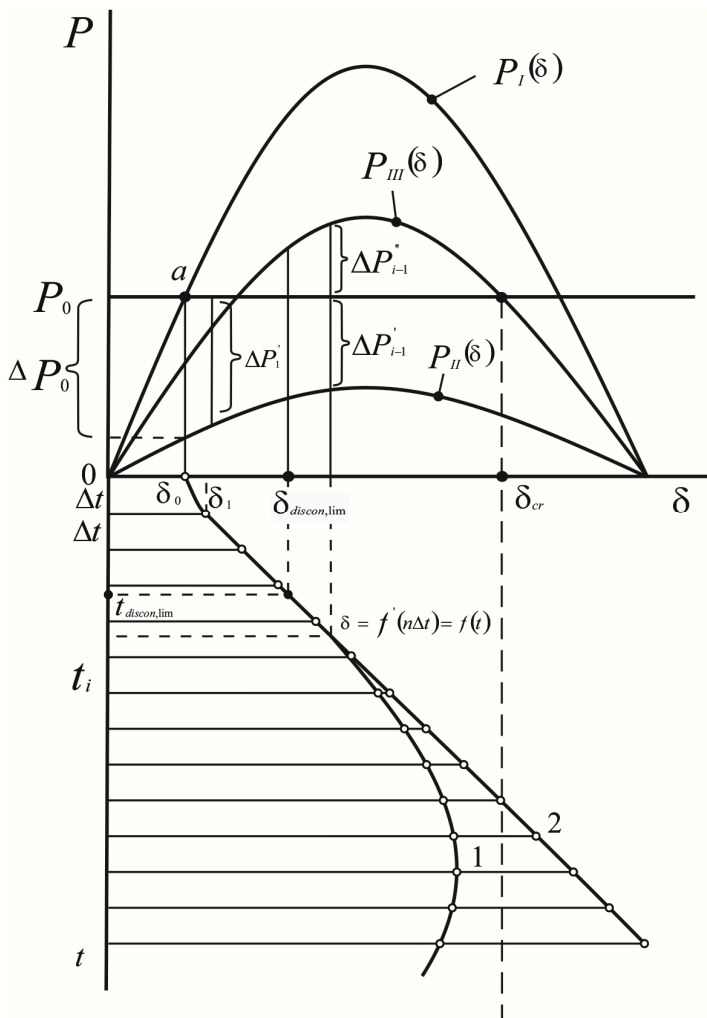


Fig. 11.7. To calculations of dynamic operation stability by the method of successive intervals

If inertia constant  $T_J$  and time  $\Delta t$  are in seconds, and angles  $\Delta\delta_1$  and  $\Delta\delta_2$  in degrees, and the constant  $k = 18000\Delta t^2 / T_J$ , is introduced, the expression (11.17) will be

$$\Delta\delta_2 = \Delta\delta_1 + k\Delta P_1. \tag{11.18}$$

By analogy with  $n - th$  time interval, it will be

$$\Delta\delta_n = \Delta\delta_{n-1} + k\Delta P_{n-1}. \tag{11.19}$$

If operation mode changes with transition from one power-angle curve to another during  $i - th$  interval (Fig 11.7), the angle increment is determined with the help of expression

$$\Delta\delta_i = \Delta\delta_{i-1} + 0,5k(\Delta P'_{i-1} + \Delta P''_{i-1}). \tag{11.20}$$

Calculation of the curve  $\delta = f'(n \cdot \Delta t)$  points should be performed until the angle  $\delta$  starts decreasing by curve 1 (Fig. 11.7), that meets condition of stability retention, or until it is found out that the angle continues to increase by curve 2 and  $\delta > \delta_{cr}$  that is in accordance with condition of stability violation.

Limiting time of SC disconnection can also be determined by the curve  $\delta = f(t)$  using the values of critical angle of SC disconnection calculated by (11.8).

11.3. Estimation of dynamic operation stability of complex electric power system

Usually EPS of an enterprise is energized by some independent sources. With it the external electric power supply system can be considered as complex one. EPS with two-side energizing is prevailing.

Consider electric power system including two sources, elements of links between them, and loads (Fig 11.8, a). Performance of the sources generators is described with the equations:

$$\begin{aligned}
 T_{J1} \cdot d^2 \delta_1 / dt^2 &= P_{10} - P_1 = \Delta P_1; \\
 T_{J2} \cdot d^2 \delta_2 / dt &= P_{20} - P_2 = \Delta P_2.
 \end{aligned}
 \tag{11.21}$$

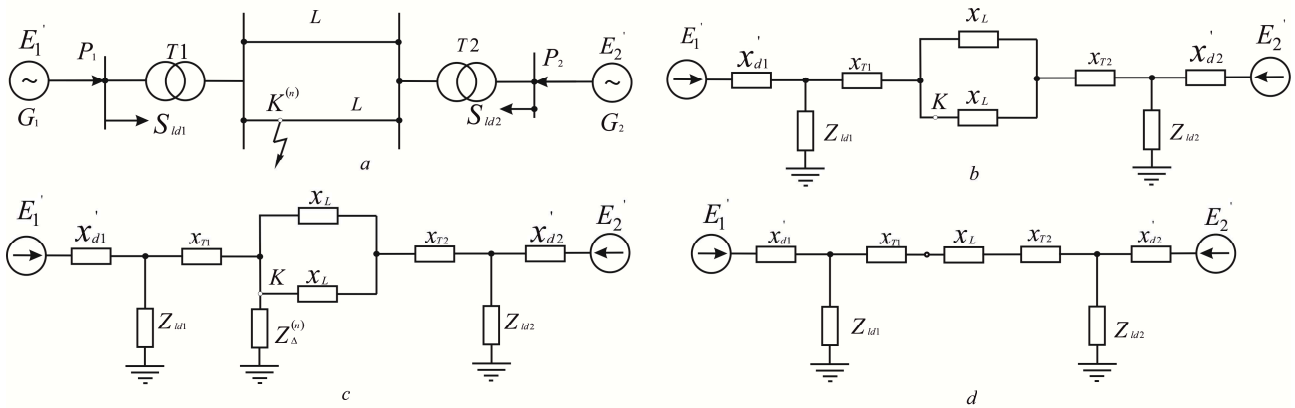


Fig. 11.8. Electric power system with two sources: a – design circuit; b, c, d -equivalent circuits for operating modes at normal, emergency, and post-emergency operation respectively

Introduce the load model into equivalent circuits of electric power system for different operating modes as complex resistances (Fig. 11.8, b-d). With it, link between the electric power sources  $G_1$  and  $G_2$  is formed by passive elements, and it can be expressed with the help of self- and mutual admittances with account of the load resistances. Then the power-angle curves of generators, on the basis of equation (11.14), can be expressed as

$$\begin{aligned}
 P_1 &= E_1' E_1' y_{11} \sin \alpha_{11} + E_1' E_2' y_{12} \sin(\delta_{12} - \alpha_{12}); \\
 P_2 &= E_2' E_2' y_{22} \sin \alpha_{22} + E_1' E_2' y_{12} \sin(\delta_{12} + \alpha_{12}).
 \end{aligned}
 \tag{11.22}$$

Power-angle characteristics with account of the power transmission direction from the sources to the network for the modes under consideration have been shown in Fig. 13.9, a; they had been plotted against common variable – the mutual angle of the sources generator rotors displacement  $\delta_{12}$ .

At the instant of SC occurrence shedding the active power taken from generators takes place, and working operation points move from  $a$  and  $a'$  to  $b$  and  $b'$  accordingly. In this case the generator  $G_1$  rotor will accelerate owing to power excess  $\Delta P_1^{(0)} > 0$ , and the generator  $G_2$  rotor will retard owing to power excess  $\Delta P_2^{(0)} < 0$ .

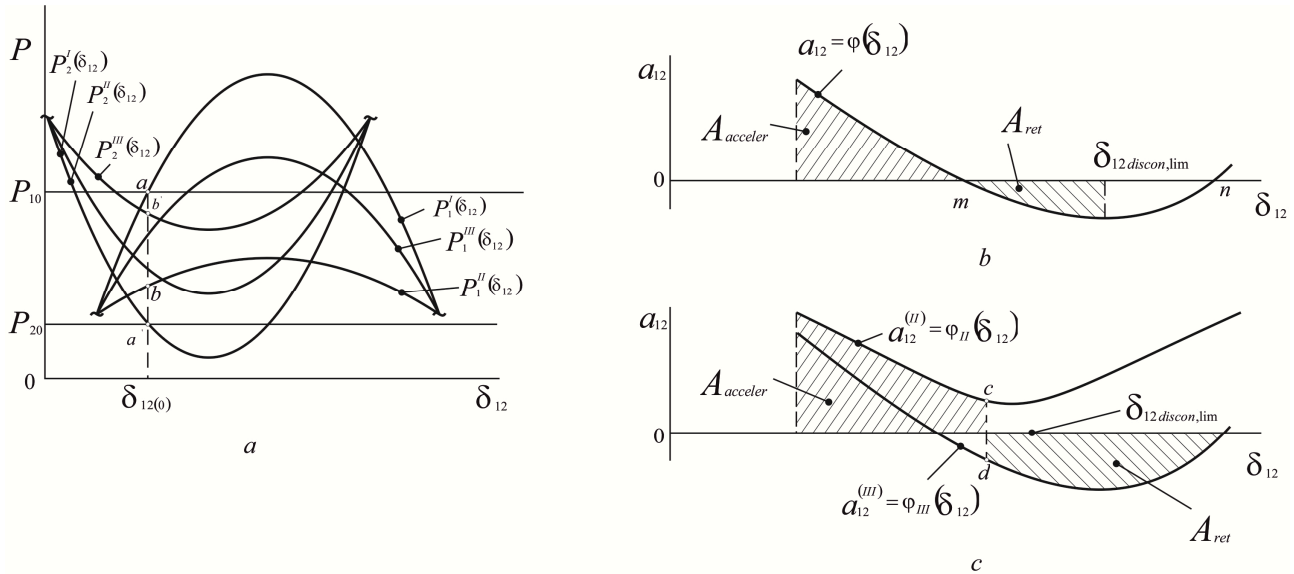


Fig. 11.9. Dependences of power and rotors relative acceleration against mutual angle of their displacement

Interrelation and signs of excess moments depend upon the system parameters, and the type of disturbance, and the time instant. It is impossible to evaluate dynamic stability by values of power excess and angle increments for every generator because velocity sign of relative displacement of the sources generator rotors is unknown (sign of velocity variation of mutual angle  $\delta_{12}$ ). This sign can be taken into account with the help of the second derivative of this variable – relative acceleration of the sources generators rotors. To do that, the equations (11.21) should be reduced to the form

$$\begin{aligned} d^2 \delta_1 / dt^2 &= \Delta P_1 / T_{J1}; \\ d^2 \delta_2 / dt^2 &= \Delta P_2 / T_{J2}. \end{aligned} \quad (11.23)$$

Subtracting equations (11.23), the equation of relative motion of the sources generator rotors is received

$$d^2 \delta_{12} / dt^2 = \Delta P_1 / T_{J1} - \Delta P_2 / T_{J2} = a_{12}, \quad (11.24)$$

where  $a_{12}$  is relative angular acceleration of the sources generator rotors.

It follows from the diagram  $a_{12} = \varphi(\delta_{12})$  (Fig.11.9, b) that the acceleration is alternating-sign. As there is dependence between the acceleration  $a_{12}$  and the velocity of rotors displacement  $\mathcal{G}_{12}$  is expressed as

$$a_{12} = d\mathcal{G}_{12} / dt, \quad (11.25)$$

the relative velocity of the rotors displacement increases under positive values of acceleration, and it decreases if values are negative.

After expression (11.25) transformation with account that  $\mathcal{G}_{12} = d\delta_{12} / dt$  we have

$$a_{12} = (d\mathcal{G}_{12} / d\delta_{12})(d\delta_{12} / dt) \Rightarrow a_{12} d\delta_{12} = \mathcal{G}_{12} d\mathcal{G}_{12}.$$

The last identity integration results in

$$\int_{\delta_{12(0)}}^{\delta_{12}} a_{12} d\delta_{12} = \int_{\mathcal{G}_{12(0)=0}}^{\mathcal{G}_{12}} \mathcal{G}_{12} d\mathcal{G}_{12} = 0,5\mathcal{G}_{12}^2 \Big|_0^{\mathcal{G}_{12}} = 0,5\mathcal{G}_{12}^2. \quad (11.26)$$

Left member of (11.26) equality determines the area bounded by the curve  $a_{12} = \varphi(\delta_{12})$  (Fig.11.9, b). The area is proportional to the velocity of relative displacement of rotors. In accordance with the value sign it is possible to assign acceleration area (increase in velocity of relative displacement of rotors), and retarding area (its decrease), and use the method of area on the basis of equality (11.26) to estimate the system with two sources dynamic stability.

The expressions  $\int_{\delta_{12(0)}}^{\delta_{12}} a_{12} d\delta_{12} = 0$  correspond to  $A_{acceler} = A_{ret}$ , and  $\mathcal{G}_{12} = 0$  (Fig.11.9, b).

The largest retarding area  $A'_{ret} = \int_{\delta_{12(m)}}^{\delta_{12(n)}} a_{12} d\delta_{12}$  can be used to estimate the dynamic stability

assurance factor  $K_{saf} = A'_{ret} / A_{acceler}$ .

When PSS operation changes the characteristics of relative acceleration, and of the sources generator rotors rotational speed are plotted for emergency  $a_{11}^{(II)} = \varphi_{11}(\delta_{12})$ , and post-emergency operation:  $a_{12}^{(III)} = \varphi_{12}(\delta_{12})$  (Fig.11.9, c). After the fault disconnected the working point moves from the point  $c$  to the point  $d$ . It is possible to mark out areas of acceleration and retardation within the graph. Their equality corresponds to limiting value of SC disconnection angle  $\delta_{12discon,lim}$ .

Limiting time of SC disconnection corresponding to the angle  $\delta_{12discon,lim}$  is found using the dependence  $\delta_{12} = f_{12}(t)$ , which diagram is plotted with the help of method of successive intervals. The main stages of its estimation are similar to those of electric power system with one generator working for buses of invariable voltage. Additionally, the increment in mutual angle of generator rotor displacement is determined for each time interval.

Within the first time interval (in brackets the upper index means the number of interval) we have

$$\delta_1^{(1)} = \delta_{1(0)} + \Delta\delta_1^{(1)}; \quad \delta_2^{(1)} = \delta_{2(0)} + \Delta\delta_2^{(1)}; \quad (11.27)$$

$$\Delta\delta_{12}^{(1)} = \Delta\delta_1^{(1)} - \Delta\delta_2^{(1)}; \quad (11.28)$$

$$\delta_{12}^{(1)} = \delta_{12(0)} + \Delta\delta_{12}^{(1)} = \delta_1^{(1)} - \delta_2^{(1)}. \quad (11.29)$$

Within the second time interval expressions (11.21) and (11.22) are used to estimate the power excess at the beginning of the interval (at the end of the first interval) for the first  $\Delta P_1^{(1)}$  and second  $\Delta P_2^{(1)}$  sources, and the angles increment:

$$\Delta\delta_1^{(2)} = \Delta\delta_1^{(1)} + k_1 \Delta P_1^{(1)} \quad \text{under } k_1 = 18000 \Delta t^2 / T_{J1};$$

$$\Delta\delta_2^{(2)} = \Delta\delta_2^{(1)} + k_2 \Delta P_2^{(1)} \quad \text{under } k_2 = 18000 \Delta t^2 / T_{J2},$$

whence

$$\Delta\delta_{12}^{(2)} = \Delta\delta_{12}^{(1)} + k_1 \Delta P_1^{(1)} - k_2 \Delta P_2^{(1)};$$

$$\delta_{12}^{(2)} = \delta_{12}^{(1)} + \Delta\delta_{12}^{(2)}.$$



For the following time intervals the increments in angles are computed on formulae:

$$\Delta\delta_1^{(n)} = \Delta\delta_1^{(n-1)} + k_1\Delta P_1^{(n-1)};$$

$$\Delta\delta_2^{(n)} = \Delta\delta_2^{(n-1)} + k_2\Delta P_2^{(n-1)};$$

$$\Delta\delta_{12}^{(n)} = \Delta\delta_{12}^{(n-1)} + k_1\Delta P_1^{(n-1)} - k_2\Delta P_2^{(n-1)};$$

$$\delta_{12}^{(n)} = \delta_{12}^{(n-1)} + \Delta\delta_{12}^{(n)}.$$

Dynamic stability of EPS with more than two sources operation is mainly studied by means of **method of successive intervals** as stated above. Such complex electric power systems are characterized by a set of independent variables – mutual angles between pairs of sources generator rotors. Availability of a number of independent variables doesn't give ability to use the method of areas to determine critical angle of SC disconnection, that's why the method of successive intervals is applied to estimate specific beforehand given time of SC disconnection.

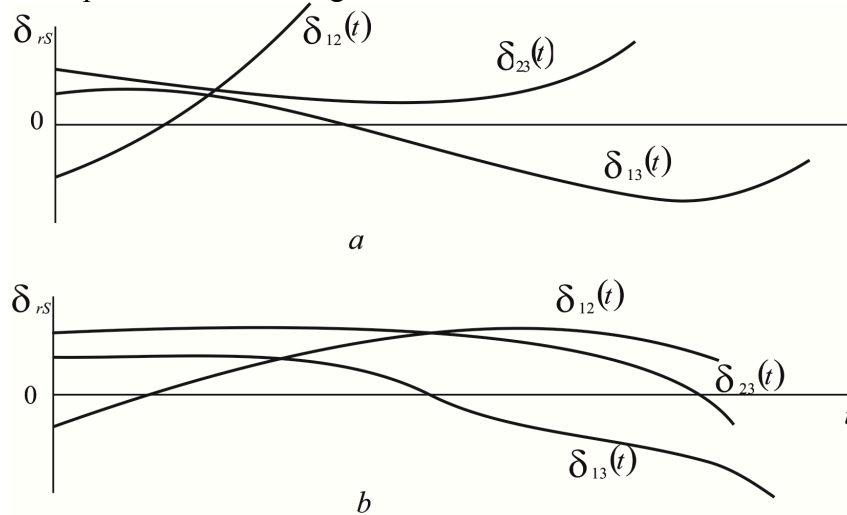


Fig. 11.10. Variation of mutual angles of generator rotors displacement: a – for system with three sources under operation stability retention; b – the same under operation stability violation

Within each interval an increment of absolute value of the angle  $\delta$  of every generator rotor displacement is estimated by formulae (11.27). Power excess on every source is easily determined with the help of equations like (11.21) and (11.22). For each interval the mutual angles between, pairs of generators  $\delta_{12}^{(n)}, \delta_{13}^{(n)}, \delta_{23}^{(n)}, \dots, \delta_{rs}^{(n)}$  are found on absolute values of angles  $\delta$ , using formulae like (11.28) and (11.29) to plot dependences  $\delta_{12}(t), \delta_{13}(t), \delta_{23}(t), \dots, \delta_{rs}(t)$ .

EPS dynamic stability is estimated by character of time variation of the mutual angles which must be uniform and non-increasing in time (Fig. 11.10, a). Monotonic increasing character of part of mutual angles in time dependences is the dynamic stability indication (Fig. 11.10, b).

Character of variation of the graphs  $\delta_{rs}(t)$  is determined for every beforehand given time of SC disconnection. To determine limiting time of SC disconnection the described succession of calculations must be repeated for other values of SC disconnection time until the character of dependences  $\delta_{rs}(t)$  variation confirming dynamic stability or instability of EPS operations is found.

#### 11.4. Estimation of dynamic stability of operation with account of generators excitation control

Dynamic stability of operation for the simplest electric power system where generator with automatic excitation regulator of proportional type works for buses of invariable voltage should be considered taking into account the non-linearity of its elements characteristics.

Change of such electric power system operation mode can be rather completely studied on the basis of method of successive intervals with linearizing of equations of transients within each time interval. In this case dynamic stability is estimated on the character of the angle of generator rotor displacement in time variation.

Excitation forcing device in its simplest modification realizes a resistor shunting within the generator exciter field circuit under voltage on its terminals decrease to  $0,85U_{G,rated}$ . With it the current in the exciter field circuit increases, and the exciter voltage that is proportional to it, with account the time lagging  $t_{lag}$  caused by automatic excitation control device operation (about  $0,05$  s) increases up to the greatest value  $E_{qe,max}$  on exponential law with the time constant of exciter  $T_e$  (Fig. 11.11).

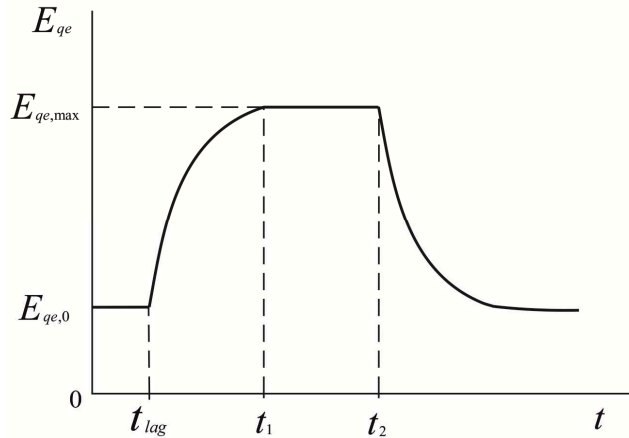


Fig. 11.11. The character of generator EMF time variation under excitation forcing

Under the steady operation the generator EMF  $E_{qe,0}$  is proportional, and as for per units is equal, to the exciter voltage. If excitation forcing takes place its increase law can be written down in such a way:

$$E_{qe,t} = E_{qe,0} + (E_{qe,max} - E_{qe,0})[1 - \exp(-t / T_e)],$$

$$t_{lag} < t < t_1. \quad (11.30)$$

Excitation forcing is available up to values of the generator voltage of  $(0,95...1,05)U_{G,rated}$ . After the forcing pulling-off, both exciter voltage and the generator EMF decrease by the law

$$E_{qe,t} = E_{qe,max} - (E_{qe,max} - E_{qe,0})[1 - \exp(-t / T_e)], \quad (11.30, a)$$

where  $E_{qe,t} \in \{E_{qe,0}, E_{qe,max}\}$ .

Having knowledge of the law of the generator's EMF  $E_{qe,t}$  variation it is possible by means of **method of successive intervals** to solve numerically the system of equations which describe the electromagnetic transient in the generator rotor

$$T_{d0}dE'_q / dt + E_q = E_{qe}(t); \quad (11.31)$$

and its relative motion under different operations

$$(T_J / \omega_0)d^2\delta / dt^2 = P_0 - E_q U \sin \delta / x_d. \quad (11.32)$$

An increment in transient EMF direct component within the estimated time interval is determined by expression

$$\Delta E'_{q(n)} = \Delta t(E_{q(n)} - E_{q(n-1)}) / T_{d0}, \quad (11.33)$$

where  $E_{q(n)}$  is taken equal to average value for the  $n$ -th time interval being calculated.

If the EMF  $\Delta E'_{q(n)}$  increment in the  $n$ -th interval is determined, it is possible to find complete value of the transient EMF direct component at the end of the interval under estimation:

$$E'_{q(n)} = E'_{q(n-1)} + \Delta E'_{q(n)}. \quad (11.34)$$

At the moment of emergency condition appearance (SC) the transient EMF direct component do not change, that is  $E_{q0} = E_{q(0)}$ . This condition is initial one for determination of the EMF  $E'_q$  variations on intervals. Under normal operation

$$E'_q = E_q - I_d(x_d - x'_d), \quad (11.35, a)$$

where the direct component of the generator current is estimated using the generator internal reactive power as well as the value of the synchronous EMF  $E_q$ :

$$I_d = E_q y_{11} \cos \alpha_{11} - U y_{12} \cos(\delta - \alpha_{12}).$$

After substitution and transformation of equation (11.35, a) it is be found that

$$\begin{aligned} E'_q = E_q [1 - (x_d - x'_d)y_{11} \cos \alpha_{11}] + \\ + U y_{12}(x_d - x'_d) \cos(\delta - \alpha_{12}). \end{aligned} \quad (11.35, b)$$

Ignoring capacitive susceptance and active resistance of the network elements, and taking into account identities  $x_d - x'_d \equiv X_d - X'_d$  and  $y_{11} \equiv y_{12} \equiv 1/X_d$ , the expression (11.35, b) is reduced to be simplified (11.69, b). Expression (11.35, b) can also be used to determine synchronous EMF  $E_q$  using the known value of the transient EMF  $E'_q$ :

$$\begin{aligned} E_q = [E'_q - U y_{12}(X_d - X'_d) \cos(\delta - \alpha_{12})] / \\ / [1 - y_{11}(X_d - X'_d) \cos \alpha_{11}]. \end{aligned} \quad (11.36)$$

The study of dynamic stability of the simplest electric power system containing generator with automatic regulator of excitation of proportional type with the use of dependence  $\delta(t)$  is performed in the following order:

1. Drawing up an equivalent circuits of the EPS for normal, emergency and post-emergency operation, and determine the mutual admittance and reactance.
2. Determine values of  $P_0, E_{q0}, E'_{q0}$  on the basis of the normal operation calculation.
3. Plot dependence of the EMF  $E_q(t)$  variation in time for the forced generator excitation using given characteristics of regulator and exciter.

4. Compute the EMF  $E_{q(0)}$  value for the first instant after emergency condition (SC) occurrence. The calculation formula is

$$E_{q(0)} = \left[ E'_{q(0)} - Uy_{12}(X_d - X'_d) \cos(\delta_0 - \alpha_{12}) \right] / \left[ 1 - y_{11}(X_d - X'_d) \cos \alpha_{11} \right].$$

5. Find an increment in the transient EMF within the first time interval with the help of (11.33):

$$\Delta E'_{q(1)} = \Delta t (E_{q(1)} - E_{q(0)}) / T_{do},$$

where  $E_{q(1)}$  is an average EMF value within the first time interval.

6. The transient EMF value at the end of the first (at the beginning of the second) time interval is calculated on formulae (11.34):  $E'_{q(1)} = E'_{q0} + \Delta E'_{q(1)}$ .

7. Active power yielded by the generator at the beginning of the first time interval is determined as

$$P_{(0)} = E_{q(0)}^2 y_{11} \sin \alpha_{11} + E_{q(0)} Uy_{12} \sin(\delta_0 - \alpha_{12}),$$

the power excess for this interval is  $\Delta P_{(0)} = P_0 - P_{(0)}$ .

8. The generator rotor angular displacement during the first interval is found as:  $\Delta \delta_{(1)} = 0,5k \cdot \Delta P_{(0)}$ .

Calculation is repeated for every successive interval in accordance with the last five items. If the power-angle characteristic of generator stays to be invariable within the  $n^{\text{th}}$  interval, then an increment in angle is

$$\Delta \delta_{(n)} = \Delta \delta_{(n-1)} + k \cdot \Delta P_{(n-1)},$$

and if it varies then  $\Delta \delta_{(n)} = \Delta \delta_{(n-1)} + 0,5k(\Delta P'_{(n-1)} + \Delta P''_{(n-1)})$ .

The generator voltage is calculated for each time interval on formulae (10.70, b) with the use of the synchronous EMF value found prior on (11.36). The instant of excitation forcing removal is controlled by the calculated values of the generator voltage.

Under estimation of dynamic stability of electric power system which generators have ordinary exciters it can be supposed roughly that forcing of the generators excitation lasts up to the moment when angle  $\delta$  achieves its maximum value. In this case, necessity of the generator voltage determination falls away only for the first step.

Consideration of automatic excitation regulator of intense action in estimation dynamic stability of the system is connected with study of characteristic equations of higher order than in case of consideration an automatic excitation regulator of proportional type. They are studied with the help of *method of D-partition* in the plane of coefficients of the variables deviation operation parameters, or with use either physical modeling, or computers.

Introduction of derivatives of operation parameters into the excitation control law provides extension of the stability area to larger values of the angle  $\delta$  than with automatic regulator of proportional type up to the limiting value. With it, the limiting power supplied to the network increases greatly (Fig.10.15, c, curve 3). If under use of the automatic regulator of proportional type, the limiting power is not more than value obtained under the EMF  $E'_q$  constancy, then while using intense automatic regulator the limiting power corresponds to its angular characteristic when  $U_G = const$ . With the help of intense type automatic regulator, the influence of own generator impedance on dynamic stability of the system operation is eliminated.

Thus, availability of different limits of power supplied to the network with the use of different generator excitation control devices helps to represent generator in equivalent circuit in different ways while calculating dynamic stability of an electric power system:

- a generator without excitation control is replaced by the synchronous direct EMF component  $E_q = \text{const}$  and the synchronous reactance  $x_d$  (Fig. 11.12, a)
- a generator with automatic excitation regulator of proportional type can be replaced by the source of EMF  $E'_q = \text{const}$  and the transient reactance  $x'_d$  (Fig. 11.12, b)
- a generator with automatic excitation regulator of intense type providing the voltage at the generator terminals stabilization is the source of the invariable voltage  $U_G = \text{const}$  which is considered as independent parameter of operation (Fig. 11.12, c).

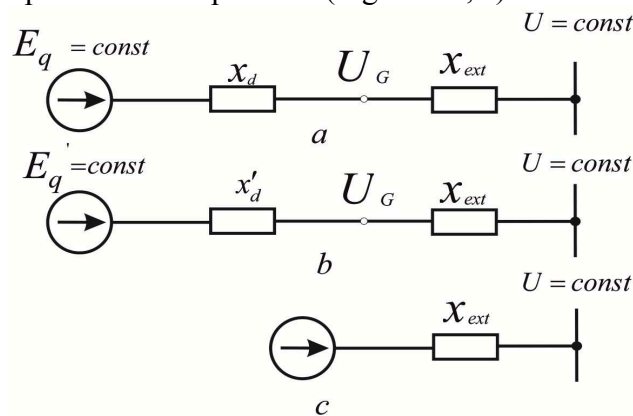


Fig. 11.12. Equivalent circuits for generator with account of automatic load transfer

### 11.5. Resulting operation stability

**Resulting stability** of operation is ability of an electric power system to restore on its own the synchronous operation mode of sources after short-time disturbance.

If the fallen out of step generators pull in synchronism after elimination the reason of dynamic stability disturbance, it is said that the system with generators connected to it possesses the resulting stability. In analysis of transients caused by dynamic stability disturbance, the concepts of asynchronous operation and of asynchronous torque are used.

Asynchronous operation is the operation mode taking place when deviations the rotational speed from the synchronous speed of motor or generator is great. Generator falling out of step is accompanied by sharp increase in rotational speed of rotor.

Under asynchronous operation if rotational speed is greater than synchronous one, the generator operates as asynchronous, and generates active power called asynchronous power. Reasons of asynchronous operation can be the following: absence of the field current; dynamic instability due to large disturbance; static instability of heavily overloaded PSS under small disturbance.

Asynchronous condition causes different abnormalities of PSS operation:

- periodical voltage lowering causing motors retarding and starters switching off in networks 0.4 kV. Stability of parallel operation of generators drops in PSS parts performing synchronously;
- protection discrimination can be violated due to the voltage decrease and increase of the current;
- oscillations of active power under which alternating-sign torque occurs on the turbine shaft resulting in extra mechanical forces;
- resonance oscillations can arise being dangerous for equipment and being able to violate synchronous operation of EPS system parts;
- deficit of active power within receiving part of the system increases if great active resistance is available between separate parts of EPS.

Asynchronously operating generator develops the asynchronous torque caused by free current that appear in the field winding and damper circuits in addition to the torque stipulated by its excitation. This additional torque occurs due to rotor motion relatively to the magnetic fields induced by the external EMF.

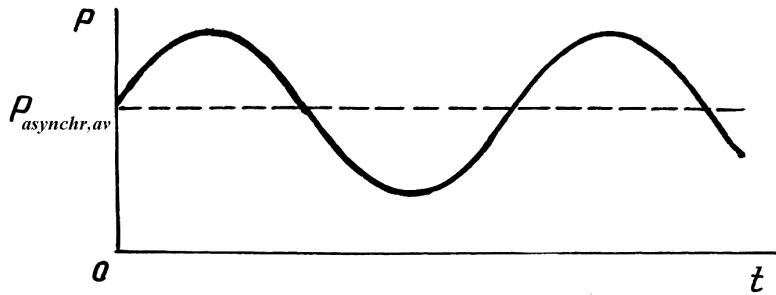


Fig. 11.13. Asynchronous power pulsation near a mean value

The generator asymmetry (salient-pole rotor, single-axis effect of the field circuit etc.) gives rise to asynchronous power pulsation near a mean value (Fig. 11.13). Reactive power varies in similar way as well as the voltage.

Thus, asynchronous torque of generator can be represented with two components: mean component and alternating-sign one. The first component depends upon type and construction of a generator as well as on mean slip, the second component does not influence greatly on the process of asynchronous operation, and it is neglected while calculating.

The mean asynchronous torque of the generator connected directly to buses of invariable voltage is determined by the expression

$$M_{asynchr} = U_C^2 \left\{ sT_d' (x_d - x_d') / [(1 + (sT_d')^2)x_d x_d'] + sT_d'' (x_d - x_d'') / [(1 + (sT_d'')^2)x_d x_d''] + sT_q'' (x_q - x_q'') / [(1 + (sT_q'')^2)x_q x_q''] \right\} / 2, \quad (11.37)$$

where  $s$  is the slip (in contrast to induction motors the positive slip here is the slip taking place when rotational speed of rotor being greater than the synchronous speed);  $T_d'$  is the time constant of the field winding under the stator winding closed;  $T_d''$  and  $T_q''$  are sub-transient time constants of generator on the direct and quadrature axes;  $x_d, x_d', x_d''$  are synchronous, transient and subtransient direct-axis reactance;  $x_q, x_q''$  are synchronous and sub-transient quadrature-axis reactance.

From (11.37) it follows that asynchronous torque of generator has the three components:

$$M_{asynchr} = M_d' + M_d'' + M_q'', \quad (11.38)$$

stipulated by action of excitation winding ( $M_d'$ ) as well as by damper direct-axis ( $M_d''$ ) and quadrature-axis ( $M_q''$ ) windings.

Dependences of the generator asynchronous torque and its components on the slip are shown in Fig. 11.14. Component  $M_q''$  has the greatest effect on the asynchronous torque.

The slip values that correspond to the greatest values of asynchronous torque components can be determined if expressions

$$sT_d' / (1 + (sT_d')^2), \quad sT_d'' / (1 + (sT_d'')^2) \quad \text{and} \quad sT_q'' / (1 + (sT_q'')^2)$$

are differentiated on slip, and the derivatives are equated to zero.

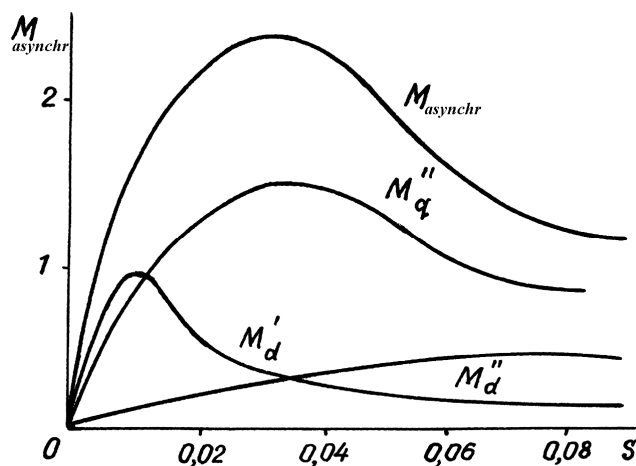


Fig. 11.14. Dependence of generator asynchronous torque and its components on slip

Then, from the equation

$$\partial M'_q / \partial s = \{T'_d [1 + (sT'_d)^2] - sT'_d \cdot 2s (T'_d)^2\} / [1 + (sT'_d)^2] = 0$$

find that

$$s = 1 / T'_d,$$

and from similar equations  $\partial M''_d / \partial s = 0$  and  $\partial M''_q / \partial s = 0$  have  $s = 1 / T''_d$  and  $s = 1 / T''_q$ .

It follows from (11.39) that the more time constant of circuit in which the free currents flow, the less the slip corresponding to the greatest asynchronous torque is.

### 11.6. Estimation of resulting operation stability

The study of resulting operation stability consists in determination conditions under which standard operation is restored in the case of short-time asynchronous operation of the PSS separate components occurrence. With it there is necessity to study as the process when unloaded synchronous machines pass from asynchronous operation to synchronous one, i.e. **the process of synchronization**, as well as the process of coming into step loaded synchronous machines which fell out of synchronism before and work asynchronously, i.e. the process of resynchronization. The study is necessary to clear up the reasons of asynchronous operation, and ways of its consequences elimination, and also for taking measures which facilitate the restoration of synchronous performance of electric installations.

Consider conditions of generators and motors synchronous performance restoration. As it was mentioned above, asynchronous operation of synchronous machines is the result of different reasons: disturbances of static and dynamic stability; excitation loss; asynchronous start; short-time interruptions of energizing; voltage lowering stipulated by SC, action of automatic re-closure or automatic transfer devices; self-starting of synchronous motors after power supply restoration.

Asynchronous operation is not dangerous for many synchronous generators but with it the active power produced by them decreases (this power is called the asynchronous active power). Usually, such generators consume great reactive power from the system which is necessary for making electromagnetic fields needed for asynchronous operation that can result in imbalance of reactive power within PSS as well in the voltage decrease in the nodal points, in disturbance of steady operation of other generators and motors. Partially, the risk of such effects is removed with the help of correct selection of reactive power sources and their location within the PSS and by use of regulating devices.

Permitting transition to asynchronous operation it is necessary to estimate possible consequences such as increase in mechanical efforts in the generator rotor when their rotational speed increases under asynchronous operation, and rise of stator current due to reactive power consumption from the network, decrease in generated active power etc. If variations of the indicated quantities are within permissible limits, the normal operation of generator can be restored without its disconnection. In this case it is supposed that the system maintain stability.

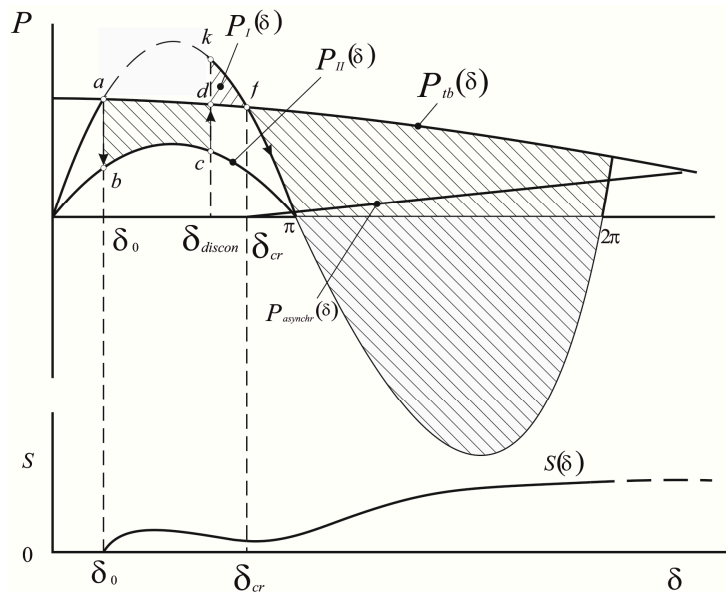


Fig. 11.15. Process of generator out of step falling

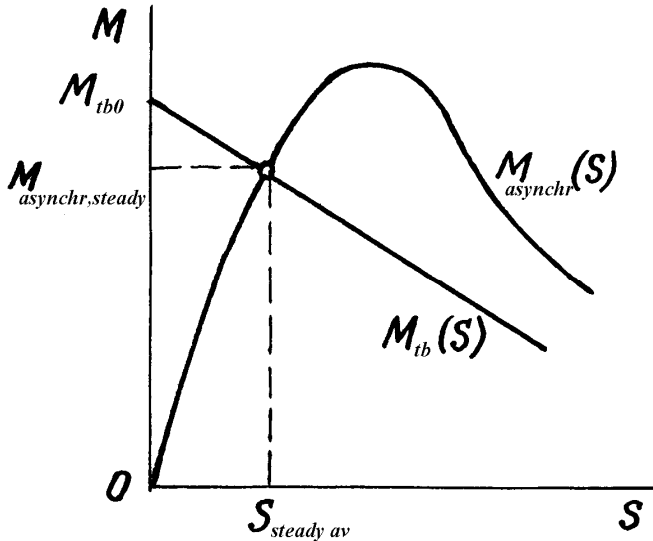


Fig. 11.16. Dependence of turbine and generator torque as a function of slip under asynchronous conditions

The process of a generator transition to asynchronous operation is shown in Fig. 11.15. The fact of dynamic operation stability disturbance (after the emergency condition disconnection, and transition to the power angle characteristic of initial operation with the help of automatic circuit re-closure) is illustrated by means of method of areas. Future progress of the process is characterized by increase in the slip and asynchronous torque (or the power ( $P_{asynchr}(\delta)$ )).

The torque of turbine is decreased owing to the effect of the prime mover rotational speed regulator. If pulsations of synchronous torque  $M_{synchr}$  are not taken into

consideration the condition of steady asynchronous operation  $M_{turb}(s) \approx M_{asynchr,steady}$  under which increase in rotational speed ceases is provided (Fig. 11.16).

The steady asynchronous condition is characterized by average values of asynchronous torque  $M_{asynchr,steady}$  and slip  $s_{steady\ av}$ . In this case the asynchronous active power by generator is

$$P_{*asynchr} \approx M_{*asynchr} \quad (11.40)$$

and reactive asynchronous power is

$$Q_{*asynchr} \approx P_{*asynchr} s / s_{cr}. \quad (11.41)$$

Under actual operating conditions the circuit between the generator and the system buses of invariable voltage decreases asynchronous torque to the value being determined by the expression  $M_{asynchr} = M_{asynchr(x_{ext}=0)} (x'_d / x_{ext})^2$ .

To determine admissibility of the asynchronous operation the greatest values of asynchronous active and reactive power that fit the greatest value of the slip  $s_{max}$  being determined by the synchronous torque pulsation are found.

Transition to the resynchronization process is possible under further decrease in the slip values caused by rotational speed of prime motors regulators or similar control action of operating staff. Slip can be determined at any instant by solution of the equation of relative motion of the generator rotor re-arranged to the form

$$sT_J \cdot ds / d\delta = M_{turb} - M_{synchr} - M_{asynchr}, \quad (11.42)$$

where  $s = d\delta / dt$ .



Integrating (11.42) within the slip and angle variation in the limits  $[s; s_{\max}]$  and  $[\delta; \delta_{\max}]$  respectively get  $0,5T_J(s_{\max}^2 - s^2) = \int_{\delta}^{\delta_{\max}} (M_{\text{turb}} - M_{\text{synchr}} - M_{\text{asynchr}}) d\delta$ ,

whence

$$s = \sqrt{s_{\max}^2 - (2/T_J) \int_{\delta}^{\delta_{\max}} M_{\Sigma} d\delta}. \quad (11.43)$$

If average value of the slip is decreased under steady asynchronous condition, then its transition through the zero value is possible. This necessary condition corresponding to the re-synchronizing process is met if

$$s_{\max}^2 = (2/T_J) \int_{\delta}^{\delta_{\max}} M_{\Sigma} d\delta \quad (11.44)$$

or

$$s_{\text{adm,av}} = \sqrt{(1/T_J) \int_{\delta}^{\delta_{\max}} M_{\Sigma} d\delta}. \quad (11.45)$$

Resynchronization takes place if

$$s_{\text{steady av}} < s_{\text{adm,av}}, \quad (11.46)$$

where  $s_{\text{steady av}}$  is average slip value under the steady asynchronous operation; and  $s_{\text{adm,av}}$  is average slip value under the resynchronization.

Resynchronization condition (11.46) is necessary but insufficient one. It means that to make the resynchronization successful under  $s=0$  it is necessary to observe the relation for torques:

$$M_{\text{synchr}} \geq M_{\text{turb}}. \quad (11.47)$$

Thus, successful resynchronization can be provided by the prime motor rotational frequency and torque control towards their decrease, or by the generator synchronous torque increase (by means of excitation control).

It determines the technique configuration needed for resynchronization process. In many cases these components can perform resynchronization of generators at thermal power stations of industrial enterprises. Specific measures to restore synchronism can be needed if connection of a power system parts is characterized by small assurance of operation stability.

The considered rough quantitative estimation of resulting stability of generators makes possible to perform only its qualitative study, and can be used for practical evaluations to determine whether resynchronization is possible. The key issues of more accurate study of the resulting stability of generators are described in specialized literature. The resulting stability can be accurately examined by computer-aided calculations.

Restoration of synchronous motors synchronous operation is performed for important mechanisms which operation maintenance is necessary by conditions of production technology, and required by safety conditions. It can be carried out by means of resynchronization; by resynchronization with automatic short-term unloading of working mechanism (if it is allowable on conditions of technological process) to the degree at which the electric motor pulls in step; or by electric motor disconnection, and its automatic restart.

The process of resynchronization is influenced by the following factors: characteristics of motor, its excitation system, level of loading, dependence of technological mechanism drag torque on slip, inertia moment of motor-mechanism unit, voltage on the motor terminals, and duration of supply interruption.

Process of a motor resynchronization can be conventionally divided into two stages: acceleration under  $M_{asynchr} > M_{WM}$  up to sub-synchronous rotational speed, and pulling into synchronism. Within the first stage the equation of synchronous motor motion is similar to that of asynchronous one. Motor accelerates up to sub-synchronous slip which is determined by equality of torques  $M_{asynchr}(s) = M_{WM}(s)$ . Within the second stage excitation is applied to a motor. Total electromagnetic torque produced by the motor is represented for practical calculations as

$$M = M_{asynchr} + M_{sf} + M_{sr} + M_{br}. \quad (11.48)$$

The torque components in equation (11.48) are:  
synchronizing torque which is determined by excitation

$$M_{sf} = E_q U \sin \delta / x_d; \quad (11.49)$$

reaction synchronizing torque stipulated by the rotor magnetic asymmetry

$$M_{sr} = 0,5U^2(x_d - x_q)\sin(2\delta) / (x_d x_q); \quad (11.50)$$

and retarding moment occurred due to currents induced in the stator winding when the motor works with excitation

$$M_{br} = (E_q / x_d)^2 (r_{st} + r_{net}) / (1 - s). \quad (11.51)$$

Here  $r_{st}$  and  $r_{net}$  are resistances of the stator winding, and the energizing network.

The process of motor resynchronization is estimated on equation (9.1) with account of (11.48) using the method of successive intervals. For rough estimation of the motor ability to come into step the following quantities are compared:

1) The drag torque of working mechanism under slip  $s = 0,05$  and the motor asynchronous torque under the same slip value with account of the voltage on motor terminals and of excitation system condition. With it such an inequality should be satisfied

$$M_{asynchr}(s=0,05) > M_{WM}(s=0,05). \quad (11.52)$$

The asynchronous torque of the motor under  $s = 0,05$  and  $U_* = 1$  is given in catalogues, and the drag torque of mechanism can be estimated on formula (9.8). Evaluation by condition (11.52) is approximate one (great reserve is available);

2) Definite critical slip  $s_{cr}$  under which the motor synchronization is still possible in the stage of starting or at self-starting with steady asynchronous operation slip. With it the following condition should be satisfied:

$$s_{cr} > s_{steady\ av}. \quad (11.53)$$

Estimation of resynchronization conditions for start and self-start of synchronous motors is described in chapter 13.

### 11.7. Computation facilities application to study operation stability

It is very difficult to describe operation and transients in complex PSS due to variety of interconnected indices as well as factors of influence. For purposeful study of transients it is simplified by means of separation of important factors and indices. The model of the studied effect is created on their basis.

The study of peculiarities of different models taking into consideration assumptions accounting properties of actual processes in a PSS permits to accomplish multiple-factorial and clearly evident investigation. It too makes possible automation of computing process and solving systems of equations describing both transient and steady processes with accuracy being sufficient for practice.

Depending of complexity of processes, different types of models are used which determine application of specific computational facilities.

When static operation stability is studied by “drifting” with the use of criterion of equality to zero of the characteristic equation absolute term or equal to it practical criteria, and to study and estimate dynamic stability within one or two cycles of oscillations by the method of successive intervals, static and dynamic models of alternating current are used.

The alternating current static model is a set of components (generators transformers, linear and loading components) which permits to simulate a single-phase equivalent circuit of the system at a definite scale.

The dynamic model of alternating current consists of more perfect elements modeling generators, motors, and load. It reproduces automatically both steady and transient processes in the system, and registers their indices automatically.

To evaluate static stability by the transient character under small disturbance of the system, and to evaluate static stability by the character of relative angles of synchronous machines and voltage variation, and to study all types of stability of the system with small number of sources, and taking into account influence of generators rotation speed and excitation regulators and their characteristics computers may be used. These are continuous machines which consist of a set of operation elements (direct-current amplifiers) and perform operations of addition, multiplication, integration, differentiation, functions transform, etc.

Time is independent physical variable for analog computers. System of equations describing transient within the considered electric system with account transients in its regulating devices is entered into such a machine. If solution is performed in real-time scale, the real regulators can be connected to the computer. Increase in number of sources in the system under study results in great increase in the number of operational elements of the model. It makes input difficult, and reduces accuracy of the solution.

To estimate stability of electric power systems on different criteria the computers are applied. They meet high requirements of accuracy with account of large number of factors. The only requirement for their application is availability of mathematic description of the transients and acting factors. The use of computers is also expedient in the case of wide range of the system indices and calculation conditions variations, and when estimation of taken assumptions is made.

To study stability of operation and analyze processes having complicated mathematical description the physical models can be applied. They are used to study technical means for the processes stability increase new regulators and for devices of automatic control development. Physical modeling is performed to reproduce physical phenomena with the help of model similar to those taking place within real system under study. Physical model is a decreased copy of the system where all elements (generators, transformers, PTL, loads, etc.) are made physically similar corresponding to real ones.

Combination of computations with the help of alternating current models, analog and digital computers with experimental investigations of physical models and actual PSS is the most effective way to study transients and operation stability.

**Test questions**

1. What equivalent circuits of synchronous generators are applied to estimate operation stability?
2. What are the peculiarities of PSS dynamic stability study at SC of different types?
3. What are the distinctive indications of static and dynamic stability of PSS?
4. What mode of generator operation is called the asynchronous one?
5. What is the asynchronous torque of generator?
6. Which assumptions are basis of the simplified methods of PSS dynamic stability evaluation?
7. What is the task of PSS dynamic stability study?
8. In what way can limiting angle and SC disconnection time be determined?
9. In what way is PSS dynamic stability evaluated by the angle  $\delta$  variation in time?
10. How is it possible to check PSS dynamic operation stability under initial operation retaining by means of automatic re-closure?
11. What are the peculiarities of PSS dynamic operation stability analysis when it has several power sources?
12. What are the types of automatic load transfer? What is the field of their application?
13. What are the reasons of synchronous machines asynchronous operation occurrence?
14. What is the synchronous generator and motor re-synchronizing?
15. What are the conditions for evaluation of motors pulling in synchronism possibility?

**Topics for essay**

1. Study of PSS dynamic operation stability with two-way feed.
2. Evaluation of PSS dynamic operation stability of on basis of refined methods.
3. Conditions of synchronous machines of technological plants re-synchronizing and ways of its performance.

## **CHAPTER 12: STABILITY OF LOAD NODES OPERATION UNDER SMALL DISTURBANCES**

12.1. Starting positions

12.2. Operation stability of induction and synchronous motors analysis

12.3. Account of influence of electric line parameters on stability of load node operation

12.4. Influence of reactive power compensation on stability of load node operation

12.5. Use of load static characteristics

Test questions

Topics for essay

### 12.1. Starting positions

Points of PSS, in which power takeoff and power distribution for energizing groups of electric consumers (electric motors, lighting equipment, furnaces, capacitor installations etc.) with different design characteristics and operation regimes take place, are called *the nodal load points or the load nodes*.

While studying PSS stability the loads can be represented as different analytical models in the system equivalent circuits depending on peculiarities of the considered problem. Completeness of the electric load mathematic description has essential influence on the results of electromechanical transient calculations.

To determine the completeness of load account it is necessary to base upon the required accuracy of final results, and cutting down the background information and computations.

Small disturbances of EPS operation may be initialized by the feeding power (variation in voltage and frequency) as well as changes in operation of the PSS itself and its using equipment (starts, torque fluctuations and motors overload stipulated by technological conditions, variations in number of feeding lines, control of some operation parameters values; routine switching within distribution network etc.). Under such conditions of electric power supply, properties and types of the load node using equipment have considerable effect on the operation stability.

If the load node is comparable by total consumed power with power of the energizing PSS, or is electrically remote from the power sources, its operation can be unstable under perturbations.

Operation stability of an industrial load node is studied in such a sequence:

- 1) The load node is substituted for its analytical model, and its parameters are determined;
- 2) Important parameters and a stability criteria for the given electric power supply circuit are selected;
- 3) The node marginal state and stability is estimated by critical values of essential variables.

The real load node substitution for the analytical model (equivalent representation) under the analysis of small disturbances is based on identity the transient indices being estimated with use of factual and equivalent parameters for the present process stage.

The load node with induction motors is substituted for analytical model as an equivalent induction motor which performance is described with the help of the same equations as of the actual motors. Substitution error depends upon the way it is carried out. By the results of substitution criteria analysis the following three groups were selected [10]:

- 1) Motor parameters averaging under each similar slip value proceeding from the assumption that actual motors have equal slip in the same instant of transient;
- 2) Replacement by the criteria of coincidence of transient active and reactive power consumed from network by the group of actual motors and their equivalent;
- 3) Replacement keeping invariable limits of the group of actual motors and their equivalent dynamic stability.

Mathematically, motor load is described in different ways depending on the number of electric motors forming the load node:

- In the load nodes with small number of electric motors each of them is taken into account by its equations and motion parameters, and is directly introduced into computations;
- In the load nodes containing groups of different electric motors which belong to one production enterprise, the groups are substituted for a small number of equivalent electric motors with parameters which are estimated according to definite rules using the data of actual motors;
- Large load nodes are described with use of a number of specific data concerning load components and supply network parameters as well as initial information being the result of probabilistic and statistical analysis.

Mutual influence of electric motors is one of important factors in study the operation stability of large load nodes. Calculation accuracy is not practically decreases if series of simplifications of equations describing each electric motor is assumed. For example, it is possible to ignore losses in the stator of electric motor, and consider the active power consumed from network as equal to the motor electromagnetic power. This power proportional to the torque of the motor if energizing

voltage has constant frequency. All mechanical losses can be related to a driven mechanism. A number of components of transient in synchronous, and especially induction motors can be ignored.

Under such simplifications, the equations of electric motors motion being used to describe large load nodes can differ from equations according to which electromagnetic transients within single electric motors are estimated. But it has not great influence on errors in final results of PSS load stability estimation.

The choice of the equivalence criterion depends on final purpose of the problem and its solution accuracy required. As for rough computation, the statistic parameters of a large load node model can be taken as parameters of an equivalent induction motor.

Diversity of types of asynchronous motors within load nodes is not significant, and it gives ability to take them into consideration using their actual parameters, and parameters for normal conditions. Determination of equivalent for large and different by technological purpose groups of synchronous motors is performed separately for salient- and non-salient-pole machines because of difference in their asynchronous characteristics, mechanical inertia constants, and characteristics of driven mechanisms.

Weighted mean values of synchronous motor parameters are used to perform rough computations of the load nodes stability.

As for salient-pole motors they are the following: the starting torque ratio is  $M_{*start} \approx 0,8$ ; the power factor is  $\cos \varphi_{rated} \approx 0,9$ ; the direct and quadrature axes synchronous reactance are  $x_{*d} \approx 1,3$  and  $x_{*q} \approx 0,85$ , the stator leakage reactance is  $x_{*\sigma} \approx 0,15$ ; the field leakage reactance and its time constant, if other windings or circuits are open, are  $x_{*f\sigma} \approx 0,21$  and  $T_{f0} \approx 2,4$ s; the leakage reactance and time constant of the damper winding on direct axis, if other windings are open, are  $x_{*cd} \approx 0,12$  and  $T_{cd0} \approx 0,08$  s; the leakage reactance and time constant of the damper winding on quadrature axis, if other windings are open, are  $x_{*cq} \approx 0,09$  and  $T_{cq0} \approx 0,06$  s; the load ratio is  $k_{lfact} \approx 0,85$ .

A load node with induction and synchronous motors is represented with the complete design model. Its parameters are determined with the help of equivalent for typical components of load, and described by means of static or dynamic load characteristics (see Art.12.5).

For estimation of the load nodes operation stability the static load properties and weighted mean values of complex estimation model parameters may be used. They are given in Tables 12.1 and 12.2 (power of motors is indicated in per units relatively the rated load node power). Approximate data concerning structure of load nodes with respect to the consumed power are:

- for industrial consumers: motors – 55%, static load – 45%;
- for municipal and residential consumers: induction motors – 30%, static load – 70%;
- for agricultural consumers: motors – 5%, static load – 95%.

Table 12.1  
Weighted mean values of node design model components with complex load

Load components	$S_{rated}$	$P_{rated}$	$\cos \varphi_{rated}$	$k_1$
	with account of synchronous motors	without account of synchronous motors		
Induction motors	0.80	1.00	0.80	0.70
Synchronous motors	0.14	–	0.90	0.85
Static load	0.55	0.54	0.81	1.00

Table 12.2  
Static load operation parameters

$U_*$	0.20	0.40	0.60	0.70	0.75	0.80	0.85	0.90	0.95	1.0	1.05	1.10
$P_*$	0.05	0.18	0.41	0.53	0.60	0.68	0.75	0.83	0.91	1.0	1.09	1.18
$Q_*$	0.03	0.12	0.27	0.34	0.39	0.43	0.52	0.62	0.77	1.0	1.35	1.90

Stability of load node operation is studied using the whole PSS equivalent circuit and its operation parameters. Depending on specific conditions the design circuit of power supply is reduced to one of the major types (Chapter 10). It makes possible to use practical stability criteria.

As a result, the four different design load node models are obtained. They differ in stability criteria used for analysis (Fig.12.1):

- The model in which voltage in the load node is independent variable that is invariant to using equipment conditions. This model permits to estimate operation stability independently for each of specific groups of using equipment (Fig.12.1, a) by their basic criteria.

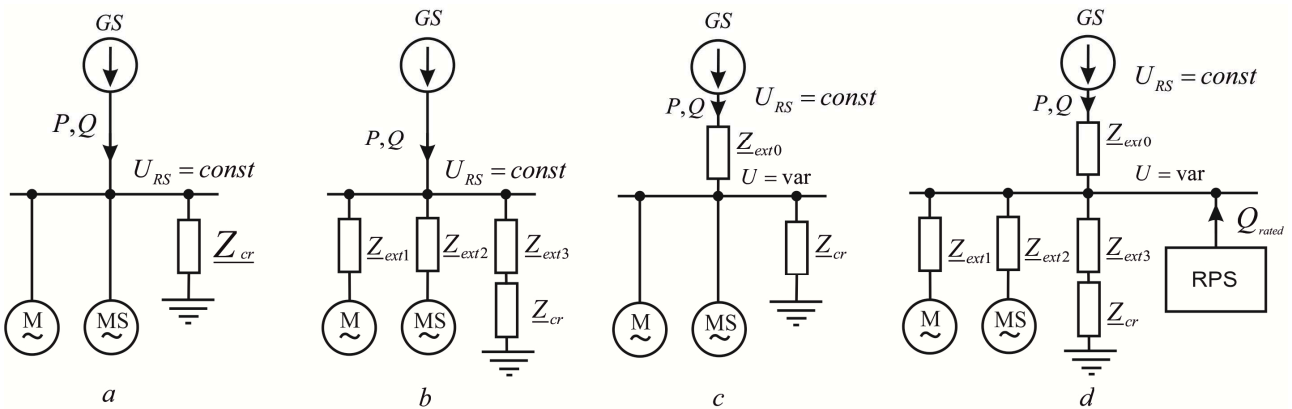


Fig. 12.1. Design models of load node

- The model in which specific groups of using equipment are radially connected with buses of load node by means of external resistances (Fig.12.1, b) voltage on which is independent operation variable.

- The model in which specific groups of using equipment are connected with load node by means of common external resistance, and independent variable of operation is EMF of power source (Fig.12.1, c).

- The model in which load node has all specific components including a reactive power source (RPS) (Fig.12.1, d).

Below, the procedure of stability estimation for each of the four design models of a load node is described.

### 12.2. Operation stability of induction and synchronous motors analysis

Estimation of a load node operation stability taking selected groups of induction and synchronous motors separately is performed for design models of the load node according to Fig.12.1(a or b ). Voltage on the load node buses is the essential independent variable. To estimate stability the key stability criteria (10.12) and (10.23) are applied. With it, there is necessity to take into consideration the characteristics of mechanisms as well as their loading.

The main condition of stability keeping for providing operation stability of induction motors (or their equivalent) is inequality  $d(M - M_{WM})/ds < 0$ . The equality  $d(M - M_{WM})/ds = 0$  corresponds to its limit. Under  $M_{WM} = const$  and direct connection of the motors to the load node



buses, the critical parameters corresponding to its static stability limit are determined with the help of expressions (10.24) – (10.26). When motors are connected to a load node by means of specific external resistances (Fig.12.1, b), the estimation of critical parameters of operation as well as stability margin is performed in similar way, but these resistances should be taken into consideration.

If active resistances are neglected, then  $x'_s = x_s + x_{ext}$ . In these cases, the following expressions are used:

$$\left. \begin{aligned} s'_{cr} &= r_2 / (x_s + x_{ext}) = s'_{cr} / (1 + x_{ext} / x_s); \\ P'_{max} &= U_{RS}^2 / [2(x_s + x_{ext})] = P_{max} / (1 + x_{ext} / x_s); \\ U'_{RS,cr} &= \sqrt{2mP_{rated}(x_s + x_{ext})} = U_{RS,cr} \sqrt{1 + x_{ext} / x_s}; \\ K_{saf} &= (s_{rated} - s'_{cr}) 100 / s_{rated}; \\ K_{vaf} &= (U_{RS} - U'_{RS,cr}) 100 / U_{RS}. \end{aligned} \right\} (12.1)$$

Availability of external resistances through which the induction motors are connected to the load node reduces the slip value being limiting by condition of static stability.

If the characteristic of the loading mechanism  $M_{WM} = f(s)$ , is assigned, the critical operation parameters are found solving system of equations:

$$\left. \begin{aligned} M &= M_{WM}; \\ dM/ds &= dM_{WM}/ds. \end{aligned} \right\} (12.2, a)$$

With account expressions (11.6) and (12.1), after relevant transformations, equations (12.2, a) take on form

$$\left. \begin{aligned} 2M_{max} U_{RS,cr}^2 s'_{cr} s_{URS,cr} / \left[ (1 + x_{ext} / x_s) (s'^2_{cr} + s_{URS,cr}^2) \right] &= \\ = M_{WM,st} + (M_{WM0} - M_{WM,st}) (1 - s_{URS,cr})^p / (1 - s_{rated})^p; \\ 2M_{max} U_{RS,cr}^2 s'_{cr} (s'^2_{cr} - s_{URS,cr}^2) / \left[ (1 + x_{ext} / x_s) (s'^2_{cr} + s_{URS,cr}^2) \right] &= \\ = -p (M_{WM0} - M_{WM,st}) (1 - s_{URS,cr})^{p-1} / (1 - s_{rated})^p. \end{aligned} \right\} (12.2, b)$$

After solving equation system (12.2, b), it is possible to determine the parameters of operation  $U_{RS,cr}$  and  $s_{URS,cr}$  for limiting by stability condition where corresponding to the static stability bound slip  $s_{URS,cr}$  will be more than the critical slip  $s'_{cr}$  under condition that  $M < M_{max} / (1 + x_{ext} / x_s)$ .

It is assumed that  $s_{URS,cr} = s'_{cr}$  when computations are approximate. After solving equation system (12.2, b) it is obtained:

$$U_{RS,cr} = \sqrt{\left[ M_{WM,st} + (M_{WM0} - M_{WM,st}) (1 - s_{URS,cr})^p / (1 - s_{rated})^p \right] \times (1 + x_{ext} / x_s) / M_{max}} \quad (12.3)$$

When  $p = 0$ , the particular case  $M_{WM0} = const$  takes place, and expression defining the critical voltage is transformed into corresponding expression following from (12.1).

The critical voltage on the motor terminals is determined under the assumption of equality of the stator current  $I_1$  and the reduced rotor current  $I'_2$ . As

$$I_1 = U_{RS,cr} / \sqrt{(r_2/s'_{cr})^2 + (x_s + x_{ext})^2},$$

and taking into consideration (12.1) that

$$I_1 = U_{RS,cr} / \sqrt{2} (x_s + x_{ext})$$

the expression determining the critical voltage on the motor terminals can be written in the form of:

$$U_{M,cr} \approx I'_2 \sqrt{(r_2/s'_{cr})^2 + x_s^2} = U_{RS,cr} \sqrt{1 + 1/(1 + x_{ext}/x_s)} / \sqrt{2}. \quad (12.4)$$

Static stability of synchronous motors connected up to the load node having constant values of voltage and frequency is broken under limiting condition  $d(M - M_{WM})/d\delta = 0$ . Taking into account dependences (10.8) and (9.8) this condition can be represented as

$$d(M - M_{WM}) / d\delta = \left[ \begin{array}{l} (dE_q/d\delta) U_{RS} \sin \delta + \\ + U_{RS} E_q \cos \delta \end{array} \right] / (x_d + x_{ext}) = 0. \quad (12.5)$$

If there are no automatic load transfer devices for motors, the derivative  $dE_q/d\delta = 0$  and the limiting by static stability operation corresponds to the angle value  $\delta = \pi/2$  then

$$\left. \begin{array}{l} P_{\max} = U_{RS} E_q / (x_d + x_{ext}); \\ U_{RS,cr} = m P_{rated} (x_d + x_{ext}) / E_q. \end{array} \right\} \quad (12.6)$$

Electromotive force  $E_q$  in parts by its no-load value is determined with the help of the expression:

$$E_{*q} = \frac{U_{RS}^4 - U_{RS}^2 Q (X_d + X_q) + (P^2 + Q^2) X_d X_q}{U_{RS} \sqrt{U_{RS}^4 - 2U_{RS}^2 Q X_q + (P^2 + Q^2) X_q}}, \quad (12.7)$$

where  $X_d = x_d + x_{ext}$ ;  $X_q = x_q + x_{ext}$ .

If automatic controller of excitation of proportional type is available the synchronous motor on the analogy of generator can be put equal to the transient reactance  $x'_d$  and EMF  $E' = const$ . The latter quantity can be found by the formula

$$E' = \sqrt{U^4 - 2U^2 Q x'_d + (P^2 + Q^2) x_d'^2} / U.$$

In this case the critical voltage at terminals of the motor is expressed by means of the dependence

$$U'_{RS,cr} = mP_{rated} (x'_d + x_{ext}) / E' \quad (12.8)$$

and it is always less than critical voltage which is determined with the help of expression (12.6) when automatic load transfer is not available as the transient reactance is much less than synchronous one.

Availability of external resistance connecting the load node with induction and synchronous motors reduces limiting values of the largest by static stability active power and increases value of the critical voltage of the load node. It toughens requirements for stability of the energizing voltage.

### 12.3. Account of influence of electric line parameters on stability of load node operation

If typical groups of load node motors are connected to energizing centre with the voltage  $U_{RS} = const$  via electric power network (Fig.12.1, c), the conditions of load node operation stability considerably depend on parameters of the network ( $\underline{z}_{ext}$ ) and condition of all the consumers. In this case, voltage in the load node varies, and its values depend on variation of the indicated factors. Therefore stability operation of a load node is estimated on the basis of independent variable – the voltage of the load node using the indirect criteria

$$dE_{equiv} / dU > 0 \quad (12.9)$$

or

$$d\Delta Q / dU < 0. \quad (12.10)$$

While using criterion (12.9) the analytical dependence between operation variables can be established with the help of static characteristics of load node  $P_{ld} = F_1(U)$  and  $Q_{ld} = F_2(U)$ . It is

$$E_{equiv} = \sqrt{(U^2 + P_{ld}r_{ext} + Q_{ld}x_{ext})^2 + (P_{ld}x_{ext} - Q_{ld}r_{ext})^2} / U. \quad (12.11)$$

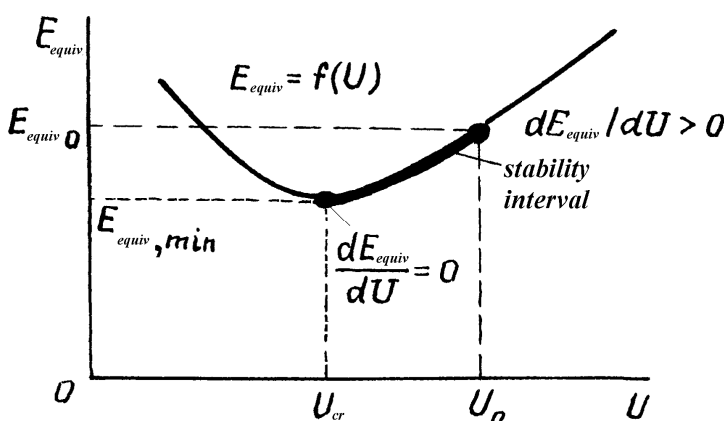


Fig. 12.2. Evaluation of load node static stability by the criterion  $dE_{equiv} / dU > 0$

This expression study within the domain of the functions  $F_1(U)$  and  $F_2(U)$  has its aim to determine coordinates  $E_{equiv,min}$  and  $U_{cr}$  of function (12.11) minimum corresponding to the limit of static operation stability  $dE_{equiv} / dU = 0$  (Fig.12.2).

According to criterion (12.10) the static operation stability is evaluated by reactive power in the load node imbalance caused by the voltage decrease.

For the point of operation balance, the condition  $Q_{RS} = Q_{ld}$ , and within its neighborhood – inequality  $d(Q_{RS} - Q_{ld}) / dU > 0$ , should be met. The way of the reactive power increment

$$\Delta Q = Q_{RS} - Q_{ld} \quad (12.12, a)$$

study depends on information available concerning the load node. If static characteristics of the load are known, the conditions of static stability keeping are determined according to recommendations given in Chapter 10. Otherwise, the expression

$$\Delta Q = Q_{RS} - (Q_{SM} + Q_{IM}) \quad (12.12, b)$$

is analyzed graphically by dependences of its right side components on the load node voltage. The aim of graphical and analytical study is to determine the boundary of static stability operation keeping on the basis of condition  $d\Delta Q/dU = 0$ . In this case, the study method is similar to that in Fig.10.9.

The components of expression (12.12, b) are determined in the following way. Dependence of reactive power incoming from EPS against voltage of load node is described by means of equation (10.19), and if  $Q_{ld}r_{ext} \approx 0$  – using equation obtained by appropriate transformation of (12.11):

$$Q_{RS} = \left( -U^2 - P_{ld}r_{ext} + \sqrt{E_{equiv}^2 U^2 - P_{ld}^2 x_{ext}} \right) / x_{ext}. \quad (12.13)$$

Reactive power consumed by equivalent induction motor that is determined by means of expression (10.29) can be calculated by the formula:

$$Q_{IM} = Q_{IM,rated} \left[ c Q_{\mu} / Q_{\mu,rated} + (1-c) Q_s / Q_{s,rated} \right],$$

where

$$c = Q_{\mu,rated} / Q \approx 1 - 1 / \left[ \left( m_{\max} + \sqrt{m_{\max}^2 - 1} \right) \operatorname{tg} \varphi_{rated} \right]. \quad (12.14)$$

If the ratios  $Q_{\mu} / Q_{\mu,rated}$  and  $Q_s / Q_{s,rated}$  under the condition of frequency constancy are replaced by the values

$$Q_{\mu} / Q_{\mu,rated} \approx U / U_{rated} = U_*;$$

$$Q_s / Q_{s,rated} = \frac{m \left( m_{\max} + \sqrt{m_{\max}^2 - 1} \right)}{m_{\max} U_*^2 / m + \sqrt{\left( m_{\max}^2 U_*^2 / m \right)^2 - 1}},$$

we get expression for determination of reactive power basing on voltage of the load node (catalogue data with account of external resistances are recalculated by (12.1)):

$$Q_{IM} \approx \frac{c U_*^2 + (1-c) m \left( m_{\max} + \sqrt{m_{\max}^2 - 1} \right)}{m_{\max} U_*^2 + \sqrt{m_{\max}^2 U_*^4 / m^2 - 1}} Q_{IM,rated} \quad (12.15, a)$$

or

$$Q_{IM} = Q_{IM,rated} \left[ \frac{c U_*^2 + \frac{m \cdot \operatorname{ctg} \varphi_{rated}}{\left( m_{\max} U_*^2 / m + \sqrt{m_{\max}^2 U_*^4 / m^2 - 1} \right)}}{\left( m_{\max} U_*^2 / m + \sqrt{m_{\max}^2 U_*^4 / m^2 - 1} \right)} \right]. \quad (12.15, b)$$

On the threshold of static operation stability  $s = s_{cr}$  and  $m = m_{\max} U_{*cr}^2$ . With it, critical value of terminal voltage of the induction motor is determined by expression:

$$U_{*cr} = \sqrt{m/m_{\max}}, \quad (12.16)$$

and critical value of the reactive power consumed by it – with the help of the expression:

$$Q_{IM,cr} = Q_{IM,rated} \left[ cm/m_{\max} + (1-c)m \left( m_{\max} + \sqrt{m_{\max}^2 - 1} \right) \right]. \quad (12.17, a)$$

The latter expression is simplified, taking into account (12.14), and takes on the form:

$$Q_{IM,cr} = Q_{IM,rated} m \left( c/m_{\max} + 1/\operatorname{tg} \varphi_{rated} \right). \quad (12.17, b)$$

Stability of a synchronous motor operation can be studied without taking into account its saturation variations and difference in armature reaction by direct and quadrature axes of the salient-pole rotor ( $x_d \approx x_q$ ).

Taking into account automatic load transfer and its adjustment which influences both value and direction of exciting current the reactive power of synchronous motor can be determined on the formula:

$$Q_{SM} = U \left( U - E_{q0} \cos \delta \right) / x_d, \quad (12.18)$$

where  $E_{q0}$  is EMF of no-load condition stipulated by the field current and determined with the expression:

$$E_{q0} = \sqrt{U^2 + \frac{2U_{rated}^2 x_{*d} Q_{SM}}{S_{rated}} + \frac{U_{rated}^4 \left( P_{SM}^2 + Q_{SM}^2 \right) x_{*d}^2}{S_{rated}^2 U^2}}. \quad (12.19)$$

If  $E_{q0} < U$ , the motor consumes reactive power from network; if  $E_{q0} = U$  it operates with  $\cos \varphi = 1,0$  and  $Q_{SM} = 0$ ; if  $E_{q0} > U$  (it is achieved by the field current increase) the motor generates reactive power into network.

For the loaded synchronous motor the EMF produced in the stator winding by the resulting magnetic flux in the air gap can be determined by the formula:

$$E_{\delta} = \sqrt{U^2 + \frac{2U_{rated}^2 x_{*\sigma} Q_{SM}}{S_{SM,rated}} + \frac{U_{rated}^4 \left( P_{SM}^2 + Q_{SM}^2 \right) x_{*\sigma}^2}{S_{SM,rated}^2 U^2}}, \quad (12.20)$$

where  $x_{*\sigma}$  is the reactance of the motor (in the case of salient-pole motor  $x_{\sigma} = (0,6...0,7)x_d''$ ).

Reactive power produced by a synchronous motor depends greatly on the short-circuit ratio which is a design value of a motor (in calculation it is assumed to be  $1/x_{*d}$ ), and on the ratio of the field current change. The field current relation with the essential variable (the network voltage) while studying stability of load node operation is predetermined with by structure of synchronous motor excitation system under the conditions:

- The field current does not depend on the network voltage if automatic or manual control of excitation is not available, and if the motor winding is fed by the exciting machine;
- If the field winding is energized by a converter and automatic field control is not available, the field current is proportional to the network voltage;
- If automatic field control is available, the field current increases when the armature voltage reduces.

Dependence of the armature EMF against the field current of synchronous motor is determined by the expression:

$$\begin{aligned} E_{q0} / U_{rated} &= k_{\mu} I_{\epsilon} / I_{\epsilon,x} = (I_f / I_{f,rated}) / (I_{f,no-load} / I_{f,rated}) = \\ &= k_f / k_{f,no-load} \end{aligned} \quad (12.21)$$

where  $k_{\mu}$  is the factor characterizing the saturation grade of the armature core ( $k_{\mu} = 1$  if saturation is not taken into account);  $I_{f,no-load}$  is the field current at no-load;  $I_{f,rated}$  is the rated field current;  $k_f$  is the field current ratio depending on the excitation system;  $k_{f,no-load} \approx 1 / (x_d m_{\max} \cos \varphi_{rated})$  is the same at the motor no-load condition.

Under the field current variation the reactive power of synchronous motor changes that can be easily seen if expressions (12.19) and (12.21) are compared:

$$\frac{I_f}{I_{f,no-load}} = \frac{\sqrt{U^2 + \frac{2U_{rated}^2 x_{*d} Q_{SM}}{S_{SM,rated}} + \frac{U_{rated}^4 (P_{SM}^2 + Q_{SM}^2) x_{*d}^2}{(S_{SM,rated} U)^2}}}{U_{rated}}$$

or

$$\begin{aligned} & \left( Q_{SM} / S_{SM,rated} \right)^2 (x_{*d} / U_*)^2 + \left( Q_{SM} / S_{SM,rated} \right) 2x_{*d} + \\ & + U_* + \left[ x_{*d} P_{SM} / (S_{SM,rated} U_*) \right]^2 - \left( k_f / k_{f,no-load} \right)^2 = 0 \end{aligned} \quad (12.22)$$

where  $U_* = U / U_{rated}$ .

Solving equation (12.22), it is obtained:

$$\frac{Q_{SM}}{S_{SM,rated}} = \frac{\left[ \sqrt{\left( U_* k_f / k_{f,no-load} \right)^2 - U_*^2} - \left( x_{*d} P_{SM} / S_{SM,rated} \right)^2 \right]}{x_{*d}} \quad (12.23)$$

This dependence can be used to study the reactive power of synchronous motor as a function of the voltage on terminals under known law  $k_f = f(U_*)$  of the field control.

For the limit at which the load node static operation stability is kept the critical voltage on the synchronous motor terminals, with account of (12.6) and if the field current is controlled according to (12.21), is determined by means of the expression:

$$U_{*cr} = P_{SM} x_{*d} / \left[ S_{SM,rates} (k_f / k_{f,no-load}) \right] \quad (12.24)$$

After this expression has been substituted into (12.23) it is obtained:

$$\begin{aligned} (Q_{SM} / S_{SM,rates})_{cr} &= -U_{*cr}^2 / x_{*d} = \\ &= -P_{SM}^2 x_{*d} / \left[ S_{SM,rates}^2 (k_f / k_{f,no-load}) \right] \end{aligned} \quad (12.25)$$

It follows from equality (12.25) that at the limit of the load node static operation stability the synchronous motor consumes reactive power from the network.

#### 12.4. Influence of reactive power compensation on stability of load node operation

Reactive power enters the load node from the PSS but it can be generated by the local RPS such as a capacitor bank, synchronous condenser, valve RPS as well as by the synchronous motor (Fig. 12.1, d).

Reactive power of a capacitor bank depends greatly on voltage of the node (Fig.12.3, curve 1):

$$Q_{CB} = Q_{CB,rated} \left( U/U_{rated} \right)^2 = Q_{CB,rated} U_*^2, \quad (12.26)$$

where  $Q_{CB,rated} = \omega C_{CB} U_{rated}^2$  is rated power of the capacitor bank;  $Q_{CB}$  is the resulting battery capacitance.

Synchronous condenser (synchronous motor) can generate and consume reactive power which is equal to

$$Q_{SC} = Q_{SC,rated} \left( U_* E_{*q} - U_*^2 \right) / x_{*d}, \quad (12.27)$$

where  $Q_{SC,rated}$  is the rated power of the synchronous condenser, and  $E_{*q}$  is its synchronous EMF.

Operation condition of a synchronous condenser is determined by its excitation system according to V-curve (Fig.12.4): rising branch corresponds to excitation at which  $E_q > U$ , and reactive power is generated (Fig.12.3, curve 2); descending branch corresponds to excitation at which  $E_q < U$ , and reactive power is consumed from the network (Fig.12.3, curve 3).

A static rectifier valve RPS consists of an uncontrollable capacitor bank, a controllable choke, and a control unit or capacitor bank regulated by a thyristor switch.

Reactive power of the RPS with a controllable choke, having parallel connection of power elements (Fig. 12.5, a, b), is determined with the help of the expression:

$$Q_{LC} = Q_L - Q_C = Q_C (x_C/x_L - 1), \text{ at } Q_{LC} \in \{0; Q_{LC}\} \quad (12.28,a)$$

where  $Q_L = U^2/x_L = \text{var}$ ;  $Q_C = U^2/x_C = \text{const}$ , and in the case of series connection (Fig.12.5, c, d) - by the expression:

$$\begin{aligned} Q_{LC} &= U^2 / (x_C - x_L) = \\ &= Q_C / (1 - x_L/x_C) \end{aligned} \quad (12.28, b)$$

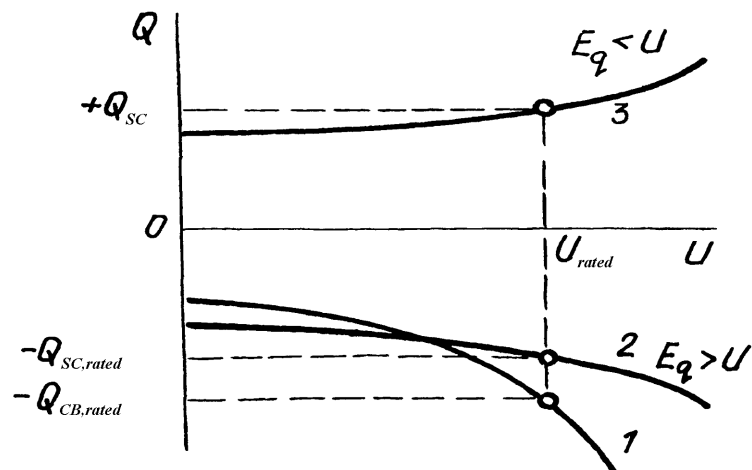


Fig. 12.3. Dependence of reactive power on voltage of a node: 1 – for a capacitor bank; 2 and 3 – for a synchronous condenser

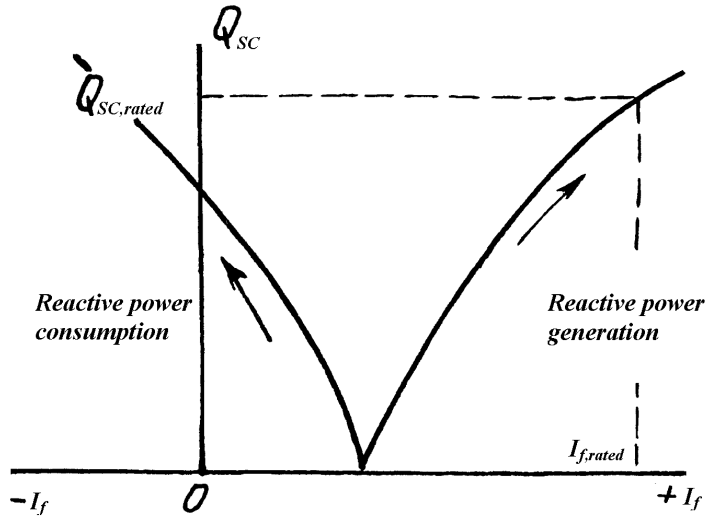


Fig. 12.4. V-curve of synchronous condenser

It can vary from the rated value  $Q_C$  to the greatest one of

$$Q_{LC \max} = U^2 / [x_C (1 - x_L/x_C)].$$

Upper limit depends on the greatest permissible voltage value on the capacitor bank (generated reactive power increases if the voltage grows; the voltage increase is provided by creation conditions close to the voltage resonance when  $x_L \approx x_C$ ).

Reactive power of the RPS with thyristor switch can be smoothly regulated depending on the current  $I_{CB}$  flowing through the capacitor bank:

$$Q = I_{CB}^2 / (\omega C_{CB}).$$

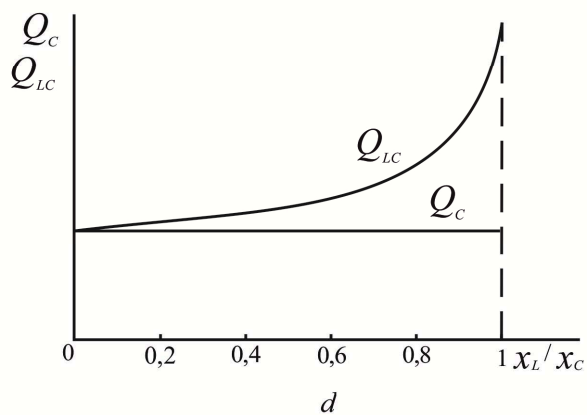
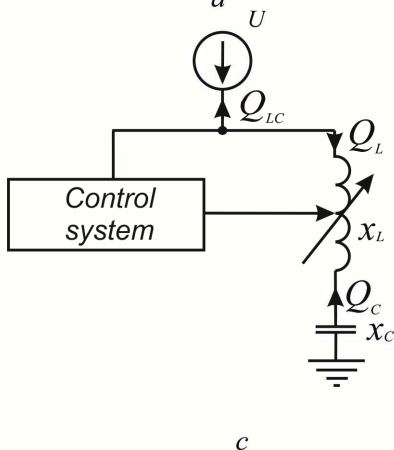
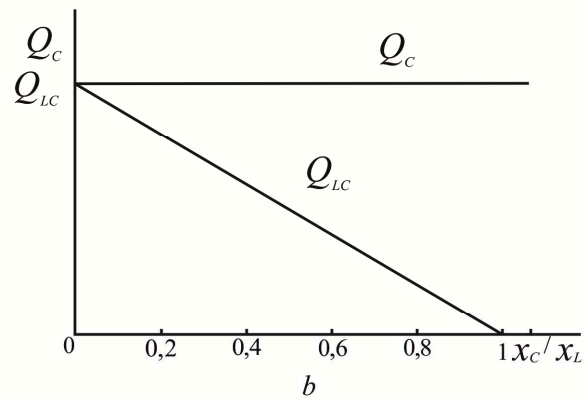
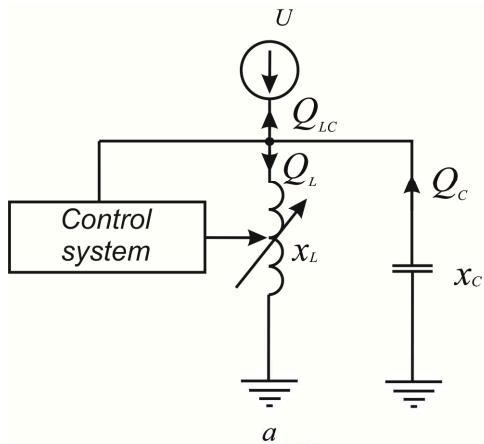


Fig. 12.5. Circuits and characteristics of static valve reactive power source

The main purpose of the local RPS is unloading the feeding and distributive networks of an industrial enterprise from the flow of reactive energy. The capacitor bank can be located at any point of an electric power network. The synchronous condenser is placed in a large load node having voltage of 6 to 10 kV, and it is usually concurs with a district substation of EPS. The static valve RPS with parallel connection of power elements is applied in an electric power supply system feeding powerful electric using equipment with sharply varying load where it is used as balancing, filtering and compensating or filtering and balancing device.



Owing to the local RPS the power loss in the network decrease, and voltage level and its quality at consumers improve, and conditions of stability of the load node operation vary.

For assessment the load nodes operation with reactive power compensation, location of compensating devices at the nodes with induction motors and application of a local PRS at the node with complex load is important.

When the RPS is located at the node of asynchronous load the conditions of external electric power supply vary, and it results in electric using equipment voltage increase due to reduction of the voltage drop in the network. The voltage drop is determined by the expression:

$$\Delta \dot{U} = \left\{ Pr_{ext} + (Q_c - Q_{rated})x_{ext} + j \left[ Px_{ext} - (Q_c - Q_{RPS})r_{ext} \right] \right\} / U. \quad (12.30)$$

Equivalent parameters of the external network when synchronous condensers or motors generating reactive power have been connected up (Fig.12.6, a, c), can be determined by the formula:

$$\begin{aligned} U_{RS,equiv} &= \left( U_C / x_{ext} + E_{q0} / x_d \right) / \left( 1/x_{ext} + 1/x_d \right) = \\ &= U_{RS} \left[ 1 + \left( E_{q0} / U_{RS} \right) / \left( x_{ext} / x_d \right) \right] / \left( 1 + x_{ext} / x_d \right); \quad (12.31) \\ x_{equiv} &= 1 / \left( 1/x_{ext} + 1/x_d \right) = x_{ext} \left( 1 + x_{ext} / x_d \right). \end{aligned}$$

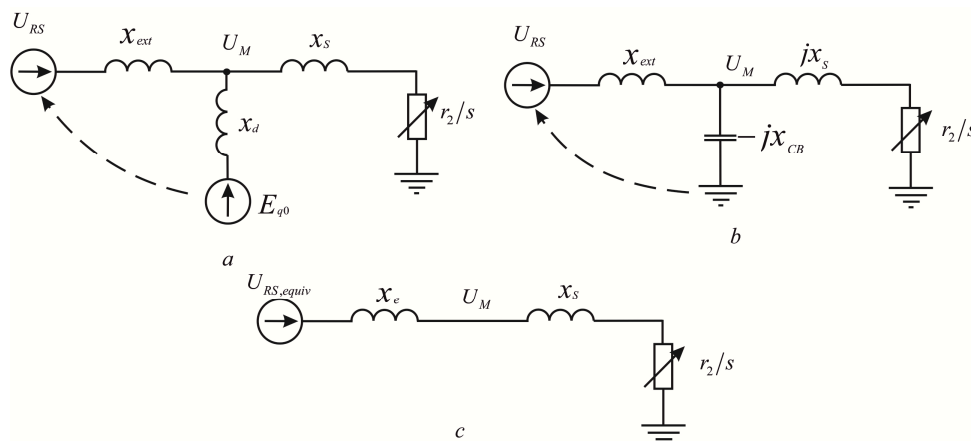


Fig. 12.6. Equivalent circuit of induction load node: a – with a synchronous condenser or motor; b – with a capacitor bank; c – general equivalent circuit

It follows from (12.31) that if  $E_q > U_{RS}$  the conditions of stability of a load node operation improve owing to increase the values of critical parameters determined by equation (12.1) as  $U_{RS,equiv} > U_{RS}$  and  $x_{equiv} < x_{ext}$ .

When a capacitor bank is connected (Fig.12.6, b) the equivalent parameters of the external network are determined by the expressions:

$$\left. \begin{aligned} U_{RS,equiv} &= U_{RS} / \left\{ jx_{ext} \left[ 1 / (jx_{ext}) - 1 / (jx_{CB}) \right] \right\} = \\ &= U_{RS} / \left( 1 - x_{ext} / x_{CB} \right); \\ x_{equiv} &= 1 / \left[ 1 / (jx_{ext}) - 1 / (-jx_{CB}) \right] = x_{ext} / \left( 1 - x_{ext} / x_{CB} \right), \end{aligned} \right\} \quad (12.32)$$

This means that the voltage of using equipment at the node increases ( $U_{RS,equiv} > U_{RS}$ ) as well as the coupling resistance of the node with buses of infinite power ( $x_{equiv} > x_{ext}$ ). It results in variations of critical parameters. The critical slip (12.1) decreases up to the value of

$$s_{equiv,cr} = s_{cr} / \left\{ 1 + x_{ext} / \left[ x_s (1 - x_{ext} / x_{CB}) \right] \right\}, \quad (12.33)$$

and the critical voltage increases up to

$$U_{RS,equiv,cr} = U_{RS,cr} \sqrt{1 + x_{ext} / \left[ x_s (1 - x_{ext} / x_{CB}) \right]}. \quad (12.34)$$

In this case the static stability margin of load node reduces.

The greatest value of active power taking into consideration (12.32) and (12.1) can be determined by the formula:

$$\begin{aligned} P_{equiv,max} / P_{max} &= \\ &= \left\{ U_{RS,equiv}^2 / \left[ 2(x_s + x_{equiv}) \right] \right\} / \left\{ U_C^2 / \left[ 2(x_s + x_{ext}) \right] \right\} \quad (12.35) \\ &= (1 + x_{ext} / x_s) / \left\{ \left[ 1 + (x_{ext} / x_s)(1 - x_s / x_{CB}) \right] (1 - x_{ext} / x_{CB}) \right\} \end{aligned}$$

It means that conditions of operation stability of the load node by this variable depend on relations between  $x_{ext}$ ,  $x_s$ , and  $x_{CB}$ . Increase of the capacitor bank power has negative influence on stability of asynchronous load node as  $x_{CB}$  value decreases.

If a node of complex load is located far from the source of invariable voltage and has a local RPS (Fig.12.1, d), static stability of such a node should be studied by criterion (12.1). In this case, the equation of reactive power imbalance for the node is

$$\Delta Q = Q_c + Q_{RPS} - Q_{ld}. \quad (12.36)$$

Function (12.36) is investigated for extremum graphically. When the capacitor bank or the synchronous condenser (synchronous motor) with  $E_q > U_{RS}$  is connected the summary characteristic of the load and RPS reactive power becomes quieter (Fig.12.7).

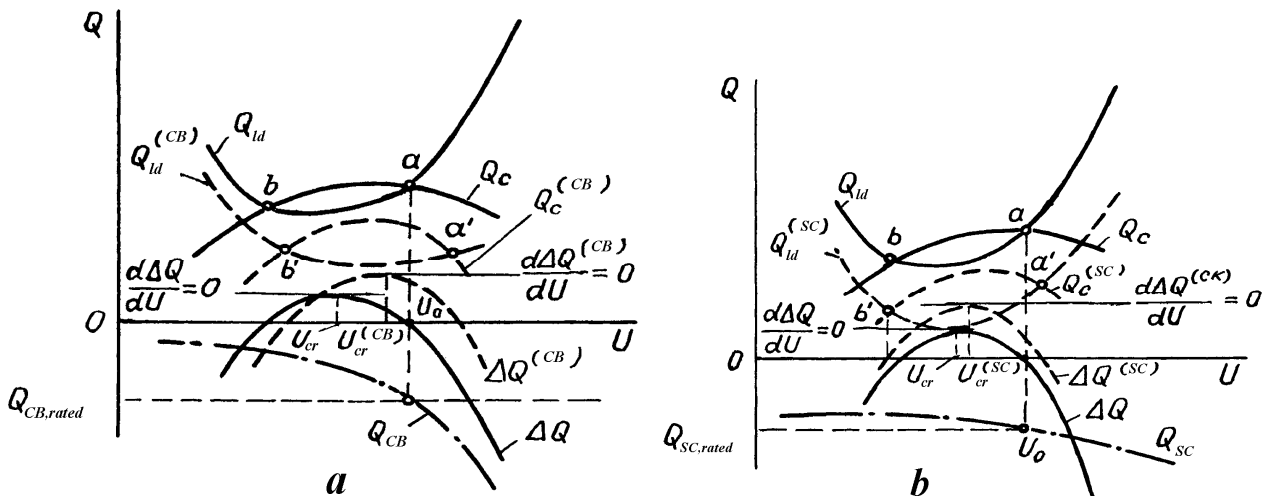


Fig. 12.7. Components of complex load reactive power variation ( $Q_{cb,nom} = Q_{sc,nom}$ ) for the cases: a – a capacitor bank is connected; b – a synchronous condenser is connected

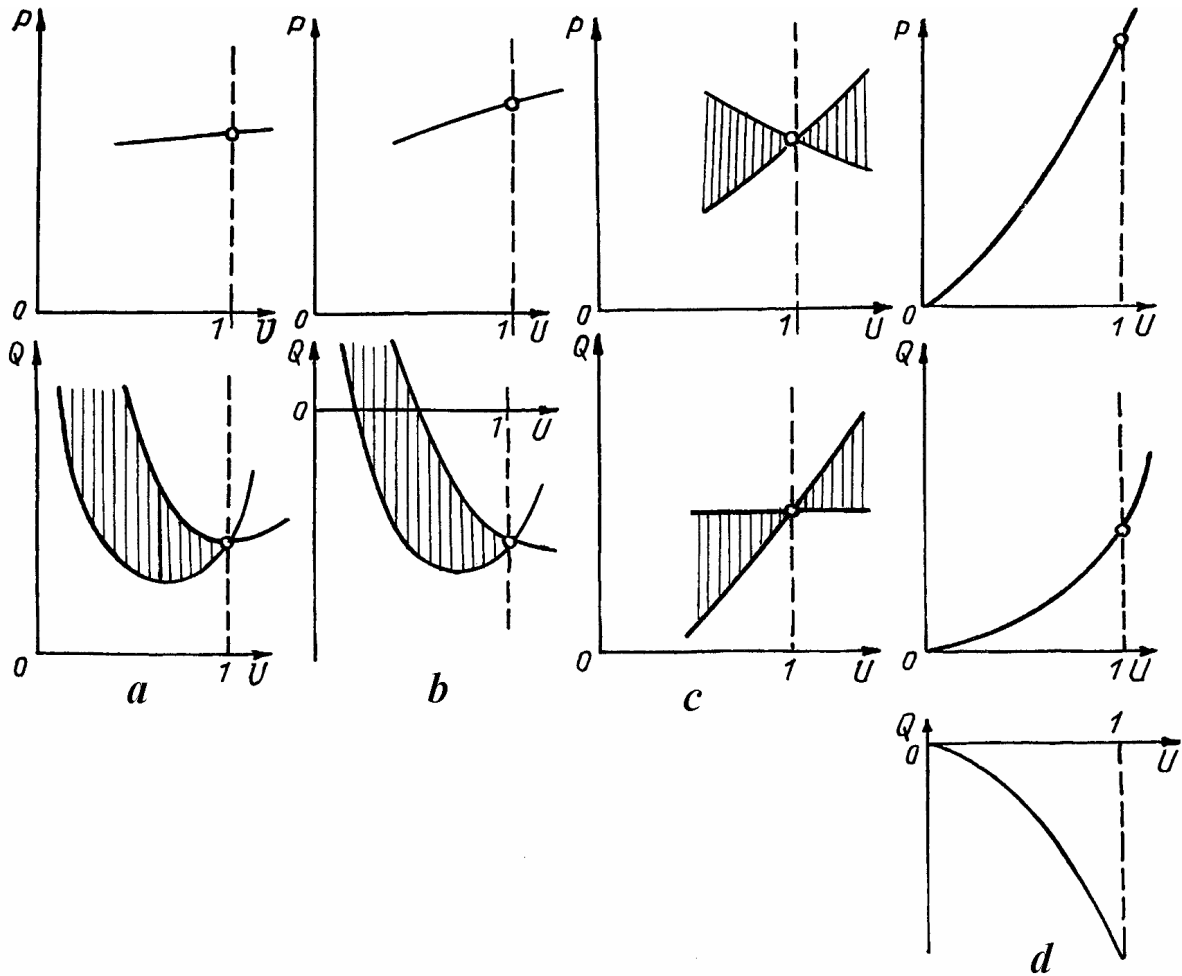


Fig. 12.8. Static characteristics of electric power using equipment: a – an induction motor; b – a synchronous motor; c – losses in series resistances; d – incandescent lamps

Stability of a load node operation takes place for the points at which  $\Delta Q = 0$  or condition  $d\Delta Q/dU > 0$  is satisfied (for example, points  $a$  and  $a'$ ). Comparison of intervals between the voltage values  $[U_{cr}; U_{cr}^{(CB)}]$  and  $[U_{cr}; U_{cr}^{(SC)}]$  shows that connection of capacitor bank causes less favorable conditions for the load node operation stability (especially when stability assurance is rather small). At the points where condition  $\Delta Q = 0$  is satisfied (points  $b$  and  $b'$ ) the stability of the load node operation is not kept because decreasing values of the voltage are combined with increasing values of the consumed reactive power which stipulates greater voltage losses within network components between the RPS and the load node. Such a voltage decrease due to increasing reactive power deficiency is progressive one. The process results in the whole load node stability violation, and is called **the voltage avalanche**. The load node with a capacitor bank is more exposed to this dangerous condition to compare with a synchronous condenser. It follows from the comparison of characteristics of the reactive power components balance (Fig.14.7) when the identity  $Q_{CB,rated} = Q_{SC,rated}$  is true.

### 12.5. Use of load static characteristics

To study the stability of PSS load nodes it is necessary to have static characteristics of major electric energy consumers. The load characteristics are determined by the using equipment parameters as well as power and voltage losses within components of distributing networks.

As it was said above, the load static characteristics are dependences between operation parameters under slow variations in processes, for example, the dependence  $P = f(\delta)$  for synchronous motors and dependence  $P = \varphi(s)$  for induction motors.

Usually load nodes are described by static characteristics being dependences of consumed active and reactive power of slowly changing voltage. Graphical form of the characteristics depends on load type (incandescent lamps, induction and synchronous motors, electric furnaces etc). Static characteristics for different types of electric using equipment are shown in Fig.12.8.

Static characteristics accurately reflect changes only under stable conditions taking place through tens of seconds after voltage change.

The latter should be taken into consideration while making calculations with successive variation of operation parameters or PSS properties. If time between the considered states amounts to minutes, the additional load variations caused by random factors as by operating staff activity such as transformers tap-change operations, synchronous motors and condensers field change can become important.

Analyzing transients in load nodes, it is necessary to take into account the influence of regulator devices. If the dead zone and discrete character of transformers with taps adjustment are neglected the voltage on buses of using equipment and active load within this range can be considered as invariable. Reactive power of load node is a sum of the reactive load of consumers and the losses in transformers with tap-changing under load which depend on the voltage in node. If reactive losses are small, the reactive power in the considered range is roughly constant.

To define the concept of power limit that is transferred in the simplest electric circuit, it was supposed that voltage on buses of consuming system is constant under all operation changes. Such an assumption can be considered as valid only when power of the supply system is 8 to 10 as much as the power of using equipment. But using equipment can have power equal to power of the feeding system. In these cases voltage on the using equipment buses depends on the feeding system condition and its load.

If it is assumed that EMF of the feeding system generators is constant, the angles  $\delta_1$ , and  $\delta_2$ , and too  $\delta_{12} = \delta_1 - \delta_2$  will vary when power transferred through a power line is changed. It will change voltage on the consuming system buses. When power transferred through PTL increases this

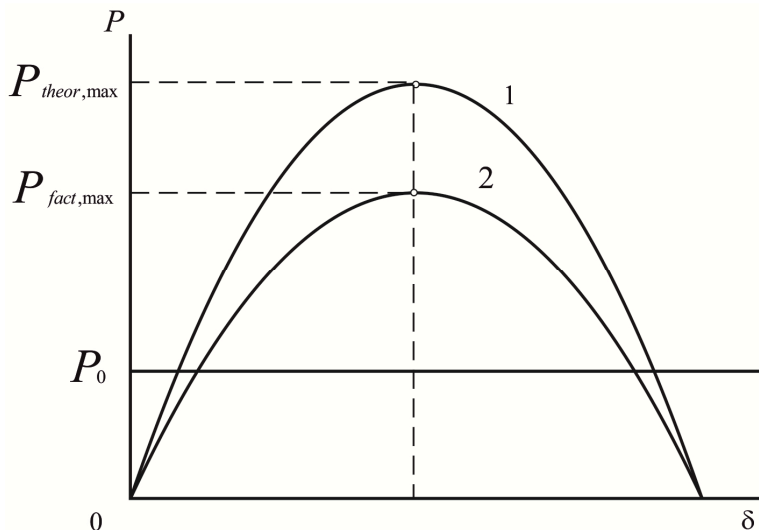


Fig. 12.9. Angular characteristics of electric system power:  
1 – ideal characteristic; 2 – actual characteristic

voltage drops, and that causes decrease of transmitted power limit  $P_{max} = EU/x_{res}$ . In this case the dependence  $P(\delta)$  can be represented by curve 2 (Fig.12.9) unlike the ideal characteristic 1 when quantities  $E$  and  $U$  are constant. The less buses voltage of using equipment under increase of power, the less is actual limit of the transferred power.

Extent of voltage on buses of consuming system decrease depends on the

properties of load connected to these buses. Influence of the load properties on the consuming system voltage is determined by **regulating effect of load** which means the phenomenon of variations of active and reactive power consumed by load when terminals voltage varies. Thus, the extent of the load active and reactive power decrease caused by the load voltage decrease can be called the regulating effect of load.

Numerically, the regulating effect is defined as variation of load active or reactive power per unit of voltage variation (Fig.12.10):

$$\begin{aligned} a_P &= \Delta P / \Delta U; \\ a_Q &= \Delta Q / \Delta U \end{aligned} \quad (12.38)$$

If infinitely small changes in voltage are considered, proceeding to limits, it is obtained:

$$a_P = dP/dU; \quad a_Q = dQ/dU. \quad (12.39)$$

In the most cases of operation stability determination, only reactance of elements is taken into consideration. The voltage reduction is mainly effected by the regulating effect of reactive power. With it, the more the slope of static characteristics in working area is, the more is the regulating effect of load.

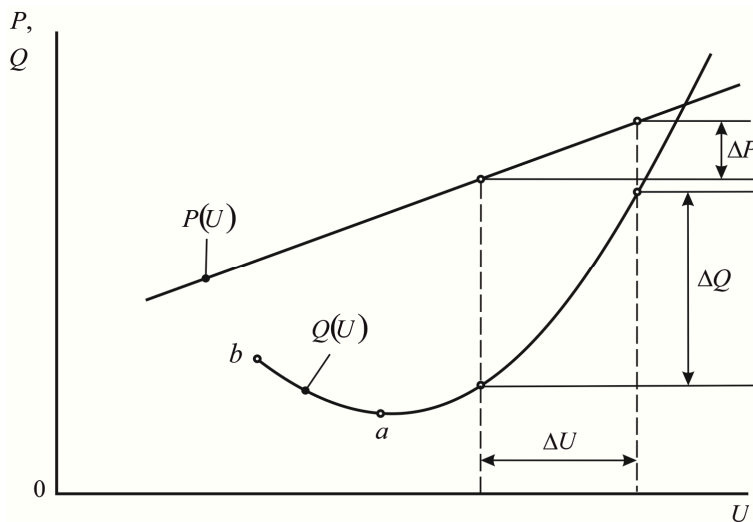


Fig. 12.10. Static characteristics of load node

But there are some other peculiarities concerning load influence on stability of a system operation. Increase of the consumed reactive power on the section *ab* of the static characteristic (Fig.12.10) caused by induction motors slip increase in the case of the reactive power deficiency, results in avalanche-like reduction of voltage accompanied by slowing down of motors. Thus, violation of the load node stability can cause operation instability of PSS.

In analyzing static stability the standard static characteristics of complex load should be used. They are compiled by designing organizations for specified groups of PSS consumers.

During great load nodes of PSS service, their circuits, power consumers and values of operation parameters can vary. Besides, purposeful control of the voltage and the balance of reactive power in the main nodes of a large industrial enterprise are accomplished. Combination of all these factors influences the load node stability. At the same time, it is very complicated to select separate components of a complex load node, and to find their deviations as for nominal values, and to reduce them to the simplest form (Fig.12.1, d). The reasons of that are:

- electric power distribution network is multi-stage one.
- there are several voltages within distributive circuit.
- controlled compensation devices are located in different points of distributive circuit.
- due to centralized automatic voltage control in points of its transformation, and to local voltage control at consumers.

- some industrial enterprises have sharply variable load curves.

As it was shown, changes of such load node operation parameters as active and reactive power, and voltage are interconnected. As the voltage is an essential variable for stability assessment, when stability of complex load node having complicated structure is analyzed, it is expedient to express the static characteristics as polynomials

$$\left. \begin{aligned} P_{*ld} &= 1 - a_P - b_P - c_P + a_P U_* + b_P U_*^2 + c_P \omega_*; \\ Q_{*ld} &= 1 - a_Q - b_Q - c_Q + a_Q U_* + b_Q U_*^2 + c_Q \omega_*; \end{aligned} \right\} \quad (12.40)$$

where  $a_P, b_P, c_P, a_Q, b_Q, c_Q$  are constant coefficients, and parameters of operation (voltage, frequency, active power, and reactive power), have been reduced to their nominal values expressed in per-units.

In the case of description of complex load node by polynomials (12.20) the accuracy is not high. But it is acceptable for analysis with successive refinement by consideration of stable conditions with intervals not more than tens of minutes.

The static characteristic of a complex load node depends on static characteristics of separate electrical consumers, their groups in flow production, and their shares in total load. Static characteristics of a load node can be obtained with the help of successive replacement of distributive network and its components which can also be given as mean (generalized) indices. Such a procedure is described in [11], its use needs knowledge of the load node components regulation effects.

Separately, it is necessary to account consumer groups supplied from transformers having devices of automatic voltage control. Static characteristics of load node with such consumers when voltage varies from  $U_{*1}$  to  $U_{*2}$  (expressions  $P_{*ld}$  and  $Q_{*ld}$  are similar by structure) are described by polynomials:

$$P_{*ld} \approx \begin{cases} 1 - a_P - b_P - c_P + a_P (U_* / U_{*1}) + b_P (U_* / U_{*1})^2 + c_P \omega_*, & U_* < U_{*1}; \\ 1 - c_P + c_P \omega_*, & U_{*1} \leq U_* \leq U_{*2}; \\ 1 - a_P - b_P - c_P + a_P (U_* / U_{*2}) + b_P (U_* / U_{*2})^2 + c_P \omega_*, & U_* > U_{*2}. \end{cases}$$

The described method for obtaining static characteristics of a complex load node is rather time-consuming.

There is an alternative way basing on application experimental data for great load nodes on the basis of which the generalized (mean, standard) static characteristics are obtained. In this case, it is convenient to write down polynomials (12.40) for small deviations of voltage and frequency using the coefficients of the load regulating effect:

$$\begin{aligned} k_{PU} &= (\partial P_{ld} / \partial U)_{U_{*=1}} = a_P + 2b_P; & k_{P\omega} &= (\partial P_{ld} / \partial \omega)_{\omega_{*=1}} = c_P; \\ k_{QU} &= (\partial Q_{ld} / \partial U)_{U_{*=1}} = a_Q + 2b_Q; & k_{Q\omega} &= (\partial Q_{ld} / \partial \omega)_{\omega_{*=1}} = c_Q, \end{aligned} \quad (12.41)$$

where  $k_{QU}, k_{PU}$  are factors of regulating effect of active and reactive power by voltage under constant frequency;  $k_{P\omega}, k_{Q\omega}$  – the same under constant voltage.

Proceeding from the experimental data study [11], the generalized static characteristics of load node can be represented numerically under the voltage change from  $1, 1U_{rated}$  to critical value, for voltage values being less than critical and for condition occurring after some of electrical loads disconnection at voltage values being greater than critical one.

If voltage is from  $1, 1U_{rated}$  to the critical value, the static characteristic of active load node is described by the equation:

$$P_{*ld} = 1 - k_{PU} + k_{PU} U_* + k_{P\omega} \omega_*, \quad (12.42, a)$$

where  $k_{PU} = 0,9 \pm 0,5$  is the factor of load which composition is close to average,  $k_{PU} = 0,6 \pm 0,3$  is the factor for industrial load nodes, and  $k_{PU} = 1,2 \pm 0,3$  for nodes not having large enterprises. Approximate values of the factor for constant frequencies are  $k_{P\omega} = 1,2 \pm 0,8$ ; the less values of  $k_{PU}$  correspond to the greater values of  $k_{P\omega}$ , and vice versa.

With account of  $k_{PU}$  and  $k_{P\omega}$  equation (12.42, a) takes the form:

$$P_{*ld} = -1,1 + 0,9U_* + 1,2\omega_* \quad (12.42, b)$$

For account the reactive load change depending on voltage it is necessary additionally to consider values of power factor  $\cos \varphi_{ld}$  as they depend on action of the RPS. Regulating effect on voltage under constant frequency of non-compensated reactive load is  $k_{QU} = 3,8 \pm 1,8$ , and for compensated load  $k_{QU} \approx (1,1 \pm 1,1)/\operatorname{tg} \varphi_{ld} + 2$ . Regulating effect of reactive power on frequency is  $k_{Q\omega} \approx (-1,5 \pm 1)/\operatorname{tg} \varphi_{ld} + 1$  under constant voltage. Taking that in consideration, the static characteristic of the reactive load node is described by the expression:

$$Q_{*ld} \approx \left[ \begin{array}{l} 5,7 - \operatorname{tg} \varphi_{ld} - 9,5U_* + \\ + (5,3 + \operatorname{tg} \varphi_{ld})U_*^2 + (\operatorname{tg} \varphi_{ld} - 1,5)\omega_* \end{array} \right] / \operatorname{tg} \varphi_{ld} \quad (12.43)$$

It is impossible to make accurate description of load node static characteristics for voltage values being less than critical one. That can be explained by the fact that it is difficult to predict reactive power consumption by loads, and voltage avalanche behavior can take place due to growing shortage of reactive power within network; the process can finish only after disconnection of some part of loads.

Therefore, under operation following after the voltage becomes less than critical (after several seconds) it is possible to use the description of static characteristics of load node as

$$P_{*ld} = 0,4U_*^2; \quad Q_{*ld} = 2,4U_*^2 \quad (12.44)$$

Under post-emergency operation when a part of loads is disconnected, and voltage values are greater than critical one, it is possible to assume that form of the node static characteristics coincides with the design characteristics that are recalculated for remained part of load (the upper index "prime" means the post-emergency operation):

$$\left. \begin{array}{l} P'_{*ld}(U_*, \omega_*) = (P'_{ld}/P_{ld})P_{*ld}(U_*, \omega_*); \\ Q'_{*ld}(U_*, \omega_*) = (Q'_{ld}/Q_{ld})Q_{*ld}(U_*, \omega_*), \end{array} \right\} \quad (12.45)$$

where  $P'_{ld}/P_{ld}$  is the relative share of the consumed active power by the remaining part of load node consumers;  $Q'_{ld}/Q_{ld}$  is the relative share of consumed reactive power under post-emergency operation.

On the basis of the considered model of load node the assessment of static stability of complex load node is performed with the help of indirect stability criteria (12.9) and (12.10) according to procedure described in Div. 12.3.

**Test Questions**

1. What design models of load node are used for analysis its static stability?
2. By what criteria the design model of a load node can be replaced by the equivalent model?
3. What is the influence of electric power network parameters on critical indices which characterize stability of electric motors?
4. What is the influence of synchronous motors automatic load transfer on their static stability conditions?
5. How does operation stability of a node having asynchronous load change when its reactive component is compensated by static capacitors and synchronous compensators?
6. What are the static characteristics of a complex load node?
7. What is the essence of idea about regulating effect of load?
8. What is the voltage avalanche, what are the reasons of its appearance?
9. What are the criteria for assessment static stability of operation of complex load node?

**Topics for essay**

1. Development of load node design model.
2. Violation of induction load node operation.
3. Influence of RPS location within distributive network on EPS operation stability.
4. Estimation of PSS static stability with voltage control by means of transformers tap-adjustment.
5. Generalized characteristics of load node.



## **CHAPTER 13: STABILITY OF LOAD NODE OPERATION AT LARGE DISTURBANCES**

- 13.1. Typical causes of large disturbances occurrence in load nodes
- 13.2. Equations of electromechanical transients in an induction motor
- 13.3. Large disturbance having the form of three-phase fault on the terminals of an induction motor
- 13.4. Large disturbance having the form of load surge on induction motor
- 13.5. Starting induction motors
- 13.6. Self-excitation of induction motors while starting
- 13.7. Self-starting induction motors
- 13.8. Equations of electromechanical transients in synchronous motor
- 13.9. Large disturbance arising in the form of three-phase fault on terminals of a synchronous motor
- 13.10. Load surge on synchronous motor
- 13.11. Starting synchronous motors
- 13.12. Self-starting synchronous motors
- 13.13. Electromechanical transients in complex load node

Test questions

Topics for essay

### 13.1. Typical causes of large disturbances occurrence in load nodes

Large disturbances in power supply networks initiate rise significant changes of a normal electric power user mode. They occur as a result of various causes: short circuits and overloads in mains and distribution networks, sharply varying drag torques on shafts of induction and synchronous motors of rolling mills and hoisting cranes, loads surge caused by arc furnaces, tube-rolling mills, and tube-welding mills. Large disturbances cause significant variations and oscillation of voltage, and changes of its phase relatively to power sources, and are the reason of other consequences as well.

Transients are of a particular importance in power supply system nodes with large electromotor load. Nowadays at major industrial enterprises induction motors with unit capacity to 5000 kW are used and unit capacity of synchronous motors reaches 63000 kW.

Asynchronous and synchronous motors of large rated power may cause the electric system node stability violation in the process of maintenance due to the motors unstable operation. Violation of induction motors operation stability is connected with possibility of the motor breakdown (braking) when voltage is reduced and for synchronous motors it can be stipulated by their step-out of synchronism that occurs when voltage in a network is low or motor excitation is changed.

Analysis of transients in power system nodes with alternating-sign motor load is reduced to the solution of the set of non-linear differential equations that describe a motor rotor motion, current and voltage changes. In this case calculation procedure is based on the following assumptions:

- The load on an electric motor shaft is changed strictly periodically, cyclically or in a random way. Transient is analyzed at constant EMF of generators  $E'_q \approx E' \equiv const$  outside the transient reactance,
- Impedance of electric motors and other loads at the instant of commutation is constant,
- Influence of damping torque of synchronous motors and rotational speed regulators of power stations turbines is not considered.

Typical is transient that occurs in power supply system using of widely spread separate distribution plant sections operation at the voltage of 6–10 kV and automatic load transfer on a sectionalizing switch (Fig. 13.1).

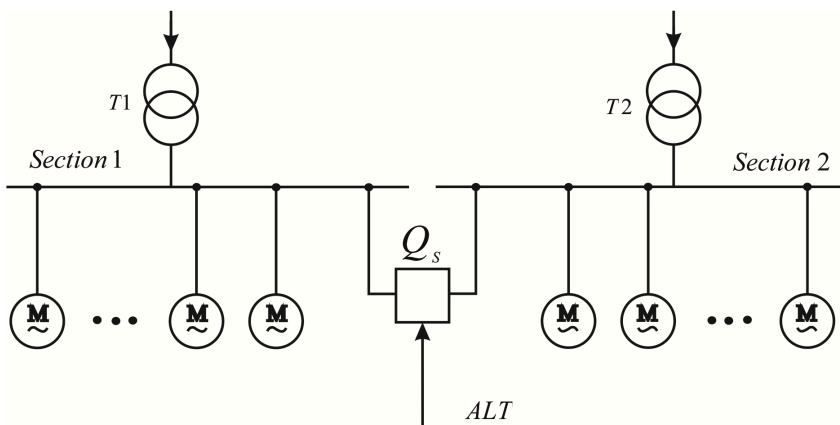


Fig. 13.1. Diagram of substation with automatic load transfer on sectionalizing switch of collecting buses

When short circuit occurs in the feed circuit of one of the distribution plant sections, motors operation stability and technological processes of continuous production can be violated. By means of automatic load transfer, motors of the faulty section are switched to the intact (undamaged) section. That can be accompanied by the node voltage decrease and as the result the motor torques reduction. Keeping of the load

node operation stability under these circumstances is possible only if self-starting of the commutated section is successful.

When powerful motor load is available, the particular short-circuit condition for which the part of the current generated by synchronous motors reaches 50–60% of short-circuit total current should be paid special attention. In this case the actual tasks are determination of the portion of the short circuit currents from synchronous motors and their optimal restriction.

Large disturbances in a power supply system are caused by the following reasons:

- Three-phase short circuits on the terminals of induction and synchronous motors;
- Throw load on the motors;
- Cutout of powerful motors or motor groups from the electric circuit,
- The motors starting,
- The motors self-starting,
- Self-excitation of induction motors.

### 13.2. Equations of electromechanical transients in an induction motor

Any operation mode of an induction motor, connected to the network with the voltage  $U$ , can be characterized by many attributes.

The most important of them are the angular rotor rotational speed  $\omega$  and the sub-transient EMF  $E''$ .

Induction motor operation is described by the differential equation set:

$$T_j (d\omega/dt) = M - M_{WM}; \quad (13.1)$$

$$T'_2 dE''_{forced}/dt + E''_{forced} = \sqrt{[U(x_1 - x'')/x_1]^2 - (T'_2 s E''_{forced})^2}; \quad (13.2)$$

$$T'_2 dE''_{free}/dt + E''_{free} = 0, \quad (13.3)$$

where  $T_j$  is electromechanical time constant of a motor-mechanism unit;  $M$  and  $M_{WM}$  are the electromagnetic motor torque and the mechanism anti-torque moment correspondingly;  $E''_{forced}$  and  $E''_{free}$  are the free and forced components of the sub-transient EMF  $E''$ ;  $x_1$  and  $x''$  are the synchronous and sub-transient reactance correspondingly;  $T'_2$  – the rotor winding time constant.

The electromagnetic motor torque is determined by the expression

$$M = P / \omega_{synchr}. \quad (13.4)$$

where  $P$  is active power consumed from the network;  $\omega_{synchr}$  – the synchronous angular frequency of the voltage on the motor terminals (for  $\omega = \omega_{synchr} = 1$ ,  $M \equiv P$  in per-units).

The mechanism anti-torque moment reduced to the motor rated power  $s_{rated}$  can be found from the equation

$$M_{WM} = [M_0 + (k_{lf} - M_0)(\omega / \omega_{steady})^p] \cos \varphi_{rated} \eta_{rated}, \quad (13.5)$$

where  $k_{lf}$  is the motor loading factor for its operation in synchronous mode ( $s = 0$ );  $\omega_{steady}$  is the rotor angular rotational speed in the stable mode;  $p$  is the exponent of power that specifies the dependence of the mechanism anti-torque moment against the rotational speed;  $\cos \varphi_{rated}$  and  $\eta_{rated}$  are the rated power factor and efficiency respectively.

As initial conditions for solving differential equations set (13.1) – (13.3) the values of the basic mode parameters could be taken:

$$\omega(0) = \omega(-0); \quad (13.6)$$

$$E''_{forced}(0) = [(x_1 - x'') / x_1][U / \sqrt{1 + [s(0)T_2'']^2}]; \quad (13.7)$$

$$E''_{free}(0) = E''(-0) - E''_{forced}(0), \quad (13.8)$$

Other motor operation parameters are determined from the expressions:  
the rotor slip

$$s = 1 - \omega_*; \quad (13.9)$$

the active power consumed from the network

$$P = UE''_{forced} \sin \delta_{UE''} / x'' + UE''_{free} \sin(\delta_{UE''} - st) / x''; \quad (13.10)$$

the reactive power consumed by the motor from the network

$$Q = U^2 / x'' - UE''_{forced} \cos \delta_{UE''} / x'' - \\ - UE''_{free} \cos(\delta_{UE''} - st)x''; \quad (13.11)$$

the stator winding current

$$I = \sqrt{P^2 + Q^2} / U; \quad (13.12)$$

the forced component of the motor EMF

$$E_{1forced} = \sqrt{\frac{(E''_{forced} x_1 / x'')^2 + [U(x_1 - x'') / x'']^2 - \\ - (2E''_{forced} x_1 / x'') \sqrt{[U(x_1 - x'') / x'' + (sT_{20} E''_{forced})^2]}}{2}}; \quad (13.13)$$

the forced component of the rotor winding current

$$I_{2forced} = E_{1forced} / x_{12}; \quad (13.14)$$

the free component of the motor EMF

$$E_{1free} = E''_{free} x_1 / x''; \quad (13.15)$$

the free component of the rotor winding current

$$I_{2free} = E_{1free} / x_{12}. \quad (13.16)$$

Equations (13.1) – (13.5) together with the initial conditions (13.6) – (13.8) and relationships that define other mode parameters through the motor ones describe the induction motor condition completely.

To calculate transients under the network voltage frequency  $\omega_{synchr}$  that differs from the rated one the same equations (13.1) – (13.5) and expressions (13.6) – (13.8) are used if the voltage, EMF, active power and reactive power have been reduced to the rated frequency.

### 13.3. Large disturbance having the form of three-phase fault on the terminals of an induction motor

In the case of three-phase fault the stator winding voltage is  $U = 0$  and as a result the forced components of the motor operation parameters are equal to zero. In this case the transient is defined only by free mode components.

It follows from (13.1 – 13.3) that under three-phase fault with  $U = 0$  and  $E_{forced} = 0$ , electromechanical transient equations are as follows

$$T_j d\omega / dt = -M_{WM}; \quad (13.17)$$

$$T_2 dE''_{rd} / dt + E''_{rd} = 0. \quad (13.18)$$

where  $E''_{rd} = E''_{free} / \omega$  is sub-transient EMF reduced to the rated frequency.

Initial conditions needed for solving equations set (13.17) and (13.18) can be determined from the mode previous to the fault

$$\left. \begin{aligned} \omega(0) &= \omega(-0); \\ E''_{rd}(0) &= E''(-0). \end{aligned} \right\} \quad (13.19)$$

In the course of the three-phase fault the motor is decelerated due to the mechanism anti-torque moment  $M_{WM}$  according to the law:

$$\omega = \omega(0) - \int_0^t (M_{WM} / T_j) dt. \quad (13.20)$$

The sub-transient EMF  $E''$  changes according to (13.18) in accordance with the dependence:

$$E''_{rd} = E''_{rd}(0) \cdot \exp(-t / T'_{2free}) \quad (13.21)$$

or in the absolute units:

$$E'' = E''(-0) \cdot \omega \cdot \exp(-t / T'_{2free}). \quad (13.22)$$

From (13.22), it follows that in the case of three-phase fault the EMF on induction motor terminals decays according to aperiodic law with the rotor winding time constant  $T'_{2free}$  that corresponds to the slip  $s = 0$  (synchronous condition of an induction motor).

The time constant  $T'_{2free}$  for synchronous operation is 4-5 times as large as for starting conditions (at  $s = 1$ ) and changes in limits of  $T'_{2free} = 0.1 \dots 0.2$  s.

At the three-phase fault, the stator winding current is equal to

$$I = E''_{rd} / x''_{subtr} = [E''(-0) / x''_{subtr}] \exp(-t / T'_{2free}), \quad (13.23)$$

where  $x''_{subtr}$  is sub-transient reactance of the induction motor at  $s = 0$ . This reactance value  $x''_{subtr}$  is by 30...35% greater than the reactance  $x''_{start}$  that corresponds to  $s = 1$ .

Therefore the motor sub-transient current

$$I'' = E''(-0) / x''_{subtr}. \quad (13.24)$$

is by 3-40% less than starting current which equals to

$$I_{*start} \approx 1 / x_{*start}'' \quad (13.25)$$

As the time between the fault in the network occurrence and the beginning of the 6 or 10 kV network breaker contacts opening is not less than 0.15-0.20 s, the fault current component caused by the induction motor decays to zero during this time. Therefore the fault current component caused by the motor is neglected while the switch breaking current is determined. The component should be taken into account in determination the surge current at the point of motor connection.

Under the three-phase short-circuit, the synchronous EMF  $E_1$  is found from the expression

$$E_1 = (x_1 / x_{subtr}'')E'' = (x_1 / x_{subtr}'')E''(-0) \cdot \omega \cdot \exp(-t / T_{2free}''). \quad (13.26)$$

At short-circuit the rotor winding current referred to the stator winding coincides virtually with the stator winding current.

As an example Fig. 13.2 gives the calculated curves of mode parameters for the motor A-12-62-10 under three-phase fault on its terminals. Before the fault the motor had normal operating condition.

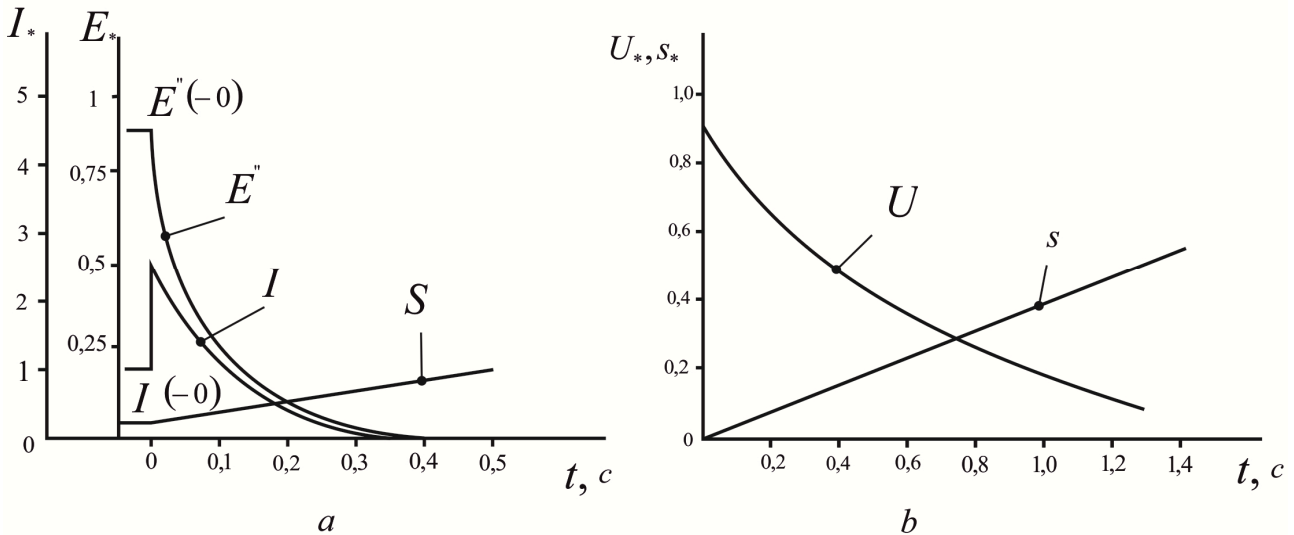


Fig. 13.2. Calculated transient curves: a) under three-phase fault on terminals of the motor A-13-62-10, b) after the motor switching off

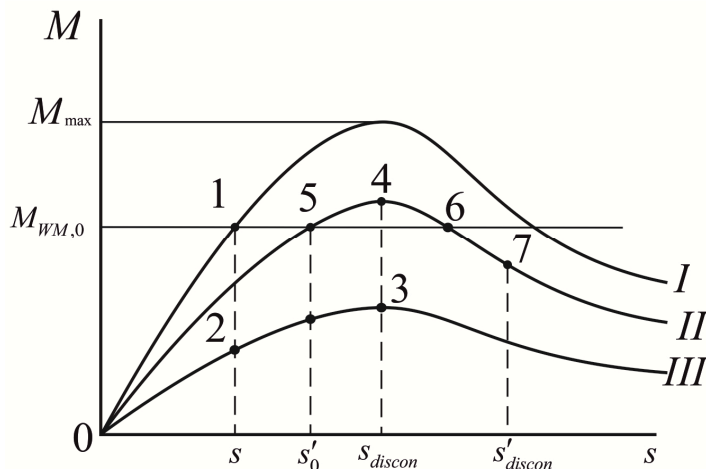


Fig. 13.3. Dependence of induction motor electromagnetic torque on slip under large disturbances

Under three-phase fault slip of induction motors of the node increase, and voltage values decrease. If the fault has been switched off, the voltage in the load node is likely not return to its nominal value. It is stipulated by the fact that in the case of the slip value increase, reactive power consumption by induction motor increases and the voltage continues to reduce.

The dependence of an induction motor torque on slip is shown on Fig.13.3. When the fault occurs under the large disturbances the motor torque reduces sharply (point 2 of the curve III), the slip increases, and the motor rotor is retarded.

If the fault have been broken up at the slip  $s_{discon}$  (point 3 of the curve III), the motor torque decreases again. At the instant of the fault breaking, the value of electromagnetic torque reaches the point 4 of the curve II, where it is higher than static load torque. In this case the rotational speed increases, the slip falls, and as a result a new steady condition is obtained (point 5 at the curve I).

If duration of operation under the fault corresponds to the motor switching off at the slip  $s'_{discon}$  (after point 6 at the curve II), the value of the motor electromagnetic torque continues to reduce. The resulting torque at the motor shaft becomes braking. The value of the motor slip increases and it stops.

### 13.4. Large disturbance having the form of load surge on induction motor

Load surge on induction motor can originate from reducing voltage in the node of electric network or from the increase of the anti-torque moment on the motor shaft and, consequently, increase of the motor slip. If the voltage changes for  $\Delta U$ , both the forced and the free components of EMF arise in the circuit of the induction motor. The transient in the circuits of an induction motor is described by general set of differential equations (13.1) - (13.3) at initial conditions (13.6) - (13.8).

Sub-transient EMF in the steady-state mode, preceding voltage reduction, is

$$E''(-0) = E''_{forced}(-0) = [(x_1 - x'') / x_1] [U(-0) / \sqrt{1 + [s(0)T_2']^2}]. \quad (13.27)$$

After the voltage decrease the forced component of the sub-transient EMF changes abruptly in proportion to the voltage change:

$$E''_{forced}(+0) = [(x_1 - x'') / x_1] [U(+0) / \sqrt{1 + [s(0)T_2']^2}]. \quad (13.28)$$

The free component of the sub-transient EMF at the instant of voltage jump compensates the abrupt change and according to (13.8) is defined by expression:

$$E''_{free}(+0) = [(x_1 - x'') / x_1] \cdot [\Delta U / \sqrt{1 + [s(0) \cdot T_2']^2}]. \quad (13.29)$$

Further change of the EMF can be described by differential equation (13.3) which solution is the following dependence:

$$E_{free} = E''_{free}(+0) \cdot \exp(-t / T_2'). \quad (13.30)$$

The forced EMF component  $E''_{forced}$  at constant voltage on the induction motor stator winding, as follows from (13.2), changes This is the result of the slip change. As speed of the slip change is determined by the time constant  $T_j$ , speed of the EMF change  $E''_{forced}$  depends on  $T_2'$ . For these two constants the inequality  $T_j \gg T_2'$  is true, and the EMF  $E''_{forced}$  virtually instantly reaches the steady-state value. After the voltage reduction, it is determined by expression

$$E''_{forced} = [(x_1 - x'') / x''] \cdot [U(+0) / \sqrt{1 + (sT_2')^2}]. \quad (13.31)$$

The total sub-transient EMF is determined by superposition of the free and forced components:

$$E'' = \sqrt{(E''_{forced} + E''_{free} \cos st)^2 + (E''_{free} \sin st)^2}. \quad (13.32)$$

At the load surge on asynchronous motor causing the reduction of feeding voltage, or at increase of the anti-torque moment on a shaft, the value of motor slip rises. If the anti-torque moment higher than the maximum ( $M_{WM} > M_{\max}$ ) at this instant, the slip of the motor increases to the value  $s = 1$  and the rotor stops. To prevent this, it is necessary to recover the rated voltage or decrease the anti-torque moment on the motor shaft in due time.

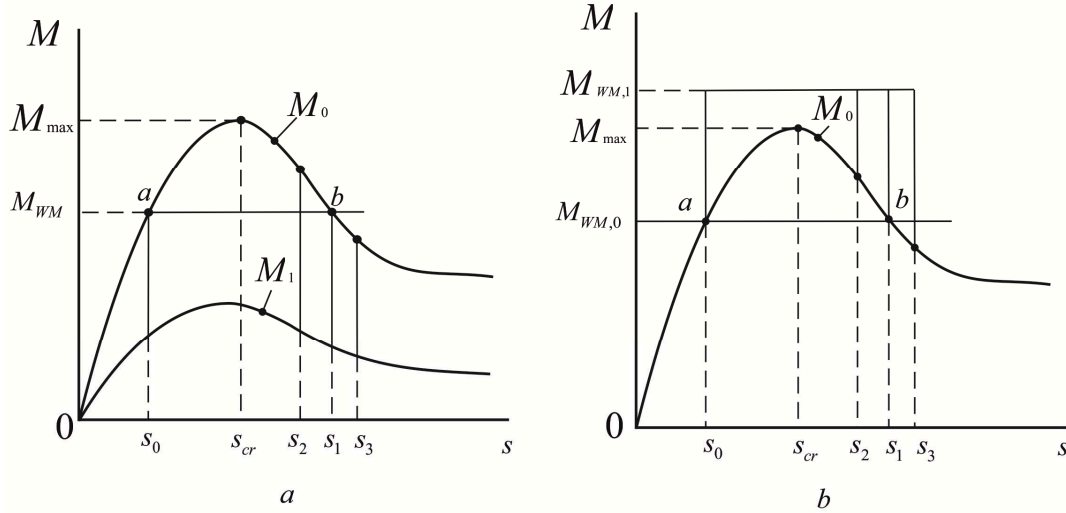


Fig. 13.4. Changes of electromechanical characteristics of unit with induction motor: *a*-at the feeding voltage reduction, *b*-at increase of operating mechanism anti-torque moment

In the normal condition, the induction motor works at the slip  $s_0$  and the torque  $M_{WM}$  (point *a* in the fig. 13.4, a). If the voltage value on the motor terminals falls down from  $U_0$  to  $U_1$ , its torque reduces and becomes equal to

$$M_1 = M_0 (U_1 / U_0)^2 = 2M_{\max} (U_1 / U_0)^2 (s / s_{cr} + s_{cr} / s). \quad (13.33)$$

At the electromagnetic torque reduction from  $M_0$  to  $M_1$ , the rotor decelerates. The time necessary for the motor shutdown and change of the slip for this time can be defined by integrating the motion equation:

$$T_j (ds / dt) = M_{WM,0} - M_1. \quad (13.34)$$

Under these conditions, it is possible to determine the critical time during which the voltage can reduce from  $U_0$  to  $U_1$  without the motor shutdown, and recovery of its normal operation after recovery of the initial voltage would take place. For that, the slip should not exceed  $s_1$  (point *b* in the fig. 13.4, a), as when the slip is more than  $s_1$  (for example,  $s_3 > s_1$ ) the working point of the motor operation shifts to the unsteady part of the curve. As a result, after the voltage  $U_0$  recovery the rotor continues decelerating and shuts down.

When expression (13.33) is substituted into equation (13.34), it can be written as

$$T_j (ds / dt) = M_{WM,0} - 2M_{\max} (U_1 / U_0)^2 / (s / s_{cr} + s_{cr} / s),$$

whence

$$dt = T_j ds / [M_{WM,0} - 2M_{\max} (U_1 / U_0)^2 / (s / s_{cr} + s_{cr} / s)].$$



After integration of the left side of this equation for the time interval from  $t = 0$  to  $t = t_{np}$ , and the right side – for the slip change interval from  $s_{*0} = s_0/s_{cr}$  to  $s_{*1} = s_1/s_{cr}$ , we determine the time value when the rotor reaches the slip  $s_{*1}$ :

$$t_{\text{lim}} = \frac{s_{cr} T_j}{M_{WM,0}} - \left[ s_* + k(s_*^2 - 2ks_* + 1) + \frac{2k^2}{\sqrt{1-k^2}} \arctg \frac{s_* - k}{\sqrt{1-k^2}} \right] \Bigg|_{s_{*0}}^{s_{*1}},$$

$$\text{where } k = \frac{M_{WM}}{M_{WM,0}} \left( \frac{U_1}{U_0} \right)^2; \quad s_* = \frac{s}{s_{kp}}.$$

The slip values  $s_{*0}$  and  $s_{*1}$  can be defined from the expression  $M_{WM,0} = 2M_{\text{max}} / (s_* + 1/s_*)$  or  $s_*^2 - 2M_{\text{max}}s_* / M_{WM} + 1 = 0$ , whence

$$s_* = M_{\text{max}} / M_{WM,0} \pm \sqrt{(M_{\text{max}} / M_{WM,0})^2 - 1}. \quad (13.36)$$

The sign “+” corresponds to  $s_{*1}$ , and the sign “-” – to  $s_{*0}$ .

Change of the motor operation condition under increase of the anti-torque moment on the shaft (fig. 13.4, b) fully corresponds to its state at reduction of the feeding voltage. At finding  $t_{\text{lim}}$  by expression (13.35),  $M_{WM,0}$  should be replaced by  $M_{WM,1}$ .

At the load surge the electromagnetic torque of induction motor can be too found as the sum of the electromagnetic torques caused by the forced and the free components using proper values of the EMF.

### 13.5. Starting induction motors

Starting of an induction motor is transition of the motor and the operating mechanism from the stationary state ( $\omega = 0$ ) to rotation with rated angular speed ( $\omega = \omega_0$ ). During starting the motor develops the electromagnetic torque needed for overcoming the anti-torque moment of the operating mechanism and storing certain kinetic energy in the rotating masses of the unit. The motor accelerates under the action of the asynchronous electromagnetic torque conditioned by the forced EMF component. Free components of the mode arise only at the time instant of the motor connection to the electric network and appear only at the beginning of starting and practically do not influence the motor acceleration. It allows not taking into account the free EMF component at the calculations of an induction motor starting.

Differential equations, describing transients of an induction motor starting could be got from the general set of equations (13.1) - (13.3), assuming  $E''_{\text{free}} = 0$ :

$$T_j d\omega / dt = T_{20} (1 - \omega) (E'')^2 / (x_1 - x'') - M_{WM}; \quad (13.37)$$

$$T_2 dE' / dt + E' = \sqrt{[U(x_1 - x'') / x_1]^2 [T_2' (1 - \omega) E'']^2}. \quad (13.38)$$

At starting from the stationary state the initial conditions for transient computations are:

$$\left. \begin{aligned} \omega(0) &= 0; \\ E''(0) &= [(x_1 - x'') / x_1] U \sqrt{1 + (T_2')^2}. \end{aligned} \right\} \quad (13.39)$$

The electromagnetic transient of an induction motor starting are defined by electromagnetic time constant equal  $T_2' = 0.02 \dots 0.1$  s. Electromechanical transient are specified by electromechanical constant equal  $T_j' = 1 \dots 10$  s. The comparison of  $T_2'$  and  $T_j'$  shows that the rate of sub-transient EMF  $E''$  change in the process of starting is by one or two orders higher than the rate of rotational frequency change. It is the basis for assumption that the sub-transient EMF reaches its steady-state value virtually instantly in comparison with the rotor speed. Then the change of this EMF can be determined from the equation of the steady state. This equation is got from equation (13.38) under condition of  $dE''/dt = 0$ :

$$E'' = [(x_1 - x'') / x_1] U \sqrt{1 + (sT_2')^2}. \quad (13.40)$$

Taking this assumption into account, the process of induction motor starting is described by single differential equation of electromechanical transient (13.37).

Asynchronous motor starting occurs as a result of the electromagnetic torque excess over the mechanism anti-torque moment (Fig.13.5). Let us examine this process calculation using equation of motion

$$T_j ds / dt = M_{WM} - M = \Delta M. \quad (13.41)$$

Equation (13.41) is studied at various values of the mechanism anti-torque moment. Values of  $M_{WM}$  and  $M$  are evaluated using values of the slip  $s$ . Static dependencies  $M_{WM}(s)$  and  $M(s)$ , and their difference  $\Delta M(s)$  are shown in Fig. 13.5.

Dividing the slip values range into a number of equal intervals

$$\Delta s_1 = \Delta s_2 = \dots = \Delta s_i = \dots = \Delta s_n$$

the motor equation (13.41) for any interval can be written as

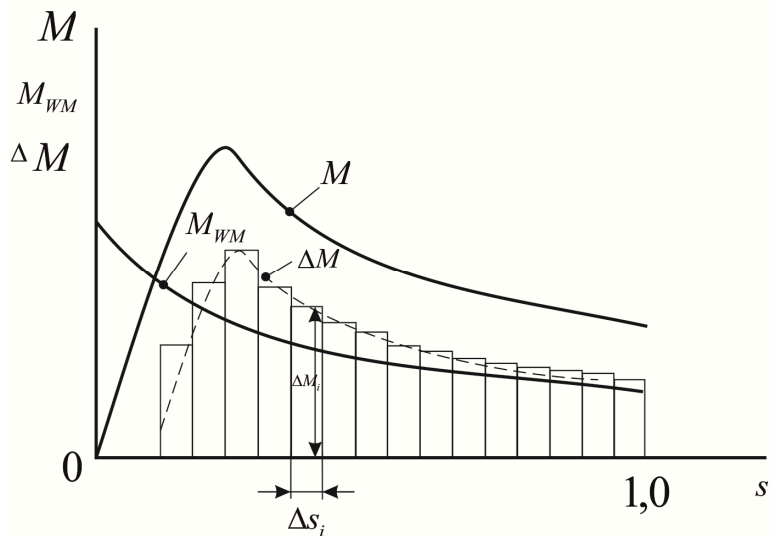


Fig. 13.5. Graphical-analytical calculation of induction motor starting

$$\Delta M_i = T_j (\Delta s_i / \Delta t_i)$$

or

$$\Delta s_i = (1 / T_j) \cdot \Delta M_i \cdot \Delta t_i, \quad (13.42)$$

where  $\Delta M_i$  is average value of the surplus torque at the interval  $\Delta s_i$ . The time from the beginning of a motor starting till the end of  $n$ -th interval is determined by the expression

$$t = T_j \sum_{i=1}^n (\Delta s_i / \Delta M_i). \quad (13.43)$$

Accuracy of computation by (13.43) increases with the interval  $\Delta s_j$  reduction and corresponding increase of the intervals number.

When  $M_{WM} = const$  during the whole process of the motor acceleration, and electromagnetic torque  $M$  is described by the expression that corresponds to the simplified equivalent circuit of the motor, then

$$M = U^2 r_2 s / (x_s^2 s^2 + r_2^2). \tag{13.44}$$

The surplus torque can be evaluated by the dependence:

$$\Delta M = M_{WM} - U^2 r_2 s / (x_s^2 s^2 + r_2^2). \tag{13.45}$$

The time of the motor pick-up in the range of the slip from  $s_1$  to  $s_2$  is

$$t = \int_{s_1}^{s_2} T_j (ds / dM) = T_j \int_{s_1}^{s_2} (x_s^2 s^2 + r_2^2) ds / [M_{WM} (x_s^2 s^2 + r_2^2) - U^2 r_2 s]. \tag{13.46}$$

Therefore if the anti-torque moment, produced by an operating mechanism is constant, the methods of the transient in a load node at an induction motor starting analysis are significantly simplified.

### 13.6. Self-excitation of induction motors while starting

When capacitor banks are connected in series into the network, compensation of the network inductive reactance takes place, and the network voltage changes depending on the network load. In this case, the following undesirable effects not provided for normal operation may happen:

- Synchronous machines hunting;
- Crawling of induction motors at a speed below the rated;
- Occurrence the resonance oscillations at under-speed that may cause sub-harmonic currents and voltages.

Probability of self-excitation of induction motors while starting can be evaluated with the help of the dependences used for synchronous motors. In this case more thorough analysis of the considered effect is needed.

Under change of angular speed  $\omega$  an induction motor as well as a synchronous generator changes its reactance according the curves 4 and 5 (Fig.13.6).

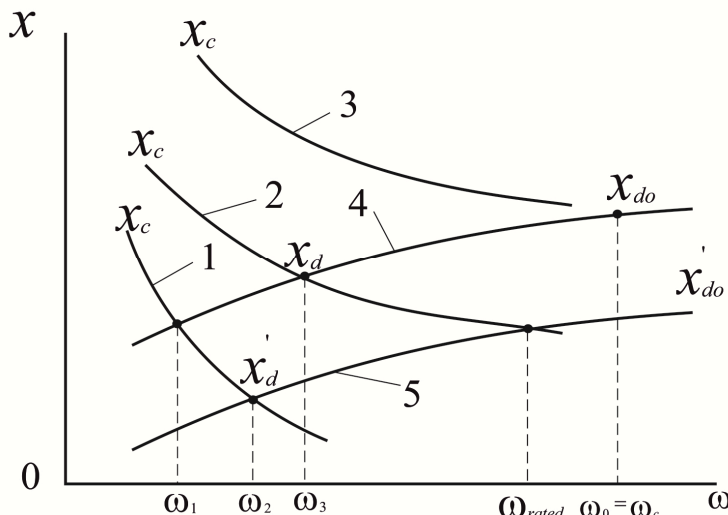


Fig. 13.6. To self-excitation induction motor consideration

If a capacitor bank of longitudinal compensation with capacitive reactance  $x_C = 1 / (\omega C_{CB})$  is connected to the network, the value  $x_C$  against angular speed  $\omega$  changes at different values of  $C_{CB}$  according to the curves 1, 2, 3.

The area between the curves  $x'_d(\omega)$ ,  $x_d(\omega)$  represent the self-excitation motor area.

When  $x_C$  changes during the time of the motor starting according to the curve 3, self-excitation doesn't occur. But when  $x_C$  changes

according to the curve 2, self-excitation occurs from the angular speed value  $\omega_3$  to normal operation with the speed  $\omega_{rated}$ . When  $x_C$  changes according to the curve 5, self-excitation of a motor during the starting can occur in the interval of the angular speed value from  $\omega_1$  to  $\omega_2$ . It can though disappear without reaching full development if the motor passes the zone of self-excitation fast. Angular speed values  $\omega_1$  and  $\omega_2$  correspond the boundary of the motor self-excitation area with  $x_C$  changing according to the curve 1.

At self-excitation of an induction motor, the following effects may occur:

- As the stator current and the power consumed from a network exceed the rated values by several times, considerable motor overheating is possible;
- The voltage on motor terminals reduces and electromagnetic torque decreases;
- The motor can “stick” at the speed below the rated one due to the electromagnetic torque reduction, that causes the current and torque beats, resulting in rotor hunting.

The following factors should be taken into account at self-excitation analysis.

The motor reactance changes from minimum to maximum value during acceleration.

The motor winding inductance at the particular rotor slip can produce resonance circuit with series compensation capacitance. As a result the motor is self-excited at the speed below rated and rotates at the speed corresponding to the frequency of free oscillations in this circuit.

In the course of the motor acceleration, negative value of the rotor slip can occur, at which the motor passes to generator condition relatively the self-excitation circuit. There are created either conditions for stable motor operation in this mode when losses in rotor circuit are equal to the power generated or conditions for motor operation with rated speed if losses exceed the generated power, and self-excitation does not occur or is unstable.

Generating circuit frequency depends on the network parameters, capacitive reactance of the capacitor bank, and the motor winding reactance. To prevent self-excitation, it is necessary either to choose an adequate compensation capacitance or to connect resistance in series or in parallel to the capacitor.

In accordance with proportion of  $x_C$  and the motor parameters three typical operation conditions can occur, that correspond to the curves 3, 4, 5 in Fig.13.6.

**In the first case** (curve 3)  $x_C \geq x_{\mu 0}$ , where  $x_{\mu 0}$  is the inductive reactance of the motor magnetizing circuit. The whole self-excitation area lies in the range of rotational speed higher than the synchronous one. In limits of rotational speed from zero to the synchronous one the motor is not excited.

**In the second case** (curve 2),  $x_s / (r_2 / r_1 + 1)^2 \leq x_C < x_{\mu 0}$  where  $x_s$  is the stator leakage reactance,  $r_1$  and  $r_2$  are resistances of the stator and rotor windings.

In this case one part of the motor self-excitation area lies in the range of rotational speed from zero to the synchronous one and another is beyond the bounds of the synchronous rotational speed. Dependence of the electromagnetic motor torque against the slip is given in Fig. 13.7 a. Self-excitation causes the motor rotational speed reduction by 35-40% from the rated value.

**In the third case** when  $x_C < x_s / (r_2 / r_1 + 1)^2$  the motor excitation starts when rotational speed is close to zero, and self-excitation area lies in the range of rotational speed from zero to the synchronous one.

Dependence of the electromagnetic torque of an induction motor against the slip at the motor acceleration is given in Fig.13.7, b. If after connection to the network the motor accelerates so slowly that the process of its self-excitation finishes during the time of self-excitation area passing, the motor sticks at reduced rotational speed and rotor hunting occurs.

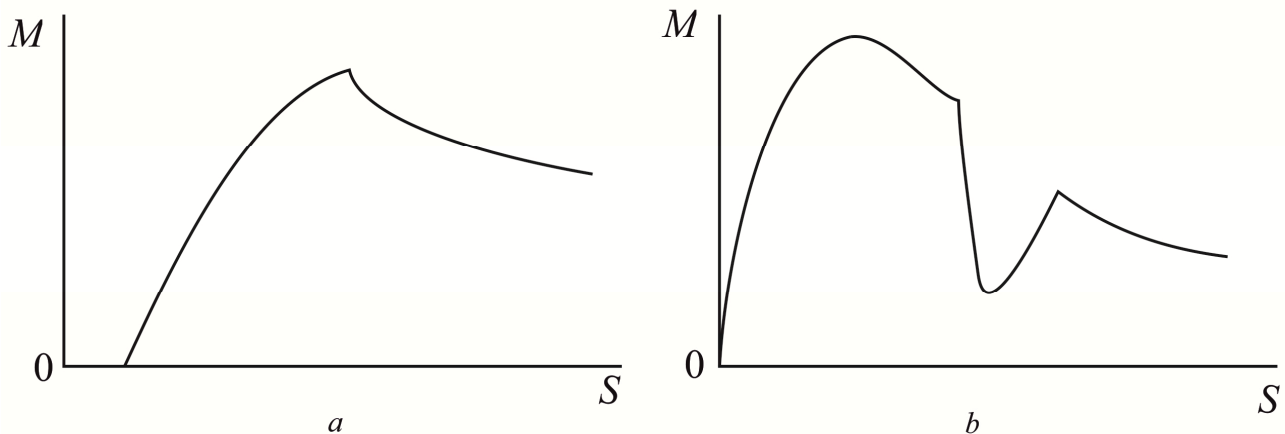


Fig. 13.7. Dependence of electromagnetic torque of induction motor against slip: a) at self-excitation; b) at acceleration

When both the motor inertia moment and the operating mechanism anti-torque moment values are low, and also when the supply voltage is high, the motor can accelerate so fast that self-excitation has no time to develop. In this case the motor passes self-excitation area fast and accelerates to the rated rotational speed. This case is represented by the curves in Fig. 13.6.

### 13.7. Self-starting induction motors

To provide operation stability and reliability of the most important power equipment supply at short-time reductions or switching-off the feeding network the self-starting of motors is used. **Self-starting** is the process of recovering of initial motor operation after reduction of its electromagnetic torque due to short-time decrease of voltage or the feeding network switching out.

At self-starting the value of residual voltage on motor terminals should be enough for obtaining the electromagnetic torque greater than the static anti-torque moment of operating mechanism. For self-starting operation only motors of the most important operating mechanisms remain switched on. Motors of equipment which self-starting is not permissible by safety conditions should be switched off.

Calculation self-starting involves determination of permissible number and total power of the fixed motors for which the level of residual voltage on the terminals is sufficient and the electromagnetic torque exceeding the static torque of operating mechanism is provided.

Self-starting is considered ensured if at the reduced voltage value and following recover of the rated voltage, the surplus torque on the motor shaft is sufficient for development their rated rotational speed under condition that for the acceleration time the windings heating does not exceed the allowed level.

Electric consumers according to the conditions of self-starting can be divided into 2 groups:

1. Consumers with constant anti-torque moment produced by working mechanisms. At short break of power supply, the motors of such consumers lose rotational speed fast, and speed up slowly at recovering initial voltage value (for example, ball mills, conveyors, rolling mills etc). Self-starting of these motors is possible under condition that a motor has torque equal to  $0,8 \dots 0,9 M_{rated}$  at the time of recovering the initial voltage value. For that, the time of supply break should be shortened to particular value determined by condition that the reduced rotational speed would not be less than permissible limit corresponding the above value of electromagnetic torque.

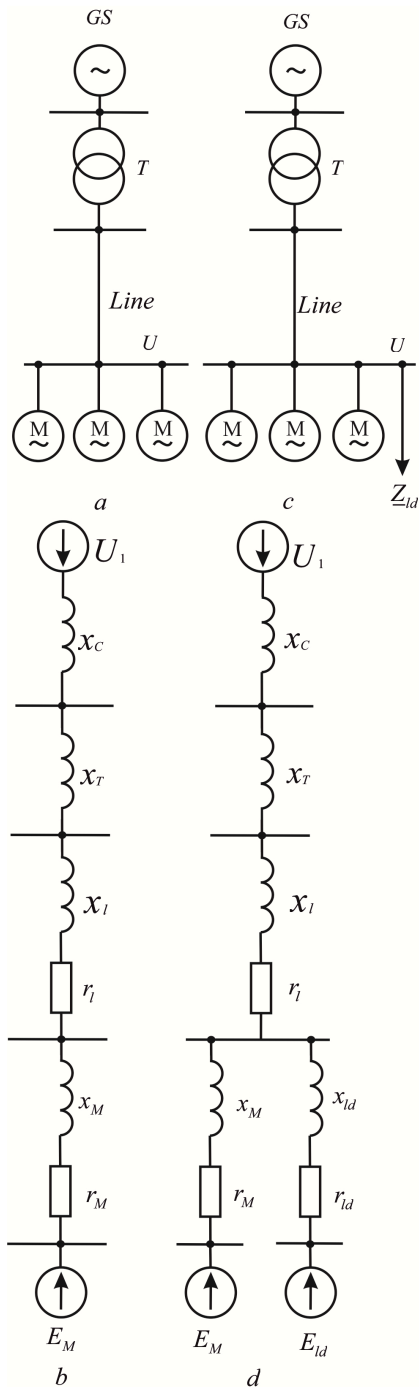


Fig. 13.8. Load node supply circuits:  
 a) design circuit for motor load;  
 b) equivalent circuit for motor load;  
 c) design circuit for mixed load;  
 d) equivalent circuit for mixed load

2. Consumers with fan mechanical characteristics (centrifugal pumps, fans, centrifugal machines etc). As their torque reduces with rotational speed reduction, self-starting of the motors is easier to provide than self-starting of equipment of the first group.

Self-starting of a group of motors can be eased by power reduction of the motors to be self-started by sectioning buses of the distribution plant powered from one feeder line.

Calculation of an induction motor self-starting lies in check of self-starting possibility. It is necessary to find out if the electromagnetic torque at residual value of supply voltage is sufficient and to determine the allowable heating for the time of acceleration of the unit consisting of the motor and the working mechanism. Typical circuits of power supply system nodes with motors and mixed consumers feeding while analyzing the conditions of self-starting are given in Fig.13.8 (a, b).

From the equivalent circuit (Fig. 13.8, b) it follows that the residual voltage on the motor terminals at self-starting is:

$$U = U_1 |z_M| / \sqrt{(r_L + r_M)^2 + (x + x_M)^2}, \quad (13.47)$$

where  $|z_M| = \sqrt{r_M^2 + x_M^2}$ ;  $x = x_c + x_T + x_L$ ,  $r_M$ ,  $x_M$  are equivalent parameters of the motors at the slip at the self-starting beginning.

For the equivalent circuit shown in Fig.13.8, c where a mixed load is represented by motors and impedance the residual voltage at self-starting is:

$$U = U_1 |z_{M,ld}| / \sqrt{(r_L + r_{M,ld})^2 + (x + x_{M,ld})^2}, \quad (13.48)$$

where  $z_{M,ld} = z_M z_{ld} / (z_M + z_{ld})$ .

In the course of self-starting, the following relationship of voltages  $U$  and  $U_1$  is kept:

$$U \geq \frac{U_1 |z_M|}{z_M + x}$$

When minimal permissible value of the voltage for self-starting is known, the permissible value of the fixed

motors power can be found. Motor impedance at the instant of self-starting is determined by expression:

$$|z_M| = S_{bv} U_{rated}^2 / (S_{ss} U_{bv}^2), \quad (13.50)$$

where  $S_{bv}$  is the base power,  $U_{rated}$  is the rated motor voltage,  $U_{bv}$  is the base voltage,  $S_{ss}$  is the rated motor power at the rated voltage and slip at the instant of self-starting.

Substituting expression (13.50) in (13.49) and using the equality sign we obtain:

$$U = U_1 S_{bv} U_{rated}^2 / (S_{bv} U_{rated} + x U_{bv}^2 S_{ss}). \quad (13.51)$$

From (13.51) we find the motors power at the instant of self-starting beginning:

$$S_{ss} = (U_1 / U - 1) S_{bv} U_{rated}^2 / (x U_{bv}^2). \quad (13.52)$$

The motors power at the instant of self-starting beginning can be too determined using the rated motor power:

$$S_{ss} = P_{rated} K_{ss} / \eta_{rated} \cos \varphi_{rated}, \quad (13.53)$$

where  $K_{ss}$  is ratio of the motor current at the slip  $s_{ss}$  at the beginning of self-starting,  $\eta_{rated}$ ,  $\cos \varphi_{rated}$  are the rated values of the motor efficiency and power factor.

The value of  $K_{ss}$  is determined by the expression:

$$K_{ss} = I_{*start} \sqrt{1 + s_{cr}^2} / \sqrt{1 + (s_{cr} / s_{ss})^2},$$

where  $I_{*start}$  is the starting current ratio.

Equating the right sides of expressions (13.52) and (13.53), the power of motors, which are not switched off at self-starting, can be found:

- for the circuit in Fig.13.8, a

$$P_{rated,ss} = \frac{(U_{rated} / U_{bv})^2 S_{bv} \cos \varphi_{rated} \eta_{rated}}{x K_{ss} (U_1 / U - 1)}, \quad (13.54)$$

- for the circuit in Fig.13.8, c

$$P_{rated,ss} = \frac{\cos \varphi_{rated} \eta_{rated}}{K_{ss}} \left[ \frac{S_{bv}}{x} (U_{rated} / U_{bv})^2 (U_1 / U - 1) - U^2 / |z_{ld}| \right]. \quad (13.55)$$

Minimal permissible value of the voltage on motor terminals is determined proceeding from possibility of self-starting:

- for units with constant anti-torque moment:

$$U^2 M_{M,min} \geq 1,1 M_{WM}; \quad (13.56)$$

- for units with fan characteristics of the anti-torque moment:

$$U^2 M_{M,max} \geq 1,1 M_{WM}; \quad (13.57)$$

where  $M_{M,min}$ ,  $M_{M,max}$  are motor minimal and maximal electromagnetic torques;  $M_{WM}$  is the static torque of the working mechanism.

### 13.8. Equations of electromechanical transients in synchronous motor

The system  $d, q$  is the most common coordinate system for recording equations of transients in motors. Park-Gorev's equations for synchronous motor are

$$\begin{cases} -u_d = \psi_q d\gamma / dt + d\psi_d / dt + ri_d \\ -u_q = -\psi_d d\gamma / dt + d\psi_q / dt + ri_q \end{cases} \quad (13.58)$$

For short-term power supply break the simplified equations are used instead of complete ones (13.38). They have been got as a result of the following assumptions:

- the EMF component that depends on rate of the flux linkage amplitude change is neglected, i.e. it is assumed that  $d\psi/dt = 0$ ;
- active resistance of the stator winding is neglected ( $r = 0$ ) or taken into account approximately.

The simplified Park-Gorev equations are as follows:

$$\begin{cases} u_d = -\psi_q \cdot d\gamma / dt; \\ u_q = \psi_d \cdot d\gamma / dt. \end{cases} \quad (13.59)$$

They allow reduce the set of differential equations describing transients in synchronous motor and to increase the step of integration by 2 or 3 times while equations solving. These equations are more obvious and make possible to exclude from consideration magnetic fluxes of the synchronous motor.

With the supply mains voltage  $U$  and the excitation winding voltage  $U_f$  the synchronous motor operation is defined by the following main parameters: the angle  $\delta$  that characterizes position of the rotor regarding synchronously rotating axis (the supply mains voltage vector), the rotor slip  $s$  or the angular rotor speed  $\omega_* = 1 - s$ , components of sub-transient motor EMF  $E_q'', E_d''$  in the axes  $q, d$ , derivative  $E_T' = dE_q'' / dt$  of the EMF  $E_q''$ .

Using these parameters, all other parameters of operation may be expressed. They are:

$P, Q$  – active and reactive power consumed by the motor from mains;

$I$  - the stator winding current;

$I_f, I_{1d}, I_{1q}$  - the currents in the field and damping windings by axes  $q, d$ ;

$E_q, E_d$  - components of the synchronous EMF. The operation condition is described by the equations:

$$d\delta / dt = 2\pi f_0 s; \quad (13.60)$$

$$T_j \cdot ds / dt = M_{WM} - M; \quad (13.61)$$

$$E_T' = dE_q'' / dt; \quad (13.62)$$

$$\begin{aligned} T_d' T_d'' \cdot dE_T' / dt + (T_d' + T_d'') E_T' = \\ -E_q'' + (T_d' + T_d'') [(x_d' - x_d'') / x_d'] (du_q / dt) + \\ + u_q (x_q - x_q'') / x_d + (x_d'' / x_d) E_{q,rated} (u_f + T_{\sigma 1d} du / dt); \end{aligned} \quad (13.63)$$



$$T'_{1q} \cdot dE''_d / dt + E''_d = u_d(x_q - x''_q) / x_q. \quad (13.64)$$

In equation (13.63)  $T_{\sigma 1d}$  is the time constant of damping winding leakage by the quadrature-axis that is defined by the ratio

$$T_{\sigma,1d} = x_{\sigma,1d} / R_{1d}. \quad (13.65)$$

The operating mechanism anti-torque moment reduced to the rated power is determined by the dependence:

$$M_{WM} = [M_0 + (K_{lf} - M_0)\omega^p] P_{rated} / S_{rated}, \quad (13.66)$$

where  $M_0$  is the initial moment (at  $s=1$ , or  $\omega=0$ );  $p$  is a power exponent that characterizes dependence of the operating mechanism torque on the rotational speed.

Electromagnetic torque developed by synchronous motor is

$$M = P / \omega_U, \quad (13.67)$$

where  $\omega_U$  is frequency of the stator winding voltage.

Active and reactive power consumed in transients are commonly expressed in terms of sub-transient EMF components and determined from the formulae:

$$P = E''_q U \sin \delta / x''_d - E''_q U \cos \delta / x''_q + (U^2 / 2)(1 / x''_q - 1 / x''_d) \sin(2\delta); \quad (13.68)$$

$$Q = -E''_q U \cos \delta / x''_d - E''_d U \sin \delta / x''_q + U^2 (\cos^2 \delta / x''_d + \sin^2 \delta / x''_q). \quad (13.69)$$

The initial conditions for solving the differential equations in transients are the following:

$$\delta(0) = \delta(-0); \quad (13.70)$$

$$s(0) = s(-0); \quad (13.71)$$

$$E''_q(0) = E''_q(-0); \quad (13.72)$$

$$E_T(0) = E'_T(-0) + (1 / T'_d + 1 / T''_d)(1 - x''_d / x''_d) \Delta u_q + (x''_d / x_d) [T_{\sigma,1d} / (T'_d T''_d)] E_{q,rated} \Delta u_f; \quad (13.73)$$

$$E''_d(0) = E''_d(-0). \quad (13.74)$$

In conditions (13.70) – (13.74) the operation parameters with the value equal to zero refer to the previous operation of the synchronous motor.

### 13.9. Large disturbance arising in the form of three-phase fault on terminals of a synchronous motor

With three-phase short circuit at synchronous motor terminals, the voltage on stator winding is equal to zero ( $U = 0$ ). Therefore the set of differential equations for transients can be represented as:

$$T_j ds / dt = M_{WM}; \quad (13.75)$$

$$\begin{aligned} T'_d T''_d dE'_T / dt + (T'_d + T''_d) E'_T + E_q = \\ = (x''_d / x_d) E_{q,rated} (u_f + T_{\sigma,1d} du_f / dt); \end{aligned} \quad (13.76)$$

$$T'_{1q} dE''_d / dt + E''_d = 0. \quad (13.77)$$

The following initial conditions correspond to its solution:

$$s(0) = s(-0); \quad (13.78)$$

$$E''_q(0) = E''_q(-0); \quad (13.79)$$

$$\begin{aligned} dE''_q(0) / dt = u_q(-0)(1 - x''_d / x'_d)(1 / T'_d + \\ + 1 / T''_d) + (x''_d / x_d) E_{q,rated} T_{\sigma,1d} \Delta u_f / (T'_d T''_d); \end{aligned} \quad (13.80)$$

$$E''_d(0) = E''_d(-0). \quad (13.81)$$

The electromechanical transient (13.73) proceeds independently of the electromagnetic transients by direct axis (13.76) and quadrature axis (13.77) (when  $U_f = const$ ).

Solving differential equations (13.76) and (13.77) allows to define the character of changes of the forced and free EMF components by direct and quadrature axes. As a result it enables calculation of in-feed current at the point of short circuit from the motor in any load node and allows calculating its influence of power supply system operation stability.

### 13.10. Load surge on synchronous motor

Load surge on a synchronous motor occurs when the motor operation is effected by one of two typical factors: reduction of supply voltage or increase of operation mechanism torque.

Let us assume that there sharp voltage falling from  $U_0$  to  $U_1$  occurs. Respectively the motor torque-angle curve changes too:  $M_0(\delta) \Rightarrow M_1(\delta)$  (Fig.13.9, a) In this case the new steady state in the point  $c$  is obtained after a hunting cycle of the rotor about this point.

Using the area method it is found that the acceleration area is less than the deceleration one. As a result the motor operation remains stable.

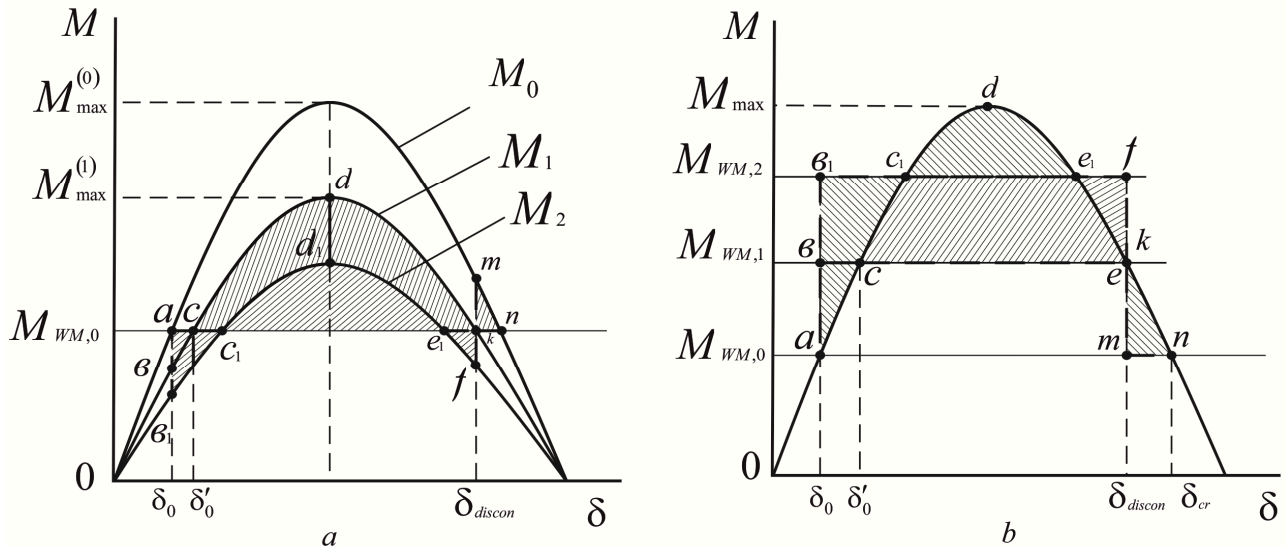


Fig. 13.9. Torque-angle curves of synchronous motor and operation mechanism: a –voltage decreases, b –operation mechanism torque increases

When voltage reduces to  $U_2$  the equilibrium point of a unit steady state moves to the characteristic  $M_2(\delta)$  (point  $c'$ ) and becomes unstable. To keep stability of the unit operation, it is necessary to increase voltage to  $U_0$ . Using the area method the value of angle  $\delta$  is chosen so that the sum of the acceleration areas is less than the sum of the deceleration areas:

$$\left| F_{ab_1c_1} + F_{e_1fk} \right| < \left| F_{c_1e_1d_1} + F_{knm} \right|. \quad (13.82)$$

When  $U$  decreases from  $U_0$  to  $U_1$  (the electromagnetic torque decreases from  $M_0$  to  $M_2$ ) the limiting angle at which the voltage should be retrieved is determined by the area method:

$$\int_{\delta_0}^{\delta_{discon}} (M_{WM,0} - M_{max}^{(1)} \sin \delta) d\delta - \int_{\delta_{discon}}^{\delta_{cr}} (M_{max}^{(1)} \sin \delta - M_{WM,0}) d\delta = 0. \quad (13.83)$$

After integration (13.83) we have:

$$\begin{aligned} (M_{max}^{(0)} - M_{max}^{(1)}) \cos \delta_{discon} &= M_{WM,0} (\delta_{cr} - \delta_0) + \\ &+ M_{max}^{(0)} \cos \delta_{cr} - M_{max}^{(1)} \cos \delta_0, \end{aligned}$$

from where:

$$\begin{aligned} \delta_{discon} &= \arccos [M_{WM,0} (\delta_{cr} - \delta_0) + M_{max}^{(0)} \cos \delta_{cr} - \\ &- M_{max}^{(1)} \cos \delta_0] / (M_{max}^{(0)} - M_{max}^{(1)}) \end{aligned} \quad (13.84)$$

In the case of load on a synchronous motor shaft increase, the braking torque surge reveals in this torque increase from  $M_{WM,0}$  to  $M_{WM,1}$ . The new stationary condition in the point  $c$  will be stable as the acceleration area is less than the deceleration one (Fig.15.9, b). With the anti-torque increase to  $M_{WM,2}$  the unit operation condition turns unstable, as the acceleration area  $ac_1b_1$  is greater than the deceleration one  $c_1e_1d$ .

To provide stable operation it is necessary to decrease the anti-torque moment of the mechanism from  $M_{WM,2}$  to  $M_{WM,0}$  at the angle  $\delta_{discon}$  value not more than the limiting value  $\delta_{cr}$  determined by the operation stability condition.

In the case of the anti-torque moment surge from  $M_{WM,0}$  to some value  $M_{WM}$  causing violation of the operation stability at constant voltage, the limiting angle of the motor switching off could be found from the equation set up by the area method:

$$\int_{\delta_0}^{\delta_{discon}} (M_{WM} - M_{max}^{(0)} \sin \delta) d\delta - \int_{\delta_{discon}}^{\delta_{cr}} (M_{max}^{(0)} \sin \delta - M_{WM,0}) d\delta = 0. \quad (13.85)$$

After the equation integration and relevant result transformation find out:

$$\delta_{discon} = [M_{WM} \delta_0 - M_{WM,0} \delta_{cr} - M_{max}^{(0)} (\cos \delta_{cr} - \cos \delta_0)] / (M_{WM} - M_{WM,0}). \quad (13.86)$$

In analysis of operation stability, it is important to know the limiting operation time at the condition of load surge on the motor that worked in the previous instant in the normal condition. It is the time during which the supply voltage decrease or operation mechanism torque increase is permissible. The limiting time can be determined by step-by-step method or simplified method of sine curve with secant line approximation that passing through the points on the torque-angle curve at the angles  $\delta'_0$  and  $\delta_{discon}$ . With use of the simplified approximation method (Fig.13.10, a) the limiting time  $\delta_{discon}$  is determined by the expression that describes transient at load surge:

$$T_j d^2 \delta / dt = M_{WM,0} - M_{max}^{(1)} \sin \delta. \quad (13.87)$$

After introduction on new variables

$$\tau = t \sqrt{M_{max}^{(1)} / T_j} \quad \text{and} \quad M_* = M_{WM,0} / M_{max}^{(1)}$$

the latter equation turns to:

$$d^2 \delta / d\tau^2 = M_* - \sin \delta, \quad (13.88)$$

where  $M_*$  is constant value of the reduced anti-torque moment of operation mechanism.

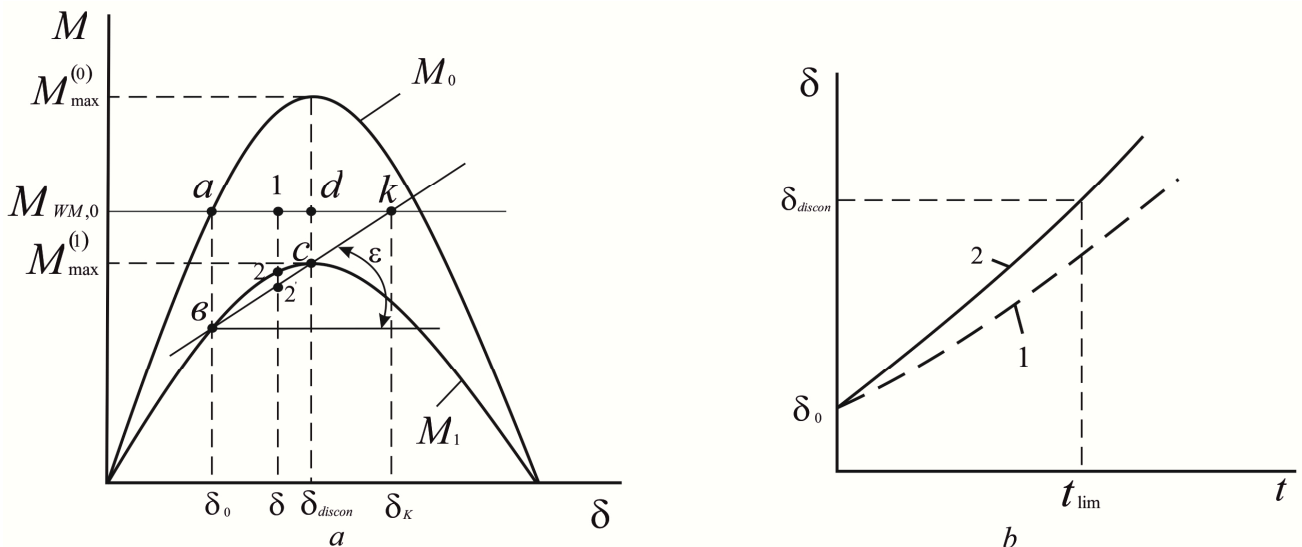


Fig. 13.10. Linearization of equation describing transient at load surge: a – torque-angle curves of synchronous motor and operation mechanism, b – dependence of angle change in time

For solving equation (13.88) it is integrated replacing the sine curve by the line segment  $bc$  (Fig.13.10, a). The difference between the reduced anti-torque moment  $M_*$  and the electromagnetic torque is equal to  $\Delta M_* = M_* - \sin \delta$  and corresponds the segment  $1 - 2$  in the Fig.13.10.

Replacing sinusoid segment  $b2c$  with the corresponding line segment  $b2'c$  we have:

$$\Delta M_* = (\delta_K - \delta) \cdot \operatorname{tg} \varepsilon, \quad (13.89)$$

where  $\operatorname{tg} \varepsilon = (\sin \delta_{discon} - \sin \delta^1) / (\delta_{discon} - \delta^1) = c$ .

Hence equation (13.88) is replaced with the equation

$$d^2 \delta / d\tau^2 = c(\delta_K - \delta). \quad (13.90)$$

After integration of this equation at initial conditions ( $\delta_0 = \delta_0^1$ ),  $\tau_0 = 0$ , it is determined:

$$\delta = \delta_K - (\delta_K - \delta_0^1) \cdot \cos(\tau \cdot \sqrt{c}). \quad (13.91)$$

or  $\cos(\tau \cdot \sqrt{c}) = (\delta_K - \delta) / (\delta_K - \delta_0^1)$ .

If  $\delta_{discon}$  is substantiated instead of  $\delta$  into the expression, the limiting operation time in the conditions of load surge can be determined:

$$\tau = (1 / \sqrt{c}) \arccos \left[ (\delta_K - \delta_{discon}) / (\delta_K - \delta_0^1) \right]. \quad (13.92)$$

Taking into account the values of  $\tau$  and  $c$ , and the equality

$$\begin{aligned} (\delta_K - \delta_{discon}) / (\delta_K - \delta_0^1) &= \\ &= (M_{WM,0} - M_{max}^{(1)} \sin \delta_{discon}) / (M_{WM,0} - M_{max}^{(1)} \sin \delta_0^1) \end{aligned}$$

that follows from the triangles  $akb$  and  $dkc$  ( $kd/ka=dc/ab$ ) similarity, we find that

$$\begin{aligned} t_{lim} &= \{T_j (\delta_{discon} - \delta_0^1) / [M_{max}^{(1)} (\sin \delta_{discon} - \sin \delta_0^1)]\} \times \\ &\times \arccos [(M_0 / M_{max}^{(1)} - \sin \delta_{discon}) / (M_0 / M_{max}^{(1)} - \sin \delta_0^1)]. \end{aligned} \quad (13.93)$$

Using the curves that characterize the angle  $\delta = f(t)$  change at load surge (Fig.13.10, b) we can assess inaccuracy of the approximation method of replacement the sine curve with the secant  $bc$ . With accuracy acceptable for engineering calculations, the allowed time of synchronous motor operation with load surge can be determined by expression (13.93).

### 13.11. Starting synchronous motors

For starting a synchronous motor is switched on to the electric network in the unexcited state ( $u_f = 0$ ). This state is determined by the set of differential equations following from the common equations set (13.60 – 13.64):

$$d\delta / dt = 2\pi f_0 s; \quad (13.94)$$

$$T_j ds / dt = M_{WM} - M; \quad (13.95)$$

$$T'_d T''_d \cdot d^2 E''_q / dt^2 + (T'_d + T''_d) \cdot dE''_q / dt + E''_q = (T'_d + T''_d)(1 - x''_d / x'_d) du_q / dt + (1 - x''_d / x'_d) u_q ; \quad (13.96)$$

$$T'_{1q} \cdot dE''_d / dt + E''_d = u_d (1 - x''_q / x'_q). \quad (13.97)$$

Taking into account the fact that the starting process weakly depends on the rotor initial position that is unknown it is taken in calculations that  $\delta(0) = 0$ . In this case all initial conditions that correspond to starting of a stationary motor are:

$$\left\{ \begin{array}{l} \delta(0) = 0; \\ s(0) = 0; \\ E''_q(0) = 0; \\ dE''_q / dt = -u(0)(1 - x''_d / x'_d)(1 / T'_d + 1 / T''_d); \\ E''_d(0) = 0. \end{array} \right. \quad (13.98)$$

In transient of starting, operation parameters include forced and free components. The basic patterns of the starting process are determined by forced components caused by the mains voltage. Free components act only at the very beginning of starting and do not virtually have impact on the motor speed-up, therefore they are not taken into consideration in analysis.

The process of synchronous motor starting can be calculated with the help of two main methods that have different accuracy.

The exact calculation method is based on solving differential equations set (13.94 – 13.98) added by expressions for braking torque of a working mechanism (13.66), electromagnetic torque (13.67) and (13.68) and the expression for determination voltage in the network. This method considers all forced and free components of operation parameters, gradual growing of the forced current in the field winding after its connection to exciter, influence of speed mode on parameter values, slip change and voltage change on the armature and field windings. The drawback of exact method is the necessity of integration with small time step at solving set of five differential equations. This fact increases time of computation.

The simplified method of starting process calculation does not consider free components and some of the forced components of the synchronous motor operation parameters. In this case starting process is considered as the process that consists of two successive stages, i.e.:

– unexcited motor acceleration to sub-synchronous rotational frequency under influence of the average induction torque  $M_{asynchr}$ ;

– pull-in of the motor into synchronism under impact of the synchronizing torque caused by excitation.

The process of the unexcited motor speed-up is calculated in accordance with the equation

$$T_j ds / dt = M_{WM} - M_{asynchr} = \Delta M. \quad (13.99)$$

At this stage the starting armature current rush and time of acceleration to sub-synchronous rotational frequency are determined. Initial value of the stator current periodical component is found by the expression

$$I = U / (x''_d + x_{ext}). \quad (13.100)$$

where  $U$  is voltage in the network point (behind  $x_{ext}$ ) where it may be taken as independent from the motor operation.

Time of speed-up to sub-synchronous rotational frequency can be determined in the same way as the time for an induction motor. When sub-synchronous rotational frequency is reached ( $s \approx 0,1$ ) the voltage is applied to the field winding. In this case there the induction electromagnetic torque  $M_{asynchr}$  arises as well as the synchronous torque  $M_{synchr}$ . As a result the motor drops into synchronism acquiring the properties of a synchronous motor.

Synchronous electromagnetic torque that acts at the second stage of the motor starting process is determined by the expression

$$M_{synchr} = (E''_{q2} U / x''_d) \sin \delta = (E_{q,rated} u_f U / x_d) \sin \delta, \quad (13.101)$$

where  $E''_{q2}$  is new forced component of the sub-transient EMF that depends on the field voltage  $u_f$ . It can be found from the equation:

$$\begin{aligned} T'_d T''_d \cdot d^2 E''_{q2} / dt^2 + (T'_d + T''_d) \cdot dE''_d / dt + E''_d = \\ = x''_d / x_d \cdot E_{q,rated} (u_f + T_{\sigma 1,d} du_f / dt) \end{aligned}$$

In the steady state this component is equal to

$$E''_{q2} = E_{q,rated} u_f x''_d / x_d.$$

Calculation of starting process at the stage of motor synchronization lies in integration of the equations:

$$\begin{aligned} T_j ds / dt = M_{WM} - M_{asynchr} - M_{nchr} \\ d\delta / dt = 2\pi f_0 \delta. \end{aligned}$$

Initial value of the angle ( $\delta$ ) is assumed equal to zero in this case. Advantages of starting process calculation by the simplified method are as follows: order of the differential equations set reduces, the step of integration is increases by two orders, computation time decreases. Nevertheless the found process of the rotational frequency, the armature winding voltage and current change, and average values of active and reactive power have sufficient accuracy.

### 13.12. Self-starting synchronous motors

Depending on the transient behavior self-starting can be divided into two types:

- self-starting with keeping dynamic stability of motor operation;
- self-starting with pulling out of synchronism (i.e. instability of the synchronous motor)

followed by resynchronization.

If the synchronous motor operates as an induction motor with certain slip to the instant of the voltage recovery, self-starting can be considered as an induction motor starting that is performed from intermediate rotational speed to which the motor had decelerated during the voltage lack.

Practically, self-starting synchronous motor with keeping its operation dynamic stability can be secured if the angle  $\delta$  value, characterizing the rotor position during regular power supply, does not exceed the critical value  $\delta_{cr} \approx 140 \dots 150^\circ$ .

The angle value is found from solution of the electromagnetic transients; it is equal to

$$\delta = \delta_0 + \pi f_0 \Delta M t^2 / T_j, \quad (13.102)$$

where  $\delta_0$  is the angle at the operation condition preceding power supply disturbance;  $\Delta M = M_{WM} - M$  is excess of the torque on the unit shaft while regular supply disturbance.

The most unfavorable disturbance of regular power supply conditions occurs at three-phase fault at small electrical remoteness from motor ( $(M \rightarrow 0; \Delta M \approx M_{WM})$ ). In this case the angle gets the value  $\delta_{cr}$  at typical initial conditions  $T_j \geq 3c$ ,  $k_3 \leq (0,7...0,8)$  for the time  $t_{cr} = 0,2...0,3s$ .

To keep dynamic stability of the synchronous motor operation,  $t_{discon}$  must be less than  $t_{cr}$ .

The critical time value  $t_{cr}$  depends significantly on the residual voltage on the motor terminals and state of its excitation system at the instant of power supply disturbance. The residual voltage value  $U_{res}$  depends on electrical remoteness of the short circuit point from the source of power supply and on the synchronous motor condition. The state of excitation system depends on a feed circuit and a type of exciter. For instance, when  $U_{*res} = 0,3$  and motor excitation is kept,  $t_{cr}$  increases in comparison with the data given above by 1.4 times, and in the case of excitation refusal increases only by 1.2 times.

The following tasks have to be solved at calculation of a synchronous motor self-starting:

- self-starting influence on regular power consumers and elements of power supply system operation is checked;
- the residual voltage in the nodes of synchronous motors connection needed for recovery of the motors regular operation is determined;
- the motor electromagnetic torque needed for its guaranteed pulling in synchronism is found;
- time of starting process and the motor overheating temperature is calculated.

Depending on load impedance, different limits of the voltage reduction in the node of a power supply system are allowed at synchronous motor self-starting:

- for simultaneous feed of lighting load and motors with frequent and long-term starts the allowed voltage is  $U \geq 0,9U_{rated}$ , and when starting and self-starting are infrequent and short-term it is  $U \geq (0,8...0,85)U_{rated}$ ;
- for isolated feed of lighting load and motors independently of frequency and time of their starting and self-starting the allowed voltage is  $U \geq (0,75...0,8)U_{rated}$ ;
- for a fluorescent lighting load  $U \geq 0,9U_{rated}$ .

When motors are powered as a transformer unit self-starting voltage is limited by minimum value of electromagnetic torque needed for the unit acceleration.

To check possibility of self-starting it is necessary to compare average asynchronous torque of the motor and operation mechanism anti-torque moment. Asynchronous torque of a synchronous motor is determined as it is done for generators.

Promising solutions in increase of synchronous motor dynamic stability in the process of their self-starting are substantiation and choice of circuits of a motor connection to the power system, i.e. sectionalizing of switch gears, use of fast automatic load transfer and automatic re-closing, parallel section work by means of bus reactor and use of motors with split stator winding.

Automatic re-synchronizing of a motor is used when self-starting is impracticable via circuitry. Pulling the motor into step in this case is to be ensured by the field forcing that increases maximum synchronous torque. To ensure resynchronization, a working mechanism unloading is provided as well as other measures affording the motor pulling in synchronism.

Time of power supply interruption during which the motor does not fall out of step is defined by the expression:

$$t_{II} \leq 0,06\sqrt{T_{j\Sigma}(M_{max} - 0,6M_{WM}) / M_{WM,0}}, \quad (13.103)$$

where  $T_{j\Sigma}$  is electromechanical time constant of the unit consisting of the motor and mechanism.



Fig. 13.11 shows dependence of the allowed time of power supply interruption on electromechanical time constant of the unit at various values of the maximum torque  $M_{\max}$  ratio and the static torque  $M_{WM,0} = 0,8M_{rated}$ .

The average critical slip at which pulling in synchronism is ensured after excitation voltage recovery under the action of the input torque is

$$s_{cr} = \sqrt{k_f M_{\max} / T_j}, \quad (13.104)$$

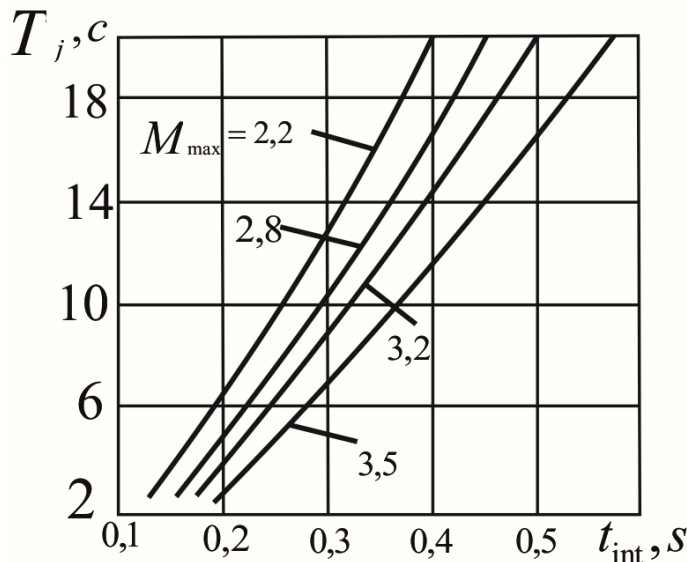


Fig. 13.11. Duration of allowable synchronous motor power supply interruption

where  $k_f$  is ratio of the field current at resynchronization; in the case of no field forcing it is  $k_f = 1$ .

Thus the critical slip of a synchronous motor corresponds to the slip at which the motor pulling into synchronism after the field energizing is ensured. The higher the value  $s_{cr}$  is the lower torque for the motor self-starting is needed.

### 13.13. Electromechanical transients in complex load node

At short-time interruptions of power supply in a complex load node group transients caused by all its power consumers occur.

To calculate and analyze transients in a complex load node a design circuit is made up (Fig.13.12, a).

In the design circuit the following components are connected to each the section:  $n_1, n_2$  – synchronous motors,  $m_1, m_2$  – induction motors,  $P_{ld1} + jQ_{ld1}$ ,  $P_{ld2} + jQ_{ld2}$  – other loads that are accounted by their static characteristics.

The design circuit of load node allows to calculate the initial operation parameters in the case of short-term power supply interruptions, running down of the motor load in the case of short circuit in the supply network and after short circuit switch off, and motor self-starting after automatic load transfer operation. At calculation transients in a complex load node, characteristics of synchronous and induction motors can be considered separately, or the characteristic of the equivalent motor for the entire section of switchgear can be used.

Parameters of synchronous motor operation are:

– the parameters that are in the set of differential equations describing the transients, such as  $\delta_i$  – the angle of displacement between quadrature axis of the  $i$ -th motor rotor and the synchronously rotating vector of the power system EMF,  $\omega_i$  – the angular speed of the  $i$ -th motor,  $E''_{qi}$  – the sub-transient EMF quadrature component,  $E'_{Ti}$  – the first time derivative of the EMF component  $E''_{di}$ ,  $E''_{qi}$  – the component of sub-transient EMF by the quadrature axis.

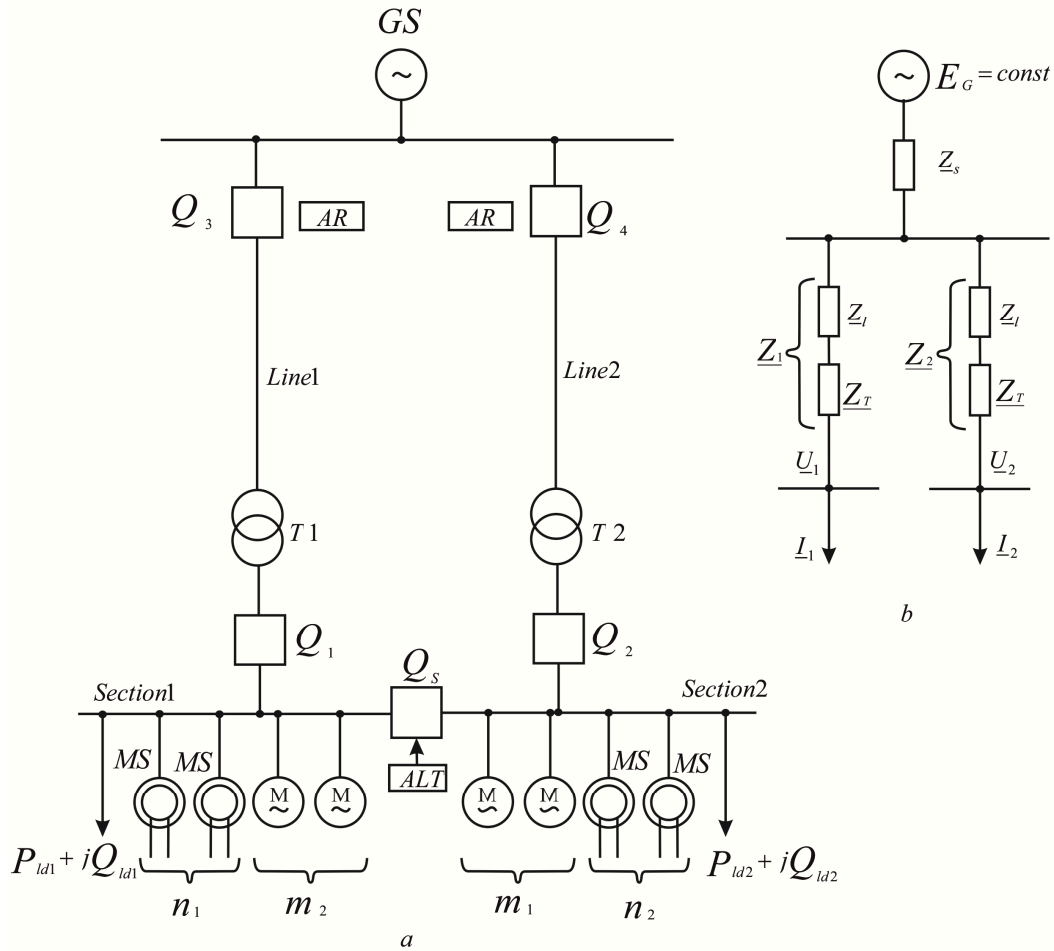


Fig. 13.12. Diagram of connection for node with complex load: a – design circuit; b – equivalent circuit

– the parameters that are found with use of algebraic dependences and remain constant at all stages of calculation: the direct-axis  $U_{di}$  and quadrature-axis  $U_{qi}$  components of voltage on the stator winding, the power active  $P_i$  and reactive  $Q_i$  components, the stator winding current  $I_i$ , the motor synchronous EMF direct-axis  $E_{di}$  and quadrature-axis  $E_{qi}$  components, the transient EMF  $E'_{qi}$ , the field current  $I_{fi}$ , the direct-axis and quadrature-axis damping windings currents  $I_{1di}$  and  $I_{1qi}$  respectively, the field winding voltage  $U_{fi}$ .

Operation parameters of an induction motor are:

– the parameters that present in the set of differential equations describing transients, i.e. the angular speed of  $i - th$  induction motor  $\omega_{IM,i}$  and the induction motor sub-transient EMF  $E''_{IM,i}$ ;

– the parameters that are present in the algebraic expressions: the active  $P_{IM,i}$  and reactive  $Q_{IM,i}$  power, the stator current  $I_{IM,i}$ , the active and reactive power of the sectional switch-gear load  $P_{SwG}$  and  $Q_{SwG}$ , the section load current  $I_{SwG}$ .

Replacement the section load with one equivalent motor becomes necessary at determining the section voltage frequency while group running-out of complex motor load after the section switching off.

In general case, when  $n$  synchronous and  $m$  induction motors are connected to a node of the network, the equivalent motor should satisfy certain conditions.

Equivalent motor rated power is determined by the sum of rated powers of all node motors

$$S_{rated,equiv} = \sum_{i=1}^n S_{SM, rated, i} + \sum_{i=1}^m S_{IM, rated, i} \quad (13.105)$$

Electromechanical time constant of equivalent unit of motor and working mechanism is

$$T_{j, equiv} = \left[ \sum_{i=1}^n (T_{j, SM, i} S_{SM, rated, i}) + \sum_{i=1}^m (T_{j, IM, i} S_{IM, rated, i}) \right] / S_{rated, equiv} \quad (13.106)$$

Active power consumed by the equivalent motor from the network is

$$P_{equiv} = \left[ \sum_{i=1}^n (P_{SM, i} S_{SM, rated, i}) + \sum_{i=1}^m (P_{IM, i} S_{IM, rated, i}) \right] / S_{rated, equiv} \quad (13.107)$$

The anti-torque moment of the working mechanism, loading the equivalent motor, is

$$M_{WM, equiv} = \left[ \sum_{i=1}^n (M_{WM, SM, i} S_{SM, rated, i}) + \sum_{i=1}^m (M_{WM, IM, i} S_{IM, rated, i}) \right] / S_{rated, equiv} \quad (13.108)$$

The equations of electromechanical transients used to determine the equivalent motor operation parameters are the following:

for a synchronous motor

$$T_{j, SM, i} d\omega_{SM, i} / dt = M_{SM, i} - M_{WM, SM, i}$$

for an induction motor

$$T_{j, IM, i} d\omega_{IM, i} / dt = M_{IM, i} - M_{WM, IM, i}$$

for the equivalent motor

$$T_{j, equiv} d\omega_{equiv} / dt = M_{equiv} - M_{WM, equiv} \quad (13.109)$$

Sum of equations for electromechanical transients of the load node motors reduced to rated power of the equivalent motor is recorded as:

$$\begin{aligned} & \sum_{i=1}^n [(T_{j, SM, i} S_{SM, rated, i} / S_{rated, equiv}) d\omega_{SM, i} / dt] + \\ & + \sum_{i=1}^m [(T_{j, IM, i} S_{IM, rated, i} / S_{rated, equiv}) d\omega_{IM, i} / dt] = \\ & = T_{j, equiv} d\omega_{equiv} / dt \end{aligned} \quad (13.110)$$

If equation (13.110) is integrated at the load node induction motors angular speed of their steady state, we get the steady state rotational speed of the equivalent motor:

$$\omega_{steady,equiv} = \left[ \sum_{i=1}^n (T_{j,SM,i} S_{SM,rated,i} \omega_{SM,steady,i}) + \sum_{i=1}^m (T_{j,IM,i} S_{IM,rated,i} \omega_{IM,steady,i}) \right] / \left[ \sum_{i=1}^n (T_{j,SM,i} S_{SM,rated,i}) + \sum_{i=1}^m (T_{j,IM,i} S_{IM,rated,i}) \right] \quad (13.111)$$

Integration of equation (13.110) at angular speed of synchronous and induction motors of the load node  $\omega_{SM,i}$ ,  $\omega_{IM,i}$  achieved in any instant of the transient allows to obtain rotational speed of the equivalent motor in the same instant:

$$\omega_{equiv} = \left[ \sum_{i=1}^n (T_{j,SM,i} S_{SM,rated,i} \omega_{SM,i}) + \sum_{i=1}^m (T_{j,IM,i} S_{IM,rated,i} \omega_{IM,i}) \right] / \left[ \sum_{i=1}^n (T_{j,SM,i} S_{SM,rated,i}) + \sum_{i=1}^m (T_{j,IM,i} S_{IM,rated,i}) \right] \quad (13.112)$$

Synchronous voltage frequency on the equivalent motor terminals is defined by the expression:

$$\omega_{synchr,equiv} = \omega_{equiv} / \omega_{steady,equiv} \quad (13.113)$$

Synchronous voltage frequency of the equivalent motor terminals in the steady state ( $\omega_{SM,i} = 1$ ;  $\omega_{IM,i} = \omega_{IM,steady,i}$ ) is equal to one.

When transients in complex load nodes are considered such common disturbances as short circuit, group motor running-out and group motor self- starting are of the great practical importance.

When short circuit occurs in a complex load node the short circuit current component from this node motors is significant and reaches 50% and more of the total current at the point of short circuit occurrence.

If synchronous motors predominate in the load node, the angle  $\gamma$  between the supply network EMF vector  $\underline{E}_{GS}$  and the node buses voltage  $\underline{U}$  is about 10...15° in the node operation conditions preceding the short circuit. The angle  $\theta$  between the quadrature-axis  $q$  of the synchronous motor and the vector  $\underline{U}$  is equal to 30..40°. At that the angle between  $\underline{E}_{GS}$  and the axis  $q$  in regular operation of load node is  $\delta = 40...50^\circ$ , and the angle between  $\underline{E}''$  and  $\underline{E}_{GS}$  reduces to 20..30° due to the direct-axis component of the sub-transient EMF  $\underline{E}_d''$ . The motor load current vector  $\underline{I}_M$  in regular conditions leads the voltage vector  $\underline{U}$  by the angle  $\varphi$ .

At the initial instant of short circuit (Fig.13.13, b) the EMF vectors  $\underline{E}_{GS}$  and  $\underline{E}''$  correspond to the previous condition, the short circuit current vectors conditioned by the network  $\underline{I}_{s,GS}$  and the

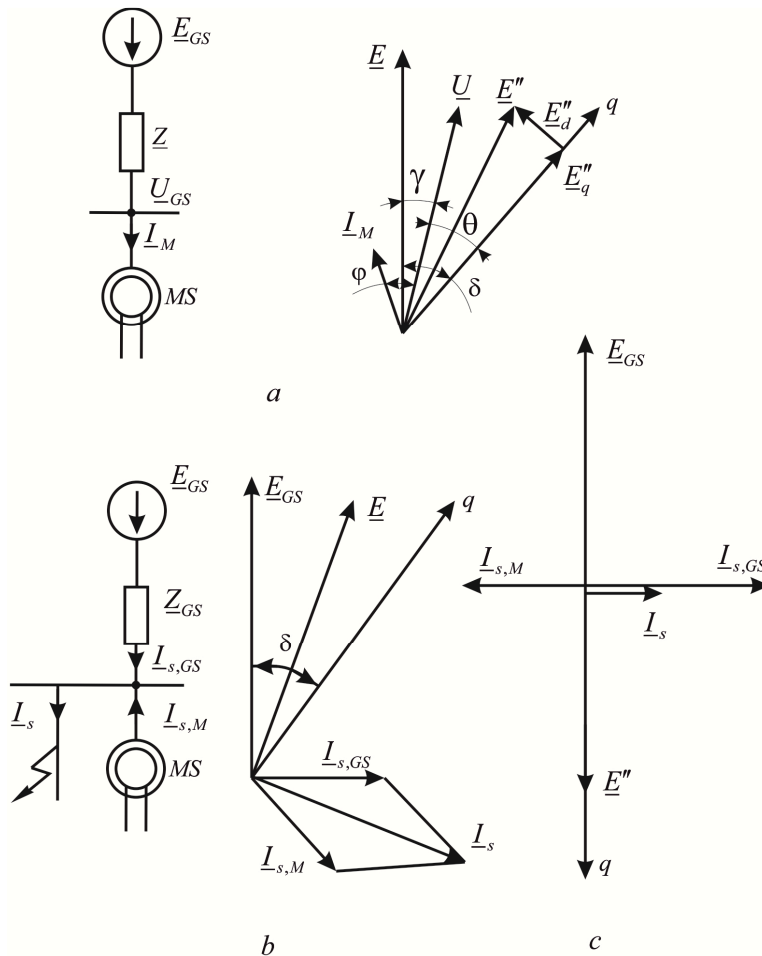


Fig. 13.13. Vector diagrams of currents and voltages in the node: a – for the initial operation; b – for the instant immediately after short circuit; c – for the case when short circuit currents caused by the system power source and the motor are in anti-phase

$I_s = I_{s,G S} + I_{s,M}$  is essentially different from the vector sum of currents  $\underline{I}_s = \underline{I}_{s,G S} + \underline{I}_{s,M}$ . At common determination of short circuit current at load node by means of algebraic summation of the currents  $\underline{I}_{s,G S}$  and  $\underline{I}_{s,M}$  significant error may occur depending of the currents ratio and the time instant. The error can reach 100% and more. To make short circuit current values more exact, computer calculation of transients in complex load nodes of power supply system can be used. That is of great importance in selection of switch gears and conductors.

When faulty section of switchgear is switched off (Fig.13.12) there occurs motor deceleration that is named a group running down of motors. Joint group running down of synchronous and induction motors during significant time interval is characterized by equal average rotational speed of synchronous motors and the slip of induction motors that has value less than critical. This kind of running down is named synchronous as speeds of dissimilar motors running down are equalized due to interchange of electromagnetic power. So motors having less time constants move to “motor operation”, and run down with smaller rate than in the case of individual running down due to additional electromagnetic torque. Motors with bigger electromechanical time constants move to “generator operation” and run down at bigger rate than in the case of individual running down.

Main parameters of synchronous group running down are residual voltage  $U_{residual}$  on the damaged section buses and its frequency  $\omega$ . In the case of joint group running down of synchronous and induction motors, voltage on the node buses is retained mainly by synchronous motors. The voltage frequency complies with average rotational speed of the synchronous motors. The residual voltage and its frequency on the switch-gear section buses in the case of run-out only synchronous

motors  $\underline{I}_{s,M}$  lag the vectors  $\underline{E}_{GS}$  and  $\underline{E}''$  by  $90^\circ$  respectively. The vector sum of currents in the point of short circuit is

$$\underline{I}_s = \underline{I}_{s,G S} + \underline{I}_{s,M}$$

that is less than the algebraic sum of these currents by 5..7%.

After short circuit occurrence synchronous motors of the node begin to decelerate, the angle  $\delta$  value increases and in certain time reaches  $180^\circ$  (Fig.13.13, c). The direct axis component sharply damps to this time instant and coincides with the axis  $q$ . The short circuit current component from feed  $\underline{I}_{s,G S}$  remains

constant and the current component from motors  $\underline{I}_{s,M}$  damps. The algebraic sum of currents

motors comply with EMF and rotational speed of the equivalent motor. The behavior pattern of synchronous motors group run-out depends on the separate motor electromechanical time constants  $T_{j,SM,i}$  relation. If the time constants  $T_{j,SM,i}$  differ, the motors run out being in transient operation.

The motors synchronous run-out takes place under condition, that the electromagnetic torque of each of them, being caused by their electromagnetic interchange, is enough to support the synchronous rotation. The synchronous group run-out existence margin is determined by the inequality

$$|M_i| \leq M_{\max,i}, \quad i = \overline{1, N}, \quad (13.114)$$

where  $M_i$  is electromagnetic torque of the  $i$ -th motor needed for the group run-out existence,  $M_{\max,i}$  is maximum synchronous torque of that motor.

At joint group of synchronous and induction motors run-out their rotational speed is other than that of the equivalent motor, and the voltage on the load node buses equals the average synchronous motors rotational speed.

Group run-out of induction motors only is synchronous at a considerable time interval. Induction motors run-out occurs in the mode of active power generation owing to significant store of kinetic power. At great store of electromagnetic power, the run-out is realized in the mode of reactive power generator. When the store of kinetic or electromagnetic energy is small the induction motors run-out occurs in the mode of active or reactive power consumption. Due to interchange with electromagnetic power the running out speed of dissimilar motors flatten resulting in their synchronous running out.

Group self-starting of complex load node motors occurs after operation of an automatic load transfer at the substation sectionalizing switch (Fig.13.12). Limiting factors of group self-starting are allowable residual voltage and motor currents at the self-starting beginning.

The allowable residual voltage across the motors terminals before switching them on for self-starting should be

$$U_{residual} \approx (0,35...0,4)U_{HOM}. \quad (13.115)$$

The allowable current at switching the motors on for self-starting is determined by their windings electro-dynamical stability.

In designing motors the winding electro-dynamic stability is calculated for the conditions of three-phase fault on motor terminals and their starting operation. The fault is characterized by rms value of the sub-transient current periodic component  $I''$ , and the starting operation is characterized by average value of the starting current periodical component  $I_{start}$ .

At starting, maximum value of the periodical current component with account of its free component at the instant of motor switching on is 30...35% more than the starting current given in catalogue data of the motor.

Therefore

$$I_{start,max} \approx (1,3...1,5)I_{start}. \quad (13.116)$$

As  $I_{start,max}$  is greater than sub-transient current of short circuit  $I''$ , the allowable initial current of motor group switching on at self-starting is

$$I_{con,perm} = I_{start,max}. \quad (13.117)$$

It is about 30% higher than the catalogue motor starting current value.

**Test questions**

1. What are the main causes of operation changes in load nodes of a power supply system?
2. Name typical large disturbances of operation in load nodes that cause a power supply system operation stability violation.
3. What equation set describes transients in induction motors?
4. How does the transient at three-phase fault on synchronous motor terminals occur?
5. What parameters specify an induction motor load surge, and how do they influence on the transient?
6. How does transient in an induction motor develop when voltage on its terminals reduces?
7. How does transient in an induction motor develop when load on its shafts is increased?
8. Give mathematical description of an induction motor starting, and explain how the time of its acceleration could be determined?
9. What is an induction motor self-starting, and how can it be controlled?
10. How is residual voltage across motors terminals determined in the case of self-starting?
11. What is the power of induction motors self-starting, and how is it determined?
12. What equations describe transients in synchronous motors?
13. How does transient at three-phase fault on synchronous motor terminals develop?
14. How does short-time reduction or de-energizing influence on synchronous motor operation stability?
15. How does transient in synchronous motor develop when working mechanism anti-torque moment is increased?
16. How is the allowable time of load surge on synchronous motor determined?
17. What methods can be used for calculation of transient at synchronous motor starting?
18. What is the aim of synchronous motor self starting, and what are the ways of its successful realization?
19. What parameters are determined to check synchronous motor self-starting?
20. What are typical major disturbances in a complex load node of power supply system?
21. What are design circuits and equivalent circuits of complex load nodes?
22. What is the equivalent motor and how its parameters are determined?
23. Describe the character of transients at short circuit in a complex load node.
24. What is the motor group running out and how does its process proceed?
25. Name characteristic properties of motor group self-starting operation.
26. What are the causes of induction motors self-excitation in the case of series compensation of reactive power?
27. What are results of induction motors self-excitation?

**Topics for essay**

1. Differential equations describing transients in induction and synchronous motors.
2. Analysis of typical great disturbances and their influence on operation stability of a power supply system.
3. Transients at self-starting of synchronous and induction motors in load nodes.
4. Transients at great operation disturbances in a complex load node.
5. Computer-aided calculation of operation stability in a complex load node.

## **CHAPTER 14: ENSURING OPERATION STABILITY IN POWER SUPPLY SYSTEMS**

14.1. General statements

14.2. Ensuring power supply system operation stability in project stage

14.3. Use of control means at electric power stations in operating conditions

14.4. Use of protection devices and system automatic equipment

Test questions

Topics for essay



### 14.1. General statements

To ensure operation stability in a power supply systems of an enterprise various measures are provided. These measures are implemented in a project stage or introduced additionally in the process of power supply system exploiting. They are aimed at reduction of rotating and retarding torque imbalance occurring on a working unit shaft during steady-state operation disturbance and in the course of transient. To reach this goal the following factors are applied: corresponding power supply system structure, technical means and emergency measures securing functioning electrical installations in conditions that reduce the length of short time disturbances of power supply and reduce sensitivity of technological electrical equipment to transients in the power supply system.

Measures of operation stability provision should be ensured in the stage of external and internal power supply circuits designing and at selection of electrical equipment, protective devices, control devices and automation as well.

A number of measures for operation stability provision are implemented in the state of power supply system exploiting through elimination of main factors that are cause of undesirable disturbance action on system operation and transition to emergency operation.

Practice of emergency measures application aimed at power supply system stability increase shows that safe and continuous work of electric power consumers in transients can be reached only at simultaneous and joint introduction of various measures as applied to external and internal power supply. Final choice of measures providing power supply system stability should be done on the base of feasible study as the set aims are reached in many cases in different ways.

According to division of electric power system elements onto main (turbines, generators, transformers, synchronous compensators, circuit breakers) and additional (control and compensating devices, system automation devices, transformer neutral resistances, resistances in generator deceleration circuit etc) two groups of measures aimed at increase in power supply system operation stability and transient quality are recognized:

- major measures, ensuring the change of power supply system parameters using its main elements;

- additional measures implemented by means of additional devices setting.

According to action on transient character there are measures providing influence on power supply system in the following ways:

- change of system parameters;
- adjusting operation parameters;
- static stability increase;
- dynamic operation stability provision.

Analyzing all the ways of power supply system operation stability increase it is necessary to aim to maximum use of automatic control and adjusting. It will lead to the cut of costs for consumer continuous power supply in emergency and post-emergency operation.

### 14.2. Ensuring power supply system operation stability in project stage

In projecting and constructing power supply system should be maximally considered requirements to the power supply system operation stability preservation and uninterrupted consumers operation.

#### *Provision of active and reactive power reserves*

Availability of a certain power reserve at generating stations and in various elements of a power supply system is one of important conditions of the system safe operation. First of all there should be provided power reserve at electric power stations. The reserve may consist of load, emergency and repair reserves. The most action on transients the revolving emergency reserve causes. The lowest needed value of this reserve is defined by probability of the most severe accidents in the system. It depends on the ways of excitation control and automatic frequency unloading.

Availability of generators active power reserve provides their work at the small angle  $\delta$  promoting both static and dynamic system stability increase. Availability of reactive power reserve causes decreasing system stability as they work at reduced values of their field currents and therefore at bigger initial angle  $\delta$ .

Optimal reserve distribution and exchange active power flows with the help of automatic frequency adjustment is of great importance for the maintenance of power supply system operation stability. To reach this aim the control systems should be based on improvement of certain processes regulation as well as upgrading connections between different devices and regulating equipment. Combining functions of regulating equipment and connections between components allows creation of united complex control means using computers, variable system structure, functional dependences etc.

#### **Change of power supply system components parameters**

Measures aimed at reduction of generators, transformers and aerial lines reactance are significant for increase stability of electric power and supply systems. In the absence of automatic generators excitation control or in the case of control with dead zone use the operation static stability depends on synchronous reactance  $x_d$ , and the dynamic stability depends on transient reactance  $x'_d$ . Synchronization and resynchronization processes and generators operation conditions in asynchronous mode also depend on sub-transient reactance  $x''_d$  and  $x''_q$  which characterize influence of damping circuits.

Effect caused by the reactance on system stability can be evaluated with the help of expressions defining maximum active generator power:

without automatic excitation control

$$P_{\max} = EU / (x_d + x_{ext}),$$

with automatic excitation control

$$P_{\max} = E'U / (x'_d + x_{ext}).$$

Dependences of the electric power system static operation stability assurance factor on change of generator and power line reactance are given in Fig.14.1 (a, b).

#### **Power line constructional changes**

Every line phase splitting into several wires results in reduction of the resultant resistance of an aerial line. The solution is especially efficient for long-distance power transmission, where it is necessary to increase transmission capacity even at ultrahigh voltage. Dependence of the system static stability assurance factor on the wires number in aerial line phases is shown in Fig.14.1, c.

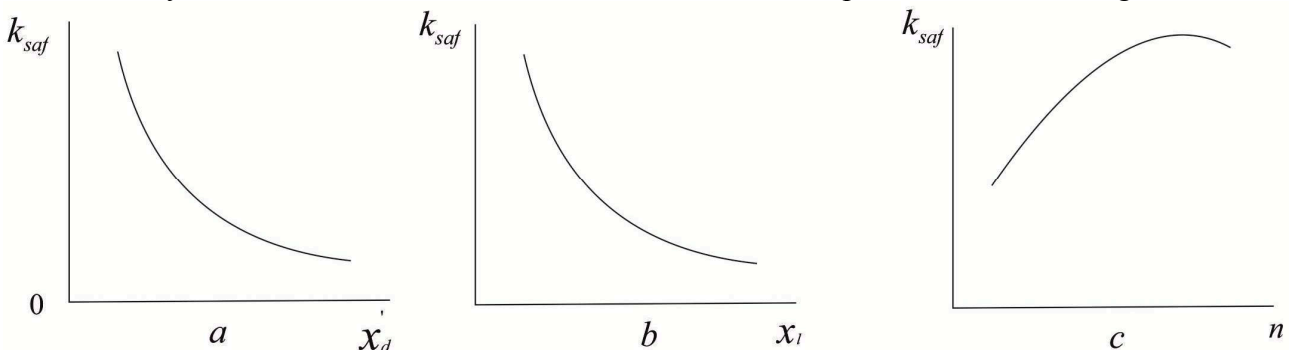


Fig. 14.1. Dependence of steady state static stability assurance factor on: a – generator transient reactance, b – line reactance, c – number of wires in a phase

#### **Increase the electromechanical constant of the “prime mover – generator” unit**

The electromechanical constant  $T_J$  effect to the reserve of dynamic system operation stability and in particular the critical time of short circuit clearing can be illustrated by the example of three-

phase fault at power station buses. In this case change in the angle  $\delta$  is characterized by expression:

$$\delta - \delta_0 = 180Pft^2 / T_J \quad (14.1)$$

From (14.1) we get that the unit inertia constant is to be increased  $n^2$  times to increase the critical time of short circuit clearing  $n$  times at the same stability factor (i.e. critical angle  $\delta_{cr}$ ). If the unit inertia constant is increased two times, the critical time of short circuit clearing increases by 45% roughly.

It should be mentioned that with change of the unit inertia constant, other parameters and characteristics are changed as well. These are constructional dimensions, costs, etc. Fig.14.2 shows dependence of a generator cost on the inertia constant.

#### **Use of more reliable circuits of supply and distribution networks**

Simplified power supply networks with substations connection by taps and use of short circuiting and isolating switches have got wide application. Maintenance of such networks has turned to be rather unreliable. Therefore design the new power supply systems should provide replacement of the simplified substations by ones using circuit breakers.

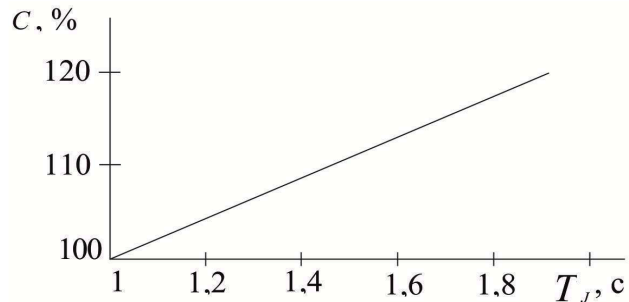


Fig. 14.2. Dependence of generator relative cost on electromechanical time constant

#### **Installation of controlled sources of reactive power**

It is recommended to use static thyristor compensators that enable to localize zones of deep voltage regulation at short circuit in extensive distribution networks. These compensators should have reactive power permitting to prevent voltage avalanche development.

#### **Substantiation of selection of meshed power network operating mode**

Meshed power networks can work in closed or open condition. If a network is closed, short circuit occurrence at any point of this network causes short-time undersupply of electric power for all consumers. But duration of power supply interruption is determined by the time of faulty section switching off  $t_{discon}$  only. When the network works in open condition that can be used in the absence of transit power flows, short circuit causes power supply interruption only of a part of consumers. Time of short circuit in this case is longer by the time of automatic load transfer operation. Therefore either closed or open network operation could be more efficient depending on conditions of protection against short-time power supply disturbance. For instance, if duration of power supply interruption is determined by consumers sticking without time delay, open condition of network is more practical, and if the critical is possibility of synchronous motors self-starting the closed condition of network is more advisable.

#### **Analysis of load supply stability for the case of capacitor banks at substations installation**

In the process of compensating devices designing it is necessary to choose optimum interrelation of the capacitor bank and synchronous motors reactive power. If capacitor banks are located in the load nodes with high share of synchronous motors it should be taken into account that generation of reactive power by capacitor banks is more economical than by synchronous motors. But when capacitor banks power is large, reserve in load node operation stability may be decreased essentially.

#### **Power transformers neutral grounding**

If a transformer neutral is grounded through little resistances which do not significantly increase neutral voltage, the insulation working conditions practically remain constant and the system stability at unbalanced short circuit improves. It can be observed on the example of single-

phase short circuit in a power supply system with transformers winding having star connection and neutrals grounded through resistances (Fig.14.3, a). The equivalent circuit for zero sequence is given in Fig.14.3, b, and the complex equivalent circuit of the power supply system at single-phase short circuit is given in Fig.14.3, c.

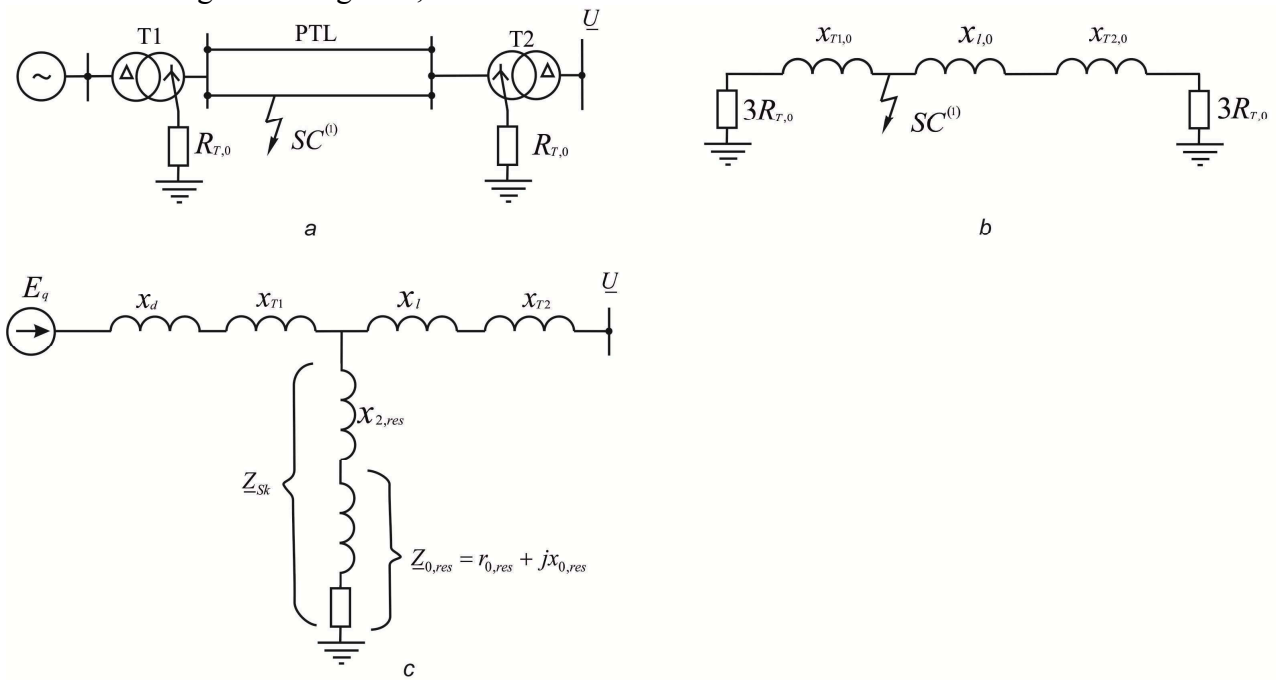


Fig. 14.3. Power supply system with grounded transformers neutrals: a – design circuit, b – equivalent circuit for zero sequence, c – complex equivalent circuit

Increase of emergency shunt impedance  $Z_{EmSh}$  which consists of the total negative sequence reactance  $x_{2res}$  and the impedance  $Z_{0res}$  of the zero sequence causes increase of maximum value of the power-angle characteristic in emergency condition and thus to rise in dynamic stability of power supply system operation.

### 14.3. Use of control means at electric power stations

Operation stability can be significantly influenced by power stations through automatic excitation control, under-frequency load shedding and emergency unloading of prime movers, i.e. turbines.

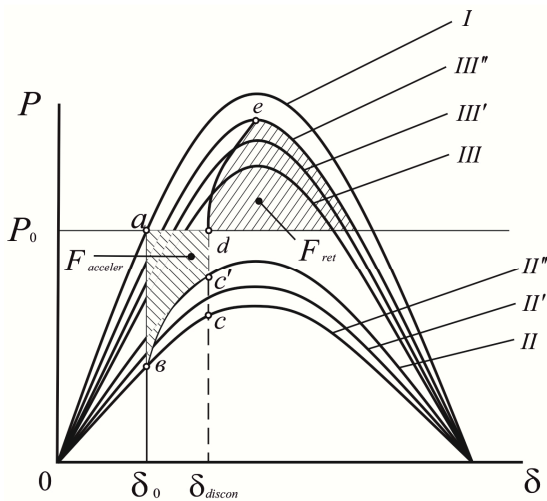


Fig. 14.4. Influence of generator automatic excitation control on dynamic operation stability

#### Automatic excitation control

When short circuit occurs in power supply system and voltage on generator buses falls down, automatic excitation control starts working. As a result of the generator field current increase its electromagnetic power in the emergency condition rises, and the operating point goes over from power-angle characteristic  $II$  to characteristics  $II'$ ,  $III'$ , etc (Fig.14.4).

Thus under the influence of automatic excitation control the generator electromagnetic power at short circuit changes not by the curve  $II$  but by the curve of gradual transition between the

curves  $II$ ,  $III'$ ,  $III''$ , etc (the curve  $bc'$ ) according to fluent increase of the EMF and the field current. After short circuit clearing, the generator electromagnetic power changes not by the curve  $III$ , but by the curve  $de$  gradually transiting from the curve  $III$  to the curves  $III'$ ,  $III''$ , etc. In this case automatic excitation control devices work to reduce acceleration area  $F_{acceler}$  and increase the probable deceleration area  $F_{deceler}$  that results in system operation stability increase.

Automatic excitation control devices can be used as multifunctional means for solution a number important tasks in power supply system:

- keeping required voltage level in the given point;
- securing high limits of static and dynamic operation stability;
- forcing of emergency operation excitation;
- limiting the machine overloading by the stator and rotor currents;
- keeping the field current unchanged during the generator running-out;
- damping small and large oscillations.

With the help of automatic excitation control devices, change the constant-error response, selection of the voltage setting for automatic synchronization and remote change of the field current adjustment limits could be done.

The adaptive excitation regulators and the regulators of variable structure using semiconductors, integrated circuits and elements of digital computing components had been developed. These automatic excitation control devices are close to analogue and digital computers by their structure.

**Under-frequency load shedding**

Frequency reduction in electric power system can cause decrease in the active generator power that disturbs operation stability. Frequency reduction causes decreasing the reactive power generation by sources and at the same time increases the reactive power consumption by the load. It results in voltage fall in load nodes and under the certain condition in avalanche of voltage and frequency when mass consumers turn off and in parallel operation of power stations and the power system violation.

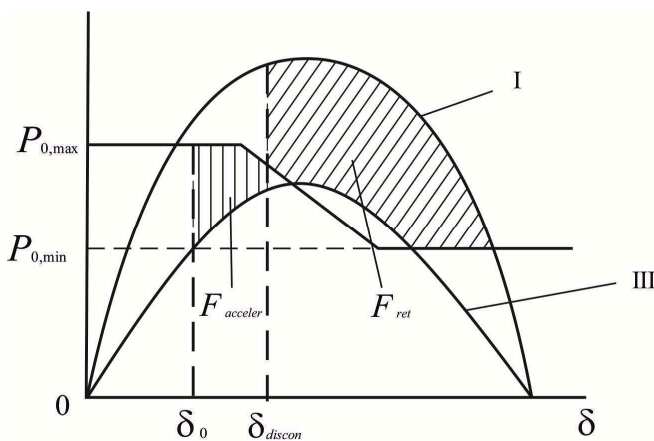


Fig. 14.5. Change of acceleration and deceleration areas at turbine output power control

The frequency reduction to unsafe limits can be prevented through introduction of spinning reserve or automatic frequency unloading when some part of load is switched off automatically. As introduction of spinning reserve can not in many cases prevent development the emergency situation due to the power units inertia, the automatic switching off the corresponding part of load is the reliable solution. Therefore nowadays great attention is being paid to rigorous technical and economical substantiation for development and selection of special load release automation. The automation use permits to increase operation stability of a

power supply system as a whole, and its load nodes stability preventing avalanche of voltage or chaotic consumers self-turning off.

**Emergency unloading of prime mover turbines**

Action of emergency turbine unloading devices lies in the fact that after some time from the short circuit occurrence, a signal for closing the gate valve for energy carrier (water or steam) introduction into the prime mover turbine is produced. Here the turbine mechanical power output

reduces from  $P_{0max}$  to  $P_{0min}$  (Fig.14.5). The acceleration area  $F_y$  reduces and probable deceleration area  $F_m$  increases resulting in rise of the system dynamic stability reserve.

After emergency elimination the prime movers reach their previous output automatically or under staff supervision.

**14.4. Use of protection devices and system automatic equipment**

The measures for change of a power supply system operation with the use of protection devices and system automation rather efficiently influence the system stability. The choice of measures depends on specific reasons causing the disturbances of industrial production processes at short-time power supply interruptions.

The major measures are as follows.

**Reduction of short-circuit duration**

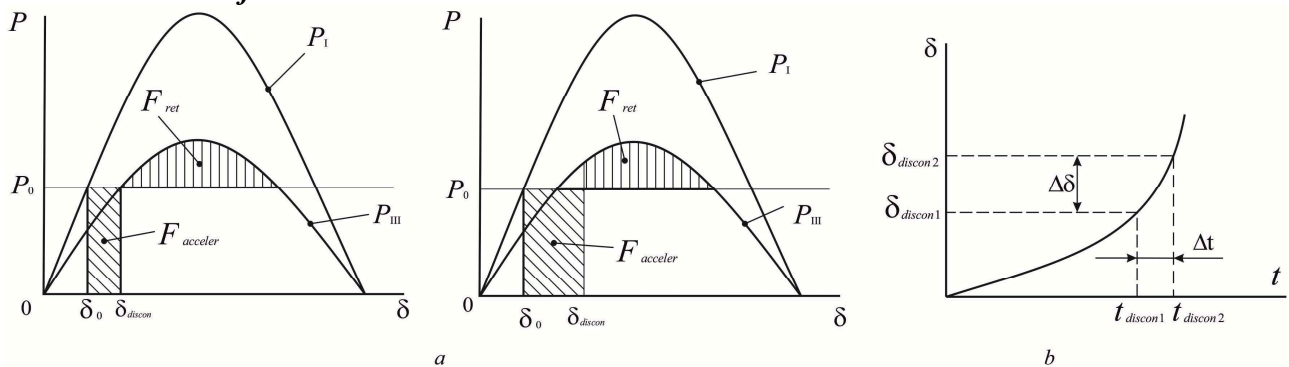


Fig. 14.6. Influence of three-phase fault duration on dynamic operation stability

Fast short circuit clearing brings to reduction of the acceleration area  $F_{acceler}$  and increase in the probable deceleration area  $F_{deceler}$  (Fig.14.6, a). As generator rotor starts rapidly accelerate in the case of short circuit occurring (Fig.14.6, b), so even small reduction of short circuit clearing time  $\Delta t$  results in significant reduction of switching off angle  $\Delta\delta$ . Fig.14.7 presents dependence of the dynamic stability

assurance factor  $K_{daf}$  on short circuit duration. The dependence also proves efficiency of dynamic stability assurance increase with reduction of short circuit clearing time.

Fastening of protection operation can be gained by refusal of lines over-current protection as it has noticeable time delay. In this case attention should be paid to differential protection, current cutoffs with reduced current settings and corresponding blockings etc.

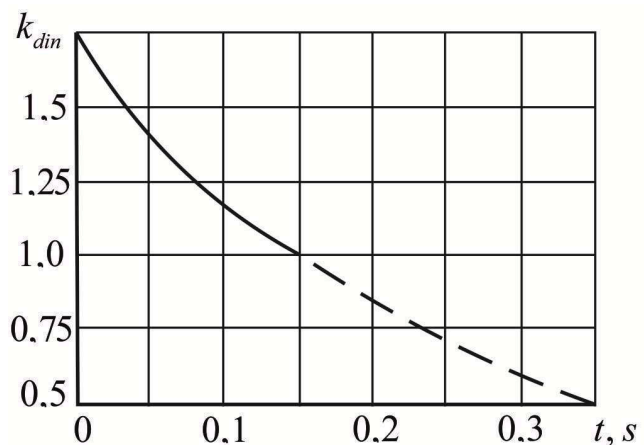


Fig. 14.7. Dependence of power supply system dynamic stability assurance factor on short circuit duration

Feed interruption in the case of typical automatic load transfer application impedes synchronous motors self-starting considerably. To eliminate the possibility of motors turning on with non-killed magnetic field that could be the cause of their damage at connection in anti-phase, the automatic load transfer response time is intentionally reduced. The time of magnetic field killing of powerful synchronous and induction motors fed from sections without static load is particularly large. Nevertheless automatic load transfer acceleration is one of the most important and effective

means of increasing dynamic and resultant stability of motors operation. Different ways are possible for automatic load transfer acceleration:

- improvement of automatic load transfer starting techniques owing to elimination of over-delay in its operation;
- acceleration of automatic load transfer through increase of its starting element voltage;
- use of leading automatic load transfer responding before damaged element switching off that leads to reduction of short-time power supply interruption up to the time that doesn't exceed short circuit duration;
- choice of the synchronous automatic load transfer device starting instant by means of automatic ensuring knowingly allowable current of non-synchronous closing and the best conditions for immediate resynchronization.

**Automatic reclosing**

The predominant part of overhead lines emergency switching off is caused by volatile faults being intermittent after short time voltage killing by line tripping. Overhead line reclosing by means of automatic devices after volatile fault (lightning surge, putting foreign objects over wires, insulation flashover, etc) recovers its regular operation.

Autoreclosing allows to eliminate the emergency fast and to recover regular operation of power supply system not only after intermittent faults but also at false operation of protection devices, spontaneous breakers switching off or faulty actions of an operating staff.

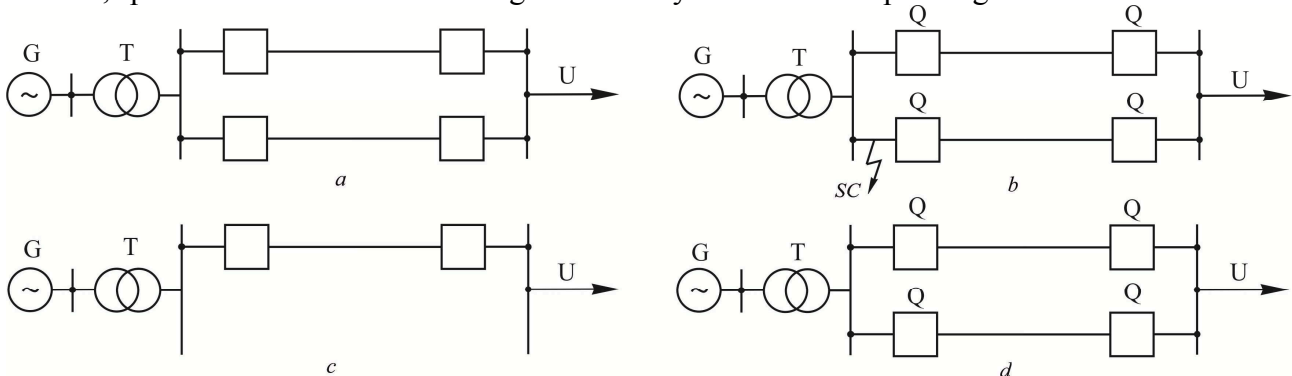


Fig. 14.9. Circuits of power supply system at different operation conditions: a – regular condition; b – emergency operation; c – post-emergency operation; d – operation after successful autoreclosing

Successful autoreclosing increases probable deceleration area  $F_{deceler}$  (Fig.14.8) facilitating keeping dynamic stability of a power system.

Fig.14.9 shows circuits corresponding to different power-angle characteristics given in Fig.14.8.

Regular operation with the power-angle characteristic fitting the curve I is characterized by parameters

$$P_1 = E_q U / x_I, \quad x_I = x_q + x_T + x_{OL} / 2$$

For the emergency condition (curve II):

$$P_{II} = E_q U / x_{II} ; \quad x_{II} = x_q + x_T + x_{OL} / 2 + (x_q + x_T)(x_{OL} / 2) / x_{EmSh}$$

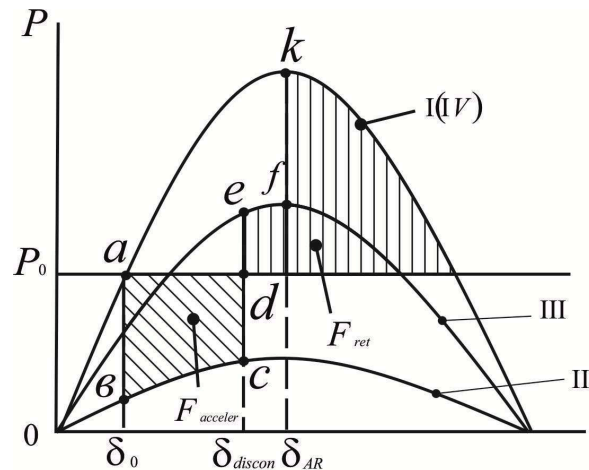


Fig. 14.8. Influence of autoreclosing to dynamic stability of power supply system operation

For the post-emergency condition (curve *III*):

$$P_{III} = E_q U / x_{III} ; \quad x_{III} = x_q + x_T + x_{OL}$$

For the condition corresponding to successful autoreclosing the curve *IV* coincides with the curve *I*. This operation is characterized by the parameters:

$$P_{IV} = E_q U / x_{IV} ; \quad x_{IV} \equiv x_I$$

Practice of auto-reclosing devices use shows that autoreclosing is one of the efficient ways of increasing operation stability of a power supply system. Successful auto-reclosing on separate feeding overhead line allows to eliminate interruption of power supply fast preventing disturbances of production process.

Self-clearing faults (apart from overhead lines) often occur in cable lines, on bus-bars of power stations and substations, in transformers and electrical apparatuses. In view of it auto-reclosing devices for transformers, cable lines and other elements of power supply system have got common in use.

For many enterprises where self-starting of induction motors and resynchronization of synchronous ones after short-time power supply interruptions are not possible, automatic motor restarting is efficient and is practically the main way of stability increase.

For a motor switched off at short-time power supply interruptions, automatic restarting may be performed in the post-emergency operation when the voltage is recovered to the value close to rated one by means of simultaneous or successive motor groups starting. In the process of automatic restarting, the voltage and, if is necessary, technological parameters are checked with the use of automation slightly different from typical circuits of auto-reclosing devices.

#### ***Voltage adjustment in nodal points of power supply system***

The voltage in nodal points of electric loads connection and on consumer terminals significantly influence insulation service life, electric power losses, stability margin of load nodes and a power supply system as a whole. Therefore the voltage in a power supply system is tended to be retained in allowable limits in the following economic ways:

- simultaneous voltage control on the feed sources buses;
- change of transformers and autotransformers ratios;
- compensators power control.

Combination of centralized and local control and control by digital techniques is the most efficient in the view of increase in operation stability of a power supply system in the process of voltage adjustment. Use of joint voltage and reactive power flow adjustment is also practical.

#### ***Increase of dynamic and resultant stability of synchronous motor operation***

This aim can be reached by:

- rapid unloading of working mechanism for the motor rotation resumption;
- automatic excitation control of synchronous motors;
- killing the magnetic field of motors that have fallen out of step to make their resynchronization easier.

#### ***Disconnection of unimportant load at the short-time disturbances of power supply***

Amount the disconnected load is determined by the calculations of transients at the most hard disturbances when conditions of motors self-starting are to be provided with the respect of keeping the most important production processes and interconnections between them.

For increasing dynamic stability of synchronous motors operation one should tend to increase speed of switching off operations. Total time from short circuit beginning to switching off breakers should not exceed 0,25 s. For asynchronous load requirements to the operation speed are less strict as self-starting of induction motors can be successful at slip exceeding significantly the value of  $s_{cr}$  at which motors consume the current close to the starting one.



**Test questions**

1. Major ways and measures of power supply systems operation stability provision.
2. What measures for operation stability increase are provided in the stage of power supply system design?
3. How does availability of active and reactive power reserve effects the operation stability increase?
4. How operation stability increase is reached by means of electric system elements parameters change?
5. Ways of operation stability increase by change of power supply system circuits and their operation conditions.
6. How does automatic generator excitation control influence on operation stability?
7. How is stability increase reached by means of the automatic frequency unloading?
8. How are protection and automation devices used to increase operation stability of power supply systems?
9. How is automatic load transfer accelerated for a load node stability increase?
10. Auto-reclosing efficiency for increase of a power supply system operation stability.
11. Does voltage control influence on a power supply system operation stability increase?
12. Ways of electric motors dynamic and resultant stability increase.

**Topics for essay**

1. Methods of load node stability increase in power supply system design.
2. Increase of power supply system operation stability by use of their elements constructional features and operational conditions.
3. Use of automation and computers for increase of power supply systems stability in the process of their operation.

## **CHAPTER 15: MONITORING AND DIAGNOSTICS OF EMERGENCY OPERATION MODES**

- 15.1. Initial statements
- 15.2. Development of software for circuit diagrams and electric power supply system operation monitoring
- 15.3. Program complex for distributive networks control
- 15.4. Monitoring of transients
- 15.5. Development of methods of short circuit points identification in high-voltage power transmission lines

### 15.1. Initial Statements

Electric power supply systems of towns, industrial enterprises and agriculture have the most extensive and extending power networks. Optimization of their operation by criteria of functioning, reliability and quality of electric power supply needs use of high-level control systems based on modern complex systems of automated supervision, electricity metering, its quality monitoring, technical service automation, and power system operation monitoring. To solve these problems it is necessary to implement the up-to-date information technology using suitable hardware and software.

The peculiarity of electric power supply system of an enterprise is its unified and integral structure. It unites electric power supply systems of mainline production and auxiliary processes, and includes generating components, emergency power sources, power transmission lines, switchgears, and loads. An automated power supply control system (APSCS) must be integral system to perform centralized control of all objects of electric power supply by means of automated workstation (AWS).

Tasks of APSCS for each energy department should be determined depending on production and economic expediency taking into account rational use and the technique functional capabilities. In APSCS should be incorporated as traditional AWS for operational dispatcher control, commercial and technical power measurement, and equipment information guide, as modern AWS for energy management and for power supply system operation control. Due to increase in electric power tariffs, the AWS of power manager becomes more and more important. Power manager planes and supervises over key economic indices such as the specific rates of power consumption per unit of production and production run optimization, monitoring and electric power supply system operation control. Automated workstation adopted for monitoring and electric power supply system operation control makes possible modification of the protection setting according to load change as well as to make effective detailed operational study of emergency situations and power failures.

To perform stated functions efficiency APSCS hardware platform must satisfy specific character of the electric power supply task put by. International Electrical-engineering Commission (IEC) carries out activity on standardization the decisions and development normative documents which regulate structure of automated control systems for electric power supply objects. The key demands of IEC concern:

- Standardization of APSCS sensing device;
- Functionality, response time, synchronization of controllers used in APSCS;
- Communication protocols of intelligent devices;
- Special algorithms of power equipment control.

As it is known, processes within power networks are characterized by high rate. Therefore, the equipment used in APSCS should meet specific requirements as for response time. Equipment for data acquisition must contain built-in clock, and have suitable speed to fix discrete events, assign time marker to the changed parameter on built-in clock reading, and have a buffer to store messages formed in such a way. Accuracy of time parameters specifying in modern protection devices is 10ms. Therefore, effective control of these means needs proper operational speed to capture data. Real accuracy of discrete state clamping on, taking into consideration transients filtering, should not exceed units of milliseconds that is commensurable with half-cycle of industrial frequency voltage. Common optimized software and hardware environment with operational engine "on events", availability of priorities of messages and commands help to process much information arriving irregularly in reliable way.

To refer parameters of all objects in the system to universal time the engine of timing based, as a rule, on GPS standard is used. APSCS architecture provides for the system server functioning as a source of universal time for all components of the system as well as mutual synchronization of field equipment clocks with accuracy up to milliseconds. The field equipment structure choice for implementation the project of electric power supply automation depends on the system functional as

well as on the requirements to electric power supply reliability and operational change of the system protection and automatics settings.

Specific controllers, which microcode software includes algorithms of data processing meeting the principles of electric power supply monitoring and control, are used for automation of electric substations. The controllers compare real time markers to measured values providing realization principle of selection before operation (SBO), taken as the basis of switching units remote control. Buffer of events excludes data loss in the case of avalanche growth of events on emergency.

To meet the advanced reliability requirements to power supply of technological equipment, reconstruction of relay protection and system automatic equipment (RPA) with microprocessor-based protection facilities (MPF) installation in switch-gears of high-voltage substations. MPF within APSCS perform remote monitoring and control of protection settings, oscillography of emergency situation development, and gathering information as well. Besides, MPF provides the commutation facilities remote control.

Automated process control systems (APCS) should provide the end user evident and informative representation of the stored information for analysis and taking decisions concerning energy-saving technology improvement.

It is extremely important for electric power supply systems to improve power quality, diminish the risks of electric power disconnection, and achieve better operating characteristics of the applied electrical equipment. It is stipulated by use a large number of power-intensive production processes at industrial regions, and high technological equipment down-time cost and the operative staff professional skills lowering. Major accidents of power systems in Georgia, the USA, Canada, Great Britain, France, and other countries bring out importance of this assertion clearly.

Solving this global problem is reduced to a number of local problems: PSS operation monitoring, search for ways of reserve putting into operation and electrical equipment removal from service for repair, fulfillment operative dispatch control at unforeseen contingencies and search for optimum topology of electric power supply system.

Speed of response at unforeseen contingencies is an important factor. When a power department dispatcher performs necessary operations according to instructions, the desirable efficiency is not always achieved. Disadvantage of this method is impossibility to consider in real time virtually all situations that could take place. For that reason, only some typical cases are considered. It also should be noted that PSS is not built up at once but is formed gradually as new consumers appear.

Real solution for such a problem is development of means for operational monitoring and control intelligence support in the course of interaction between the operator and the network. Taking decisions under service needs solving problems with account the production specific character. The last is better performed by an operator, and formal tasks are solved more efficiently with computers. Besides, only operational staff, but not computers, is responsible for the results.

Taking all the factors into consideration leads to substantial labor intensiveness growth, as examination of great number competitive versions in limited time should be done. Labor intensity is stipulated by combinatorial nature of the problem. The use of computer facilities for its solving insignificantly widens possible number of checked versions.

Owing to above reasons the requirement of the solution optimality is not obligatory, but its solution should be admissible and found in limited time. The method should allow finding the optimal solution when it is necessary. Genetic algorithm meets the requirements. It can manipulate many parameters simultaneously. The algorithm has a number of advantages: information on the response surface is not required, the method is robust at falling to a local optimum, is simple and transparent in use and reliable at solving large-scale optimization problems. The method is applicable for solving problems with varying environment.

Below practical realizations of different versions of the system of intelligent support are considered.

**15.2. Development of software for circuit diagrams and electric power supply system operation monitoring**

Program Power System Suite ([www.pss.pstu.ru](http://www.pss.pstu.ru)) program is functionally divided into several purposeful components which interaction is shown in Fig. 15.1.

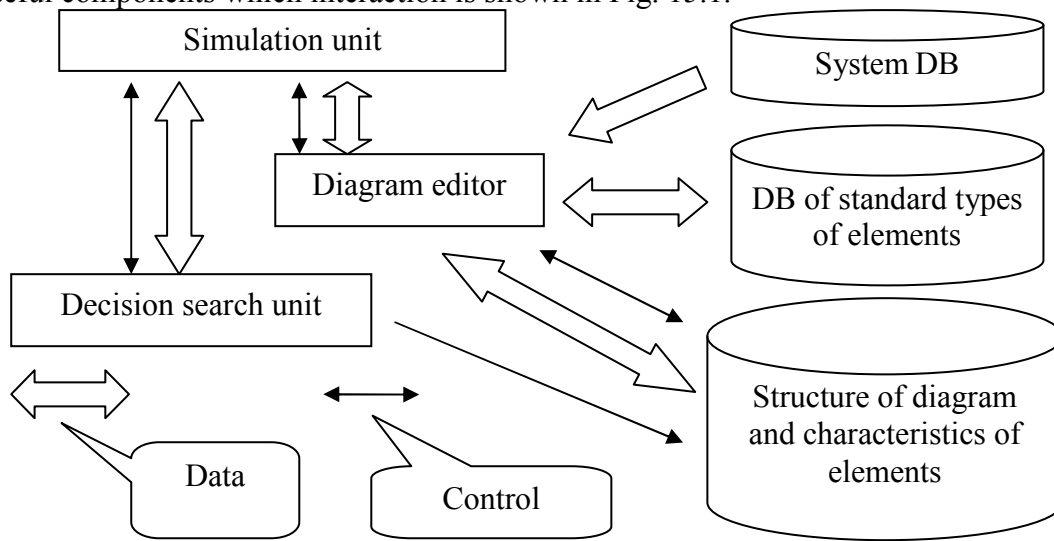


Fig. 15.1. Structure of the system and its components interaction

User's interface of the developed program is realized with the help of MS Windows standard elements. Now MS Windows system is one of the most popular environments for PC. Some components of the system are not the best ones in their category. But their close integration and prevalence make Windows the most attractive platform to develop applications.

For storage electric power supply circuit diagrams and standard electric equipment parameters the Database Management System (DBMS) InterBase is used.

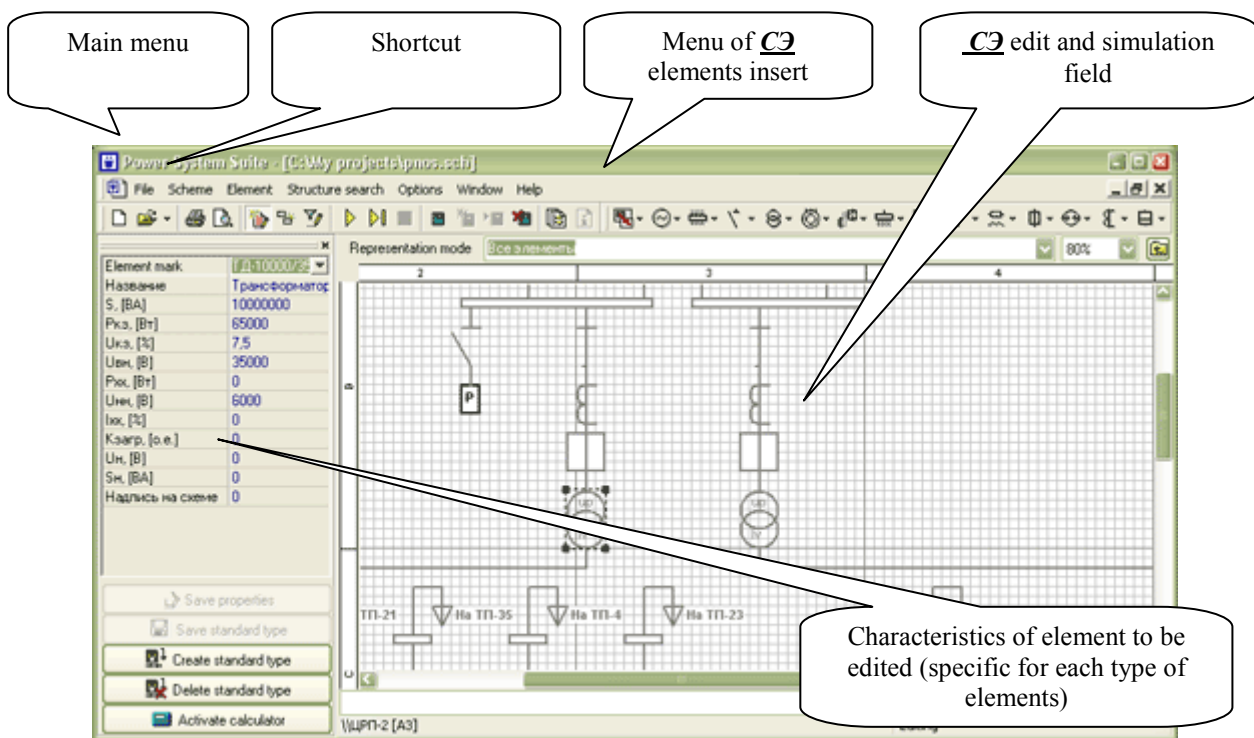


Fig. 15.2. Power system site program interface

For storage electric power supply circuit diagrams and standard electric equipment parameters the Database Management System (DBMS) InterBase is used. One of the key attributes of InterBase is its version architecture which gives exceptional facilities for the case of many users availability, because writing users never block reading ones, and it is very important if several users work with a common file of the enterprise PSS.

Within the program complex the stage of simulation and optimization follows the stage of electric circuit compilation. With the help of element base a user makes up the circuit diagram putting electric elements in the patch-board. The element base contains electric equipment being mostly spread in PSS circuits which is grouped on the principle of purpose. The circuit diagram is formed of separate components, and their characteristics are assigned. After that simulation or optimization process may be run, and values of estimated performances of PSS components are looked through.

Majority of functions controlled by the program are accessible by means of control panel and active window (Fig. 15.2).

The use of the result obtained helps power services to:

- speed up making the decisions in unforeseen contingencies and improve the decisions quality;
- make search of nominal operations of the enterprise electric complex easy;
- simulate electric power systems of arbitrary structure;
- reveal non-uniformity of load distribution between bus sections within some step-down substations of PSS;
- find solutions which help to equalize voltage levels on buses without closing all intersectional switches (it gives ability to minimize loss in the case of emergency);
- improve efficiency of decision making, and lessen the number of emergencies, and minimize damage caused by them at the expense of response time cut down;
- make optimal selection of electric equipment;
- schedule preventive maintenance;
- to compute protection settings.

The used algorithms are multi-purpose. They are not tied to specific models of electrical equipment and national specific of electric power system development. That's why they may be applied for any PSS.

### 15.3. Program complex for distributive networks control

In electric power systems supplying large consumers and power supply areas, remote control is used to gather information from objects concerning operation parameters, electric power consumption, switching units and relay protection operation etc. The information is primarily processed, and shown by individual or cooperative visualization means, and archived with the help of operative information complexes.

The use of "client-server" ideology and standard databases with free access in operative information complexes makes possible to develop unified program complex to manage distributive networks (DMS) which helps to solve problems concerning initial data processing, their storage and display. Besides, more difficult problems are solved separately. Development of a unified complex for distributive networks management creates possibility for the network management efficiency improvement due to the following factors:

- reduction of costs for software and maintenance of problems on distributive network management;
- electric power supply reliability improvement and reduction of time needed for emergencies removal;
- cutting off electric power losses in the networks by means of operation measures;
- monitoring and power quality improvement;
- electric power consumption planning within 24 hours at 30-minutes intervals;
- electric power supply system equipment certification;

- operative staff more competent training with the help of simulators using the database information;
- postponement of intended investments as the result of the network and the available equipment efficient use.

Consider the above factors in more details.

***Reduction of costs for software and for support the problems on distributive network management***

DMS system is based on general relational database in which information is organized and stored by definite rules. The first rule is not to have duplicated data. The database is managed by the administrator, but information may be put in definite sections by a process engineer. Initially, equipment and its topological connections are written. For an example if transformer is replaced within network, the new transformer data are entered to the database, and the new parameters are now used to estimate operations, short circuit currents etc. That is, the database structure is so organized that information is corrected in one point, and then it is used for many tasks. Such an approach helps to reduce the tasks maintenance expenses greatly.

For a number of calculation tasks the same functions are used. Such function could be represented as unified program modules, such as, a module of circuit design based on location of switching units. In different operational tasks the same interface representing data of circuit diagrams can be used, and data developed in the same graphics editor are applicable to these tasks.

***Improvement of electric power supply reliability and reduction the time needed for emergency conditions elimination***

For operative reliability maintenance of electric power supply, study of the network topology and design circuit formation depending on the condition of switching units, and of the power equipment parameters may be performed. Signals trustworthiness with respect to their accordance with telemetry and inhibitions for simultaneous switching on several units are checked. As the result of the network topology study, the list of turned off consumers and power units is formed. Closed paths and mainline sub-circuits are determined.

Another task, based on a design circuit assembling is to find doubtful measurements or energy meter readings considering their accordance to Kirchhoff's and Ohm's laws. Energy throughputs in connections, obtained with the help of energy meters, are compared with data obtained by integration of power telemetry during the chosen time interval (usually during half an hour). Using the refined data about energy throughputs, currents and power in the main sub-circuits, the feeder conditions can be computed automatically with definite periodicity to determine the power overflows, the voltage levels and the energy losses. The results are compared with the allowed values. If the voltage levels in some nodes are either lower or higher than the allowed ones, appropriate messages is put out. Regular control of telemetry and energy meters helps to cut commercial losses of electricity because of faster replacement of defective energy meters. Besides, thanks to data refinement in respect of operation parameters and obtaining additional parameters which are not measured, the probability of making incorrect decisions by the staff reduces. In some cases there is no necessity of installation instruments at the remote consumers, and of laying communication links.

It is possible to cut electricity losses greatly if operative staff disconnects fault sub-circuit and transfers the disconnected consumers to other power sources fast. For that, the study of the network topology and the design circuit diagram in the mode of simulation available for operative staff. They can see different variants of the disconnected loads energizing on display and compute their operation modes. With it voltage levels in the circuit nodes and admissible loading of lines may be checked.

Use of operative information complexes databases for storing characteristics of the supply systems power equipment helps to register operation of switches by the coming telemetry signals. It is also possible to divide disconnections in normal working currents and in faults if relay protection signals are put in the database. In the DMS such a registration of switch units operation and other parameters (overload on transformer current etc.) gives ability to form automatically lists of

equipment for maintenance and repair. That also improves reliability of electric power network performance.

***Cutting off electric power losses in the networks by means of operation measures***

Calculation the power losses by means of DMS may be performed automatically at regular intervals depending on operation parameter data availability. It is done in such a way. Using data of telemetry about switches state and available information concerning power equipment parameters and topology of equipment connection, equivalent circuit of the network is created. Loads in the nodes are computed with use of telemetry of power or currents in main sections of power lines. Load distribution between transformer substations is performed according to correlation factors which reflect interdependences of separate transformer substations power and power of the main power line sub-circuit. Then network operation is computed using available voltage readings in power centers, and node loads. The active power losses are determined by computation. When the data are obtained periodically the determined values of active power losses are multiplied by time interval for receiving the electric energy losses over the period. The energy losses may be too found using data of energy throughputs in the main sub-circuits of power lines for fixed periods of time, for example, for half an hour. In this case the form factors are entered, and computations are performed only for open networks. Information concerning losses is filed in database. The data are grouped by power network components, so when losses are studied the information may read out for voltage classes, lines, transformers, load losses, and no-load losses. Electric energy losses may be studied on separate connections. If electric power consumption by transformer substations connected to a power line is accounted it is possible having calculated technical losses to reveal commercial components of losses within network, that is, electric energy losses related to faulty measuring devices or electric energy theft. Such a study is very important and may be very efficient as it gives a course of search malfunction of metering instruments and electric energy theft.

Computing electric energy losses at a rate of information concerning operation arrival, the following measures for their decrease could be taken:

- optimization of voltage law control within power centers of open electric networks of 35-110 kV;
- optimization of steady operations of closed electric networks by reactive power and transformers turn ratio;
- optimization the areas of circuit breaking the ring networks with different voltage classes;
- optimization of areas of breaking the 6-35 kV networks with two-side power supply;
- disconnection of a part of transformers at small loads;
- equalization of a network load curve;
- optimal use of available reactive power compensation devices.

*When laws of voltage control are optimized* minimization of electric power losses is performed at main substation of open electric network on request of a power service dispatcher or in the case of the network design change. With it, such a law of voltage control at main substations is chosen to provide minimum electric power losses in the network, and acceptable voltage level at consumers. Control at main substation is performed by means of transformers on-load tap changers, or if it is possible, with the help of generators. Voltage regulation with the help of transformer taps change without excitation may be considered too.

***Optimization of steady operations of closed electric networks by reactive power and transformers ratio***

Closed network operations are optimized on minimum losses within network with the help of transformer tap change, and, if reactive power sources are available, by change of reactive power generation.

***Optimization the places of circuit paths with different voltages opening***

Minimum losses of active power in branches correspond to network operation in which the power distribution is proportional to their active resistance. It is known, that if closed paths with branches of different voltage classes are available the losses may be significantly reduced by opening the path on low voltage side. Of course the problem of such an opening permissibility



should be solved from the point of view of electric power supply reliability and the specified voltage provision. Optimum points of opening may be different at operation with maximum and minimum loads. If there is no ability to change opening points on-line during 24 hours, the point which guarantees minimum electric energy losses within 24 hours is selected. Optimization of the opening places should be performed in the period of season load variations or if the network changes its configuration. Mainly, the measure belongs to the network level, but it can be too applied to large enterprises having complex high voltage networks in which closed paths at voltage of 110.35 or 10kV can be formed and they are energized by several high voltage network substations.

#### ***Optimization the 6-35kV voltage lines with two-side power supply opening points***

Change of the 6-35kV line with two-side power supply opening point results in redistribution of loads between feeding substations and changes of losses within external circuit. Therefore, optimum opening point is determined considering total change of the two components: within the 6-35kV voltage lines and the external circuit.

#### ***Disconnection a part of some transformers at small loads***

Disconnection of one of the transformers operating in parallel is expedient measure when reduction of no-load losses is greater to compare with increase in load losses in transformers. Such expediency should be confirmed by means of options with minimum and maximum load computation.

*Equalization of network load curve* provides more uniform load distribution within 24 hours, and, hence, decreases the form factor and power losses. As load losses in line are proportional to the current square, the uniform load distribution during 24 hours may significantly reduce the losses in circuit components.

#### ***Optimal use of available reactive power compensation devices***

To solve the problem, it is necessary to determine the laws of compensating devices performance during 24 hours making the losses in electric lines negligible. In this case the reactive power consumption or generation volumes agreed with the power system is observed.

***Monitoring and power quality improvement in a power supply network*** Power quality parameters influence significantly on the operating conditions both the electric network and the technological equipment. Improper power quality produces the following results:

- increase of the electric power losses in lines and electric power equipment;
- increase of electric power losses in the equipment results in its extra heating, and reduction the equipment service life or makes necessary to increase its rated power;
- lowering the equipment productivity and production quality.

The first two components belong to electromagnetic damage, and the latter – to technological one. The electric power parameters as well as other factors influence on the technical and economic indices of lines and electric equipment operation, and the only way to determine the losses caused by low power quality is their calculation. For example, it is rather difficult to find how much electric power would be consumed by a specific enterprise to produce the same output at the condition of sinusoidal voltage and symmetry distortion absence and under higher level of operating voltage.

Visible consequences of electric power poor quality – the equipment failure and products reject - is always considered as the result of low used equipment quality. Thus, frequent failure of incandescent lamps is considered as caused by their producers, but in many cases it is caused by higher voltage in line.

#### ***Electric power consumption planning within 24 hours at 30-minutes intervals***

Coming of a large consumer of electric power to the wholesale market of electric power is possible if stringent requirements as for PSS operative control are met. The consumer in this case is:

- to have centralized control station;
- to have information concerning the current power flow balance between the power system and the enterprise;

- to have information concerning the power flow balance between the power system and the enterprise with 30-minutes interval;
- to plan the power flow balance and to apply for 24-hours schedule of power flow balance;
- to maintain 24-hours diagram of power flow balance and observe the current mode;
- to have direct communication channels between PSS and the power system dispatcher station.

The requirement to carry on the load diagram of enterprise is one of the most important in the list. Appropriateness of the order for consumed power within 24 hours may be very profitable. At present time, the dispatcher diagram violation from the point of power system view is deviation of 2.0% up or downward. Nonobservance of the diagram causes a fine imposing.

#### ***Certification of electric power supply system equipment***

Architecture of DMS relational database guarantees ability to expand volumes and structure of information without any radical rearrangement of database and access to data flexibility by means of use of common access to databases standards (ODBC, SQL). That's why new divisions of database which will contain description of power equipment, devices of relay protections, measuring systems, data on power transmission line supports, cable lines etc. may be added rather simply. More complicated problem is to enter the information about all equipment and to agree information with different services concerning the data which have to be stored within the database.

PSS efficiency improvement at the certification is achieved as a result of the following:

- keeping information about equipment in one database without its duplication;
- automation of the needed equipment search and making reports on the running equipment;
- fast look of equipment manuals;
- formation the equipment lists for scheduled repairs and maintenance.

#### ***Qualitative training of the operative staff with the help of simulators using the database information***

Having in DMS the database parameters of electrical equipment, data of operation parameters, structure of substations, applications on operation, electric energy losses, and short circuit current computations, on analysis of line topology, and load prognosis, one can organize effective training of operative-dispatcher staff and process engineers in operations calculation, and training of engineers in the field of relaying.

For operative-dispatch staff, it is planned to have in the DMS complex configuration a simulator of operations switching and operation simulating trainer for which data are obtained from the DMS database. Training of the process engineers in computation the operations and of engineers of relay protection service may be performed with the help of operation simulator. Simulator "Modus" may be used as simulating trainer in operational switching and as the basis for operational trainer.

Thus, the subsystem of staff training on the basis of DMS has the following possibilities:

- use the real telemetry and telemetric control signals in training simulators of operational switching;
- use the unified DMS base in simulators;
- use the unified DMS database of electrical equipment;
- use DMS software for staff training.

This line of activity is very important as electrical-engineering staff excellent training provides efficient functioning of electric power supply systems.

#### ***Postponement of intended investments as the result of the network and available equipment efficient use***

More precise study of PSS operations within long periods of time, optimization of the line design, voltage levels, and reactive power generation make possible to refuse in some cases of

- extra cable or overhead lines laying;
- replacement or new transformer substations and power equipment installation;
- new means of reactive power compensation and voltage control installation.

This component may be decisive in saving the total costs paid for PSS operation.

### 15.4. Monitoring of transients

The system of transients monitoring is the technology of transient parameters recording aimed to analysis of dynamic properties of a power grid using:

- technique of information recording and transfer;
- means of information processing and analysis;
- especially trained staff.

The transient is a quickly flowing process of transition from one steady operation to another caused by disturbance of power system (disconnection of generators, lines, short circuit etc.). Under transition the oscillations of voltage, currents, frequency, and other parameters take place. Character of oscillations may be determined if they are measured with discreteness of no more than 20ms. Telemetry system cannot guarantee discreteness necessary for power grid.

Duration of the transients is several seconds, but it is the very factor which determines further system behavior: if it transfers to another steady operation or emergency takes place.

Character of transients is determined not only by disturbance but by the system dynamic properties too. These properties study is performed by means of digital models which should include extremely accurate descriptions of the following objects:

- generator field regulators;
- velocity of turbine rotation regulators;
- dynamic load models ;
- models of relay protection and system automation devices.

Any model and especially model used for analysis of dynamic properties of power system need to be verified, that is, in comparison of transient authentic parameters measured in different nodes of the system with parameters obtained by means of computation at the same disturbance.

Study of electric power system dynamic properties is achievable if parameters in its different geographical points are measured simultaneously. The problem is solved with the help of PPS (pulse per second) signals sent by space satellites. Systems transients of monitoring are being successfully developed in all large power systems of the world and are called Wide Area Measuring Systems (WAMS).

Even two recorders give information which helps to come to a number of valuable conclusions. But to correct the system of anti-emergency automatics it is necessary to have a network of recorders covering power system as a whole. As American experts believe, the number of objects where it is necessary to record parameters of transients should be about 10% of all nodes of a network of 330-500-750kV.

WAMS introduction into power systems of the USA, Western Europe and China made possible to reveal several trends of their practical application:

1. Verification of a power grid digital model and its elements:

- models of automatic control devices;
- load models for refinement their static and dynamic characteristics;
- power grid as a whole digital model.

2. Voltage monitoring in grid nodes:

- monitoring of load stability and voltage avalanche prevention;
- formation of control actions on regulated sources of reactive power;
- visualization of the voltage levels in the system.

3. Study of the occurred emergencies: development of methodology for the system and regional emergencies study; development of standard procedures of emergencies analysis.

4. Obtaining qualitative approximation to compute operation mode in real time. Use of the voltage phasors (module and angle) at line nodes where registration equipment is installed allows to develop "pattern" of mathematic model, to simplify greatly the convergence problem and to save time of operation computation. It is extremely important for power grids because of great length of power transmission lines.

5. Identification and analysis of low-frequency oscillations. Contemporized registration of frequency variation allows observation of low-frequency oscillations when they occur during 15

minutes. Analysis of amplitudes of the oscillations and their propagation character permits to determine the source of oscillations and to develop measures for their suppression.

6. Monitoring of the line node voltage phase angles. There is possibility to refine limits of the transmit power over separate lines and cross sections by means of comparison the calculated and online measured voltage angles. Limit refinement downward allows decrease power of disconnected consumers at power shortage occurrence in the system. Besides, it is possible to identify promptly asynchronous operation of a power system.

### **15.5. Development of methods of short circuit points identification in high-voltage power transmission lines**

Tendency of industrial production concentration in immediate proximity to raw material and natural resources location is demonstrated in constant development of electric power systems and great growth of radial distributive electric power lines, and, as a result, increase of their general length. This fact determines relative complexity of quick operative fault search and a power line trouble-shooting, and further process restoration. At the majority of power objects, place of fault in high-voltage overhead power lines is determined on the basis of different fixing measuring instruments reading.

More favorable situation from the point of view of responsiveness and fault localization takes place at power objects which have subsystems of digital oscillography. In this case, the use of numerical methods and correct mathematical tools for solving the problem of a fault location helps to obtain acceptable results (as a rule, error does not exceed 2 ... 5%). But the error of these results contains rather high methodical component, which is mainly stipulated by inadequate current distribution account within electric circuit under SC (including SC through contact resistance).

Thereat for improvement responsiveness under emergency liquidation (error of the fault location diminishing) essentially new algorithm of fault area location and relevant software are developed. They are based on use of the results of two-sided monitoring of emergency operation parameters. To organize automated procedure of high-voltage power transmission line SC location on the basis of two-sided methods it is necessary to introduce into subsystem of digital oscillography power object devices which perform the lock-on all the registration means to unified astronomical time. The software and technical means providing this automated procedure of fault area location is integral part of the subsystem of information exchange between automated dispatcher workstations of a separate power object, the regional dispatch control and power system services by means of communication channels.

The algorithm of "two-sided" fault area location is developed to find out faulty branch, to determine the type of short circuit, and distance to fault point taking power transmission lines with the voltage of 35kV and higher.

According to functioning principles, the algorithm of fault area location procedure may be divided into the following automatically performed stages:

1. Data reading parameters of the electric power line covered by digital registration devices on its ends;
2. Transfer of all the necessary information by means of available communication channels and its concentration in common dispatcher center;
3. Search of common design interval of oscillogramms; with it the emergency start, finish of different types of transients, and start and end points of steady emergency operation are determined;
4. Filtering and determination of operation parameter phasors;
5. Faulty branch (PTL) determination; for each PTL the symmetrical components of phase currents are computed;
6. Determination of the short circuit (SC) type and the faulty phases; faulty phases determination (taking into account possible changes in transposition on the line route) is performed with the help of phase relations between currents of zero, positive, and negative sequences checking, and by means of comparative analysis of the phase currents peak values;

7. SC location on the basis of the registered data of emergency operation parameters on the ends of the PTL; according to determined fault type, tracing of the fault of the PTL is performed;
8. SC location on the basis of one-sided test.

The procedure of SC location by the results of the emergency operation parameters monitoring from both ends of PTL is realized as interactive software. The software helps to perform the following functions:

- automatic recognition of emergency situations taking into account possible change in a phase wires position (transposition) within PTL route;
- semi-automatic/automatic computation (by the fact of checkback appearance of the monitoring results);
- precise determination of distance up to SC on the basis of decrease methodical and instrumental components of error influence (for example, to 0,95% the PTL total length);
- improvement trouble-shooting responsiveness by well-organized fault determination.

The next stage of the process of the fault location further development may be based on implementation of methods with support of adaptive mathematic description (simulation) of PTL. To solve the problems it is necessary to enter normative and maintenance base, that is to perform entire certification of overhead lines on the results of control tests of their parameters on the ends to determine (refine) authentic parameters of PTL. Then, the obtained ratings (parameters) may be used within adaptive model of an overhead line to avoid instrumental error caused by voltage and current metering transformers.

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